A Discourse On Applications Of Lie Groups And Lie Algebras

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Abstract: In this research article, Lie Groups and Lie Algebras are projected in a distinct direction and with innovative proofs. We develop the necessary and sufficient condition for a topological group to be Hausdorff. The criteria of topological group and Haussdorff to be connected is also derived. This research article mainly explores the concept of left in variant field and presents an important hypothesis namely "any Lie group is parallelizable" with a very simple accurate logical and analytical reasoning. Moreover this paper covers the impervious of another property namely every one parameter subgroup is an integral curve of some left-invariant vector field.

Index Terms: Smooth group, morphism, manifold, Hausdoff, smooth mapping, Lie group. Manifold, smooth mapping, left-invariant, vector space, basis, module, integral curve..

1 INTRODUCTION

A group is called a lie group or smooth group if the functions $f : M \times M \rightarrow M$ defined by

$$f\left(a,b\right) = ab \quad (1)$$

and defined by

$$f\left(a\right) = a^{-1} \quad (2)$$

are smooth functions. If M and N are lie groups then the mapping $M \rightarrow N$ is called morphism of lie groups if it is their homomorphism as abstract groups and their smooth mapping as manifolds. All lie groups and all their homeomorphisms form a category devoted by GR-DIFF. Depending on D's smoothness there is a countable family of categories GR-DIFF where either $2 \le s < \infty$ or $s = \infty$ one require of the manifolds under consideration, but practically nothing depends on as any D's-isomorphic to the analytic class group. Some researchers find slight differences between Lie groups and smooth groups. The group M which is at the same time a topological space is called a topological group if the functions (1) and (2) are continuous for it. The homomorphism $M \rightarrow N$ of topological groups is called continuous if it is a continuous mapping. Topological groups and their continuous homomorphism are called the category GR-TOP. The topological space P is said to be Hausdorff or separable if any two of its different points have disjoining neighborhoods. In other words the diagonal is called in P

A topological group should not necessarily be Hausdorff $z_b\left(P\right)$ Means that a tangent space to a maniford P at a point $b \in P$. All tangent spaces $z_b\left(P\right)$, $b \in P$ form a 2m-dimensional $(m = \dim P)$ maniford $z\left(P\right)$ projecting in a usual way onto P. The projection $\rho : z\left(P\right) \rightarrow P$ associate the point of application i.e. a point $b \in P$ for which $B \in z\left(P\right)$ with every vector B so that $z\left(P\right) = \rho^{-1}\left(b\right)$. The sections of this projection i.e. the smooth mappings $Y : P \rightarrow z\left(P\right)$ and $b \in P$ for which $\rho \circ Y = id$ that is $Y_b \in z\left(P\right)$ are known as vector fields constitute in a natural way a linear (infinite dimensional) space which will be denoted by $\beta\left(P\right)$. Differentials $\left(d\varphi\right)_b : z_b\left(P\right) \rightarrow z_\varphi\left(Q\right)$ of an arbitrary smooth mapping $\varphi : P \rightarrow Q$ form a smooth mapping $z\left(\varphi\right) = z\left(P\right) \rightarrow z\left(Q\right)$ for which can have a commutative figure

$$\begin{tikzcd} T(P) \arrow[r, "T(\psi)"] \arrow[d] & T(Q) \arrow[d, "\rho"] \\ P \arrow[r, "\psi"'] & Q \end{tikzcd}$$

Fig (1)

For The mappings $P \rightarrow z\left(P\right)$ and are clearly a frontier from the category DIFF of smooth manifolds into itself. If φ is a diffeomorphism then for vector field $\varphi \rightarrow z\left(\varphi\right)\ Y$ in $\beta\left(P\right)$ a

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field $\varphi_* Y = z(\varphi) \circ Y \circ \varphi^{-1}$ from $\beta(Q)$ is defined and for any vector field. L from $\beta(Q)$ a field $\varphi_* L = z(\varphi)^{-1} \circ L \circ \varphi$ from $\beta(P)$ is defined. .It is evident that the functions φ_* and φ^* are linear and since $\varphi_* = (\varphi^*)^{-1} = (\varphi^{-1})^*$ and $\varphi^* = (\varphi_*)^{-1} = (\varphi_*^{-1})$ they are reciprocal isomorphism's of vector spaces. If particular $P = Q = H$. Where H is some Lie group then for any element $b \in H$ and any vector field $Y \in \beta(H)$ a vector field $K_b^* Y \in \beta(H)$ is defined.

2 RELATED WORK

L.D. Faddeer et.al [1] in 1988, in their research paper discussed quantum formal groups, a finite dimensional example and reviewed the deformation theory and quantum groups. J.C. Benjumea et.al [2], in 2005, in their research article presented a method to obtain the Lie group associated with a finite dimensional nilpotent Lie algebra. In 1968, J.G. Belinfante et.al [3], in their research paper classified the finite dimensional representations of semi simple Lie algebras. David A. Vogan Jr, in 1979, in his research paper presented an essentially algebraic description of the irreducible representations of connected semi simple Lie group G in terms of their restriction to a maximal compact subgroup K of G. In 1947, Claude Chevalley [4], in his research article discussed the applications which can be made of the notion of algebraic Lie algebra to the general theory of Lie algebra and particularly of semi-simple algebra.

3 LEMMA

A topological group M is Hausdorff iff its identity is closed.
Proof:
Any point in Hausdorff space is closed and hence the condition is required. But as the diagonal $\nabla \subset M \times M$ is the inverse image of the identity under the continuous function $M \times M \to M$,$(x, y) \to xy^{-1}$ it is also enough.
From this lemma one can obtain that every Lie group is a Hausdorff topological group. In defining smooth groups the condition that function (2) must be smooth is required.

4 LEMMA

Let A, B, C be smooth manifolds and let $\theta : A \times C \to B$ be a smooth mapping such that for any point $l \in C$ the function $\theta_i : A \to B, t \to \theta(t,l), t \in A$ is a diffeomorphism of A onto the manifold B then the function $\eta : B \times C \to A$ given by $\eta(q,l) = \theta_l^{-1}(q)$, where $q \in B, l \in C$ is a smooth mapping.
Proof:
The mappings $F : A \times C \to B \times C, G : B \times C \to A \times C$ be defined respectively by

$$F(t,l) = (\theta(t,l), l) = (\theta_l(t), l), t \in A, l \in C$$

and $G(q,l) = (\eta(q,l), l) = (\theta_l^{-1}(q), l), q \in B, l \in C$

In this obvious that these functions are smooth iff so are the functions θ, η respectively. So under the hypothesis the mapping F is smooth and it is required to prove that so is the function G .To this end one can observe by definition $(G o F)(t,l) = G(\theta_l(t), l) = (\theta_l^{-1}(\theta_l(t), l)) = (t,l)$.

For any point $(t,l) \in A \times C$. In the way

$$(F o G)(q,l) = F(\theta_l^{-1}(q), l) = (\theta_l(\theta_l^{-1}(q), l), l) = (q,l)$$

For any point $(t,l) \in B \times C$. This is to say that the functions F and G are inverse to each other and hence both are one - one and onto correspondences. The statement about that the smoothness of G therefore is equivalent to the statement that the smooth bijective function F is a diffeomorphism. However it is evident that a smooth one-one and onto correspondence is a diffeomorphism. Everything has thus boiled down to calculating at each point $(b,l) \in A \times C$ the differential $(dF)_{(b,l)}$ of the mapping F which can be identified as a linear function of the form

$$Z_b(A) \oplus Z_l(C) \to Z_c(B) \oplus Z_l(C)$$
(3)

Where $c = \theta(b,l)$. Every function (3) is given graphically by a matrix of the form $\begin{pmatrix} H & I \\ J & K \end{pmatrix}$
(4)
Where
$H : Z_b(A) \to Z_c(B), I : Z_l(C) \to Z_c(B)$, $J : Z_b(A) \to Z_l(C)$ and $K : Z_l(C) \to Z_l(C)$ are linear mapping defined in clear manner. In particular for the mapping $(dF)_{(b,l)}$ the mapping H is nothing but the differential at a point 'a' of the mapping $\theta_l : A \to B$, the mapping and J is the differential of the constant mapping and consequently it is a zero mapping and the mapping K is the differential of the identity mapping and so it is also an identity mapping. Hence for the differential $(dF)_{(b,l)}$ matrix (4) is of the form $\begin{pmatrix} (d\theta_l)_b & I \\ 0 & id \end{pmatrix}$. Since the differential $(dF)_{(b,l)}$ is an isomorphism by the hypothesis of the lemma it follows that the differential $(dF)_{(b,l)}$ is also an isomorphism. For every group M any element $b \in M$ defined by the formulas $P_b y = by, S_b y = yb, y \in M$ and two formulas $P_b : M \to M, S_b : M \to M$ which are called shifts by an element b (the function P_b is called a left shift and the function S_b is a right shift). The following propositions of shifts are clear.

$P_f = S_f = id$ where f is the identity in M .

$P_c \circ P_b = P_{cb}, S_c \circ S_b = S_{bC}, P_b \circ S_b = S_c \circ P_b$ .Since

$P_b \circ P_b^{-1} = P_{b^{-1}} \circ P_b = P_f = id$ and

$S_b \circ S_b^{-1} = S_{b^{-1}} \circ S_b = S_f = id$. In particular one can see that every shift is a 1-1 onto function with

$P_b^{-1} = P_{b^{-1}}, S_b^{-1} = S_{b^{-1}}$ For any element $b \in M$. If M is a topological (smooth) group then the functions P_b and S_b are continuous (smooth) and so they are homeomorphisms.

5 PROPOSITION

If for a group M which is at the same time a smooth manifold function (1) is smooth then so is function (2) and hence the group M is a Lie group.

Proof: The smoothness of mapping (1) implies the smoothness of shifts P_b and hence the fact that they are diffeomorphisms. The corresponding function $P : (y,b) \to P_b(y) = by$ is nothing but mapping (1) and is therefore smooth. Thus under the hypothesis of lemma (1) (for A=B=C=M) and consequently by this lemma the mapping $P' : M \times M \to M$ defined by the formula $P^1(y,b) \to P_b^{-1}(y) = b^{-1}y$ is smooth. To complete the proof it remains to notice that the mapping $x \to x^{-1}$ is the composition of the smooth mapping

$M \to M \times M$, $x \to (f, b)$ and of the mapping P^1.

Therefore it is also smooth.

6 OBSERVATIONS

(i) Any abstract discrete topological group is a Lie group as a zero-dimensional smooth manifold.

(ii) Any finite dimensional vector space is a Lie group under addition.

(iii) A unit circle $|z| = 1$ whose points are complex numbers $z = e^{i\theta}$ is a Lie group under| multiplication.

(iv)The discrete product $U \times V$ of two smooth (or topological) groups U, V is a smooth (respectively topological) group.

(v) Any torus, $T^n, n \geq 1$ is a Lie group

(vi) A full linear group is a Lie group

(vii)The intersection $Sp(m;\square) \cap O(2m)$ is called an orthogonal sympletic group. The Cayley images of non-exceptional matrices of this group are of the form $\begin{pmatrix} A & B \\ -B & A \end{pmatrix}$.

Where B is a symmetric matrix and A is a skew-symmetric matrix. Since matrices of this form also constitute a vector space $Sp(n;\square) \cap O(2n)$ is a Lie group of dimension m^2 .

(viii) The intersection $Sp(n;\square) \cap U(2n)$ is called a unitary sympletic group and denoted by $Sp(m)$. It is a Lie group of dimension $2m+1$.

7 LEMMA

A topological group M is connected if it contains a connected subgroup N with a connected factor M/N

Proof: The natural mapping $\theta : M \to M/N$ is open i.e. turns open sets into open sets. In fact if $V \subset M$ then by the definition of factor topology a set $\theta(v) \subset M/N$ is open iff so is $\theta^{-1}(\theta(v)) \subset M$. But it is clear that the latter is the union $\bigcup_{y \in v} yN$ of all cosets yN , $y \in V$ and hence coincides with the union $\bigcup_{n \in N} Vn$ of all shifts of V by the elements $n \in N$. So if V and hence any V_n is open then the set $\theta^{-1}(\theta(v))$ and hence $\theta(v)$ are open. Now let $M = V \cup W$, where V, W are non-empty sets, the $M/N = \theta(v) \cup \theta(w)$, where $\theta(v)$ and $\theta(w)$ are also nonempty and open. So $\theta(v) \cup \theta(w) \neq \phi$ is nonempty either (since the space M/N is assumed to be connected). Let $\theta(b) \in \theta(v) \cap \theta(w)$, the inclusion $\theta(b) \in \theta(v)$ implies that the coset $\theta(b) = bN$ intersects $\cup$ and the inclusion $\theta(b) \in \theta(N)$ implies that the coset intersects W . One can have $bN = v_1 \cap w_1$ where $v_1 = bN \cap V$ and $w_1 = bN \cap W$ are open in bN . Since bN (together with N) is connected. This is possible iff $v_1 \cap w_1 \neq \phi$ and hence $v \cap w \neq \phi$. Consequently M is connected.

8 LEFT INVARIANT FIELD

A field $Y \in \beta(H)$ is called left–invariant if

$K_b^* = Y, \forall b \in H$. That is if

$$Y_c^* = \left(dY_{b^{-1}}\right)_{bc}(Y_{bc}), \forall b, c \in H \qquad (3)$$

It is evident that all left invariant fields form a subspace of the space $\beta(H)$ of all vector fields. This sub space is denoted by g or $I(H)$. It can be easily seen that a field $Y \in \beta(H)$ is left-invariant iff $Y_b = (dK_b)Y_f$ (4)

For any element $b \in H$. In fact relation (4) is a special case (for $c = f$) of (1) and hence it is satisfied if Y is left-invariant. Conversely if (4) is satisfied then

$$Y_{bc} = (dk_{bc})_f (Y_f) = ((dk_b)_c \circ (dk_c))_f (Y_f)$$

$$= (dk_b)_c (Y_c)$$

This is equivalent to (3). From this one can see that the linear function $Y \to Y_f$ of g into a tangent space $Z_f(H)$ is an isomorphism. In fact for any vector $B \in Z_f(H)$ the mapping $b \to (dk_b)_f B, b \in H$ is clearly seen as a vector field on H, which possess principle (3) and therefore it is left variant. To complete the proof it remains to identity that the resultant function $Z_f(H) \to g$ is clearly the inverse function of $Y \to Y_f$. As a principle, one can use $Y \to Y_f$ to identify $g = I(H)$ with $Z_f(H)$. Since $\dim Z(H) = m$, where $m = \dim(H)$ One can see in particular that for any Lie group G the space $g = I(H)$ of left invariant vector fields is finite dimensional and is of $\dim(m) = \dim(H)$. Let $J(P)$ be the algebra of all smooth functions on a smooth manifold P. For any function $h \in J(P)$ and a field $Y \in \beta(P)$ the rule $(hY)_b = h(b)Y_b, b \in P$ clearly defines some field $hY \in \beta(P)$ and a straight forward checking tells that w.r.t the operation $(h, Y) \to hY$ the vector space $\beta(P)$ changes into a module over $J(P)$. If this module is a free module of rank s i.e. if there is a system $Y_1, \ldots\ldots Y_m$ of vector fields on P such that any field $Y \in \beta(P)$ is uniquely represented as $Y = h'Y_1 + \ldots\ldots + h^m Y_m$

where $h', \ldots\ldots h^m \in J(P)$ then the manifold P is said to be parallelizable.

9 PROPOSITION

Any Lie group H is parallelizable.

Proof: One can more even more namely that every basis $Y_1, \ldots\ldots Y_m$ of a vector space $I(H)$ is a basis of the $J(P)$-module $\beta(H)$. For every point $b \in H$, the vectors $(Y_1)_b, \ldots\ldots, (Y_m)_b$ form a basis of a vector space $Z_b(H)$. So the vector Y_b of an arbitrary vector field is uniquely expanded w.r.t the vectors $(Y_1)_b, \ldots\ldots, (Y_m)_b$. This means that for every vector field $Y \in \beta(H)$ there are mappings $h^j : b \to h^j(b), b \in H$ such that $Y = h'Y_1 + \ldots\ldots + h^m Y^m$ remains to show that $h^l \in J(H), \forall l = 1, 2, \ldots. m$. Let $(W, t', \ldots\ldots t^m)$ be an arbitrary chart of the manifold H. Since the fields $Y_1, \ldots\ldots Y_m$ are smooth on W there are smooth function $Y_1^{\ j}, \ldots\ldots, Y_m^{\ j}, \ j = 1, 2 \ldots., m$ such that $Y_v = Y_v^{\ j} \dfrac{\partial}{\partial t^j}; \quad v = 1, 2, \ldots m$. Besides, since every point $b \in W$ vectors $(Y_1)_b, \ldots\ldots, (Y_m)_b$ form a basis of $Z_b(H)$ one can get det $(Y_v^{\ j}) \neq 0$ on W and so on W there are smooth functions L_j^l such that $Y_v^{\ j} L_j^l = \Delta_v^l; \ j, v, l = 1, 2, \ldots, m$ under assumption $Y = h^v Y_v$ and hence $Y = h^v Y_v^{\ j} \dfrac{\partial}{\partial t^j}$. That is $Y = h^v Y_v, \ j = 1, 2, \ldots m$ are components of Y in local coordinates $t', \ldots., t^m$ and therefore smooth. But $h^l = h^v \Delta_v^j = (h^v Y_v^{\ j}) L_j^l$ and since $h^v Y_v^{\ j}$ and L_j^l are smooth and so are $(on W)\ h^l$. Being smooth functions on every coordinate neighbourhood W, functions h^l are smooth on the entire manifold H. Recall that a smooth curve $s \to \psi(s)$ on a manifold P is said be an integral curve of a vector field if $\dfrac{d\psi(s)}{ds} = Y_{\psi(s)}$ for say s. An integral curve is called maximal if it is not a restriction of an integral curve defined on a larger interval of the real axis. It easily follows from the standard theorem on the uniqueness and existence of the solution of a system of differential equations having smooth right hand sides and from elementary properties of Hausdorff space that if the manifold M is Hausdoff which is automatically satisfied for a Lie group for any point $b \in P$ there exists a maximal integral curve $\psi_b(0) = b$ .If for every point $b \in p$ the curve ψ_b defined on the entire axis $\mathbb{R}$ then the vector field Y is called complete. It is easy to see that vector field Y on a Lie group H is left –variant iff for any two points $b, c \in H$,

$$\psi_{bc} = K_b \circ \psi_c \tag{5}$$

that is $\psi_{bc} = b\psi_c(s), s \in \mathbb{R}$. In fact for any fixed the formula $\theta_c(s) = \theta_{b^{-1}c}(s)$ defines for any point $c \in H$ a certain curve $s \to \theta_c(s)$ passing for $s = 0$ through c and it is evident that by setting $L_c = \dfrac{d\theta_c(s)}{ds}$ at $s = 0$, one can obtain on H a certain vector field $L : c \to L$. Using the principles for calculating tangent vectors for smooth curves for any point $c \in H$ one can get

$$L_c = \left[\frac{d\theta_c(s)}{ds}\right]_{s=0} = \left[\frac{d\left(K_b = \psi_{b^{-1}c}(s)\right)}{ds}\right]_{s=0}$$

$$= (dK_b)_{b^{-1}c} \left(\frac{d\psi_{b^{-1}c}(s)}{ds}\right)_{s=0}$$

$= \left(dK_b\right)_{b^{-1}c}\left(Y_{b^{-1}c}\right)$

Therefore if (2) is satisfied and consequently $\theta_c = \psi_c$ and hence $L_c = Y_c$ then $Y_c = \left(dK_b\right)_{b^{-1}c}\left(Y_{b^{-1}c}\right)$.

In particular $Y_b = \left(dK_b\right)_f\left(Y_f\right)$. Consequently the field Y is left variant. Conversely if Y is left variant and hence satisfies relation (1) then $L_c = Y_c$ for any point $c \in H$ that is $L = Y$. But it is evident that $s \to \theta_c(s)$ are integral curves of L which are automatically maximal and therefore in view of the equation $L = Y$ these curves coincide with the integral curves $s \to \psi_c(s)$ of Y. Thus $\psi_c(s) = b\psi_{b^{-1}c}$.

10 PROPOSITION

Every one-parameter subgroup r is an integral curve of some left-invariant vector field Y.

Proof: A smooth curve $r : \mathbb{R} \to H$ is called a one-parameter subgroup of a Lie group H if $r(s+u) = r(s)r(u)\ \forall s,u \in \mathbb{R}$. In other words a one-parameter subgroup is a homomorphism of the additive group $\mathbb{R}$ of real numbers (which is considered) a Lie group into the Lie group H. It can be observed that a one-parameter subgroup is thus not a subset but a function. It is clear that for $s = 0$ every one parameter subgroup r passes through the identity f of M : $r(0) = f$.

The formula $\psi_b(s) = br(s), b \in s, s \in \mathbb{R}$ defines on H a smooth curve $s \to \psi_b(s)$ passing for $s = 0$ through the point b. One can set $Y_b = \left[\frac{d\psi_b(s)}{ds}\right]_{s=0}$. A direct verification proves that the function $b \to Y_b$ is smooth that is a vector field on H and those curves ψ are integral curves of that field. An integral curves thus the curve $\psi_b = r$. Finally since $\psi_{bc}(s) = bcr(s) = b(cr(s)) = b\psi_c(s)$

The field Y is left variant. The converse of this proposition is also true.

11 CONCLUSIONS

In the above research discussion the necessary and sufficient condition for a topological group to be Hausdorff is derived. In addition to this the necessary condition for a topological group to be connected has been proved. In the context of future research one can extend these ideas to prove that a Lie group is always parallelizable. In this research article the concept of left invariant field is proposed and two propositions viz "Any Lie group is parallelizable" and "Everyone parameter subgroup is an integral curve of some left –invariant vector field are presented along with innovative proofs. In context of future research these ideas can be extended to form a matrix Lie groups admitting the Cayley construction.

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