Application Of Dea In Super Efficiency Estimation

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Abstract: DEA methods classify DMUs into two sets namely efficient and inefficient ones. DEA and multiple criteria DMUs are developed independently and designed for different purposes. Data Envelopment Analysis is an LPP based model and can estimate the RE of Decision Making Units with multiple outputs and inputs. Traditional Data Envelopment Analysis can trace Decision Making Units by its own weights. Moreover it splits up into inefficient and efficient DMUs. This research article explores some envelopment problems to identify extremely efficient DMUs. The ISE and OSE problems are proposed here and manifested that under constant returns to scale these problems are always feasible.

Keywords: ISE(Input Super efficiency),OSE(Output Super Efficiency),Envelopment Problem, Linear Programming Problem (LPP),RE(Relative Efficiency),Data Envelopment Analysis (DEA), Decision Making Unit (DMU),PE(Pareto Efficiency),Reference Technology(RT)

1 INTRODUCTION:

A.A. Foroughi et.al [1] in their research paper, discriminated DMUs by comparing their optimal weight sets under the assumption that Decision Making Units with bigger set of optimal weights are preferred. J Rezaeiani et.al, [2] in their article, introduced the notion of reference frontier share and proposed a model to measure the reference of frontier share Mahmood Mehdiloozad et.al [3] conducted a systematic analysis on the measurement of super-efficiency. The measures proposed by them are advantageous because they circumvent biases in super-efficiency estimation due to input and output slacks. Chandrababu et.al [4] examined that SE is estimated easily through graphs. Rajiv D. Banker et.al [5] proposed simulation experiments to assess the act of two alternative uses of the SE process in Data Envelopment Analysis. Since most of the projected models cannot afford the projection of PE, E. Zanboori et.al [6] developed a model and proposed with their paper based on which in the projection of Pareto-efficient is got in addition to computing the amount of SE. There are number of methods for ranking Decision Making Units among which calculating SE and then ranking the units based on the obtained amount of SE are both effective and proficient. There are number of methods for ranking Decision Making Units among which calculating SE and then ranking the units based on the obtained amount of SE are both effective and proficient. Secondary task in efficiency measurement is ranking of extremely efficient decision making units. These are given ranks based upon their SE scoring rate. For inefficient units their super efficiency scores coincide with radial efficiency scores. To EEDMU from the RT. Let (x_0, y_0) be input and output vectors such that, $x_0 \in R_+^m$ and $y_0 \in R_+^s$ of an extremely efficient decision making unit. If (x_0, y_0) is deleted from the reference technology and returns to scale are constant, the SE problem is always feasible irrespective of input or output orientation.

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However, if returns to scale departure from constancy, super efficiency problem of an extremely efficient decision making unit may turn out to be infeasible. The DMU represented by B is extremely efficient, if B is dropped from the RT the relevant frontier loses AB and BC and the super-efficient frontier consists of the segment AC. In an effort to find its superefficient input and output vectors DMU B compares its performance with D and E respectively. Consequently, its superefficient problems, both input and output oriented, are feasible and the superefficient input and output scores are respectively λ^* and δ^*. $\lambda^* \geq 1$; and $\delta^* \leq 1$. To produce the input y_0 of the extremely efficient Decision Making Unit under computation, the remaining Decision Making Units require their joint effort input level as measured by $\lambda^* x_0$, to produce y_0.Thus, $(\lambda^*-1)x_0$ can be given an interpretation that, it is the input saved by the super-efficient DMU that produces y_0 using the input x_0. The referent point for DMU B in its output super efficiency evaluation is E. Its output super efficiency score is δ^*, where $\delta^*<1$. $(1-\delta^*)$ y_0, can be given an interpretation that it is excess output produced by the super-efficient DMU that produces y_0 using the input x_0.

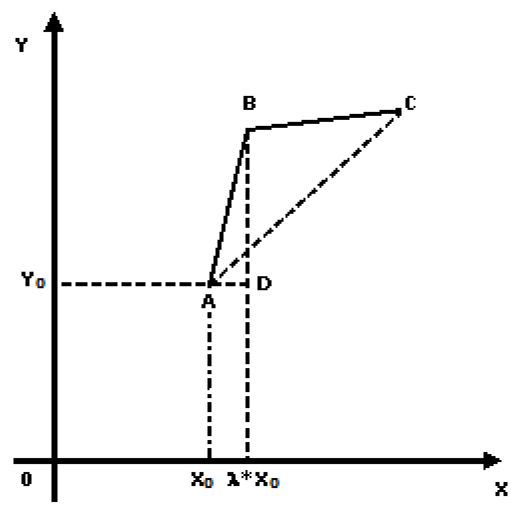

Fig (1.1)

DMU A is extremely efficient. If Decision Making Unit A is dropped from the RT, as there is no referent DMU for DMU A the output oriented super efficiency problem emerges to be infeasible. However, the input oriented SEP of DMU is feasible and the SE of DMU A is λ^* which is larger than unity.

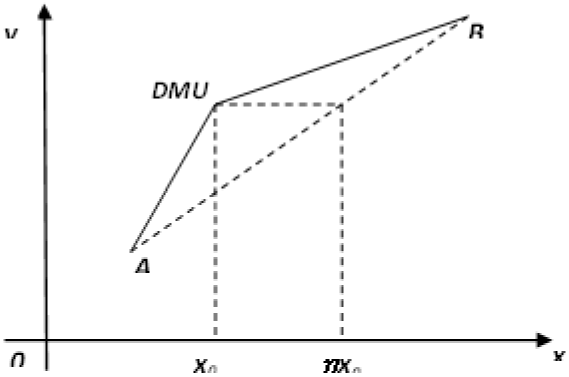

***Fig (**1.2)*

DMU C is extremely efficient. If this DMU is deleted from the RT its output problem is feasible but input one is not feasible. Thus, it is proved that if ISEP of an extremely efficient DMU is not feasible then its OSEP is feasible. Similarly, if OSEP is not feasible then the corresponding ISEP is shown to be feasible. It is desired not only to compute the SE of an extremely efficient DMU, but to know how robust is the efficiency of this DMU for increases of inputs and decreases in outputs.

2. Super efficiency estimation

Let x∈ R_m^+ , y∈ R_s^+ represent the dimensionality of the DEA problem. Let (x_i, y_j) be the input and output vectors of i^{th} DMU, j=1, 2 …n. SEPs are input and output oriented. SEs are computed for extremely efficient decision making units. To identify the extremely efficient decision making units, we solve envelopment problems, either input or output oriented. Input oriented envelopment problem: Extremely efficient DMU

$$Min\ \lambda$$

$$subject\ \ to\ \lambda_1 x_{i1}+\lambda_2 x_{i2}+....+\lambda_n x_{in} \le x_{i0}\lambda,\ \ i\in\{1,2,...,m\}$$

$$\lambda_1 y_{r1}+\lambda_2 y_{r2}+...+\lambda_n y_{rn} \ge y_{ro}, r\in\{1,2,...,,s\}$$

$$\lambda_j\ are\ \ nonnegative \qquad (2.1)$$

If DMU_0 is extremely efficient, the optimal solution takes the following form:

λ* = Min λ = 1

λ_0 =1

λ_j =0, ∀ j ≠ 0, s_i^- equals zero, $i\in\{1,2,....,m\}$ and s_r^+ equals zero, r $r\in\{1,2,....,s\}$. Where s_i^- and s_r^+ are slack variables associated with the input and output constraints respectively. Output oriented envelopment problem – Extremely Efficient DMU.

$$\max(\theta)$$

$$\sum_j \lambda_j\, x_{ij} \le \lambda\ x_{i0}\ \ ,\ \ i=1,2,....,m$$

$$\sum_j \lambda_j\, y_{rj} \ge \theta\, y_{r0}\ \ \ ,\ r=1,2,.....,n$$

$$and\ \ \lambda_j \ge 0$$

(2.2)

if DMU_0 is extremely efficient, the final solution takes the following form:

θ* = Max θ = 1, λ_0 =1, λ_j =0, and∀ j ≠ 0

s_i^- = s_r^+ =0, ∀ i Λ r

The linear programming models (2.1) and (2.2) admit constant returns to scale, but inversely related. Consequently one can have,

$$\left(\lambda_1 x_{i1}+..+\lambda_n x_{in}, \lambda_1 y_{r1}+...+\lambda_n y_{rn}\right) \qquad i\in\{1,2,...,m\},$$

$r\in\{1,2,...,s\}$

If DMU_0 is deleted from the RT, the production possibility set shrinks.

3. THE ISEP:

Input SE problem of DMU_0 is as follows:

$$\eta\ \ = Min\ \ \lambda$$

$$subject\ \ to\ \ \lambda_1 x_{i1}+.....+\lambda_n x_{in} \le x_{i0}\lambda,\ \ i\in\{1,2,...,,m\} \qquad (3.1)$$

$$\lambda_1 y_{r1}+...+\lambda_n y_{rn} \ge y_{r0},\ r\in\{1,2,...,s\}$$

$$\lambda_j\ \ \ge\ 0 \qquad \forall\ \ j\neq 0$$

λ* =Min λ ≥ 1

All the other DMUs or their linear combinations require inputs.η x_{i0}, $i\in\{1,2,...,m\}$to produce the outputs, y_{r0}, $r\in\{1,2,...,n\}$that are currently produced by DMU_0. Therefore, the inputs saved by DMU_0 are,

(η-1) x_{i0} , i = 1,2,…..,n

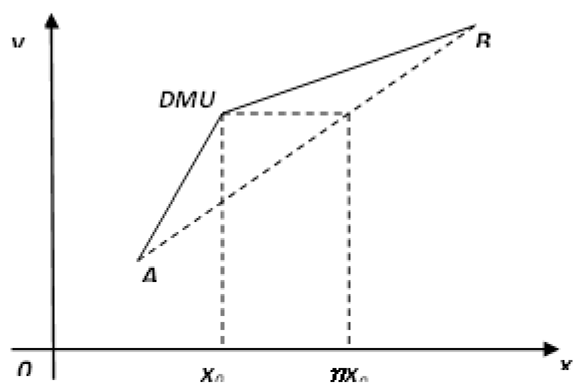

Fig: 3.1

η> 1, Input saving by DMU_0 is (η-1) x_{i0}, i = 1,2 …n

OSEP: The OSEP of DMU_0 may be formulated as follows:

$$Max\ \theta$$

$$subject\ to\ \ \lambda_1 x_{i1} + ... + \lambda_n x_{in} \le x_{i0}\ \ and\ \ \lambda_1 y_{r1} + ... + \lambda_n y_{rn} \ge \theta y_{r0}$$

$$\lambda_j\ are\ nonnegative\ and\ \text{j is nonzero} \qquad (3.2)$$

δ = Max θ , measures the output super efficiency of DMU_0.δ< 1. All the DMUs or linear combinations of them will be able to create the outputs, $\delta\ y_{r0}, r \in \{1,2,..., s\}$ .Combining inputs of DMU_0. Thus, output gains by DMU_0, due to super efficiency are, (1- δ) y_{r0}, r =1, 2 …s.

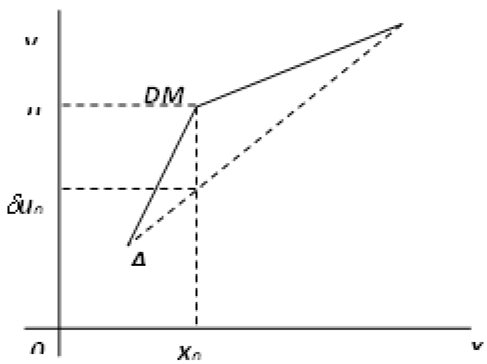

Fig: (3.2)

Outputs gain by DMU_0 are (1-δ) y_{r0} , $r \in \{1,2,..., s\}$. The SE envelopment problems (2.3) and (2.4) admit constant returns to scale. Figures (3.1) and (3.2) portray variable returns to scale. Under constant returns to scale, the input and output super efficiency problems are always feasible. It is not so always, if the frontier production function models variable returns to scale.

4 CONCLUSION AND FUTURE RESEARCH:

In the above discourse input and output oriented envelopment problems have been formulated and the EEDMUs are identified by solving the envelopment problems. Moreover linear programming problems formulated to assess SE of EE units under input and output orientation. In the context of future research one can evaluate different types of efficiency stability regions and their infeasibility in DEA .Furthermore the slack based efficiency of a DMU can be estimated.

5 REFERENCES:

[1] A. A. Foroughi, M.H. Tavassoli (2019),"Discriminating extreme efficient Decision making Units in DEA using random weight vectors", Computers Industrial Engineering, Volume128, February 2019, pages 305-312

[2] M.J. Rezaeiani, A. A. Foroughi (2018),"Ranking efficient decision making units in data envelopment analysis based on reference frontier share", European Journal of Operational Research, Volume 264,Issue 2,16January2018,pages 665-674

[3] Mahmood Mehdiloozad, Israfil Roshdi (2014). "Analysing the concept of super-efficiency in DEA: A directional distance function approach"

[4] S. Chandrababu, S. Hariprasad (2015), "A Concept of Input-Output oriented Super-efficiency in Decision Making Units", American Journal of Engineering Research (AJER), Volume 4, Issue 1, pp.76-81

[5] Rajiv D. Bankar, Hsihui chang (2006), "The super-efficiency procedure for outlier identification, not for ranking efficient units", European journal of operation research 175(2), 1311-1320.

[6] E. Zanboori, M. Rostamy- Malkhalifeh, G. R. Jahanshahloo, and N. Shoja(2014) "Calculating Super Efficiency of DMUs for Ranking Units in Data Envelopment Analysis Based on SBM Model", Hindawi Publishing Corporation, the Scientific World Journal, http://dx.doi.org/10.1155/2014/382390 .

[7] Meixue, Patrick T Harker (2002), "Ranking DMUs with infeasible super efficiency DEA Models", Management Science, Vol 48, No.5, May 2002, pp 705-710.

[8] Farell, M J, (1957), "The Measurement of Productive efficiency", Journal of Royal Statistical Society, Series-A, 120, 253-290.

[9] Timmer, C. P, (1971),"Using a Probabilistic Frontier Production Function to Measure Technical Efficiency", Journal of Political Economy, 79, 776-794

[10] Barbara A Mark, Bland Jones and Lisa Lindley (2009), "An Examination of Technical Efficiency, Quality and Patient Safety in Acute care Nursing Units", Policy Poit Nurs Pract, Pp.180-186, doi: 10.1177/1527154409346322

[11] S. Nuti, C Daraio and M Vainieri (2011), "Relationships between technical efficiency and the quality and costs of health care in Italy "International journal for Quality in Health Care, Pp. 324-330

[12] Gahe Zing Samuel Yannik, Zhao Hongzhong, Belinga Thierry (2016), 'Technical efficiency assessment using data envelopment analysis: An application to the banking sector of Cate d" in12th international journal strategic Management Conference, ISMC-2016, Antalya, Turkey

[13] Smita Verma, Ankit Kumavat and Anita Biswas [2015], 'Measurement of Technical Efficiency using DEA: A case of Indian Textile Industry', ICAESAM, LONDON (UK)

[14] Green, W H,(1980) "Maximum likelihood Estimation of Econometric Frontier Functions", Journal of Econometrics, Vol. (13). pp. 21-56

[15] Kalirajan, K (1985), 'On measuring Absolute Technical and Allocative efficiencies', Sankhya, Series-B, Vol. (47), pp. 385-400

[16] B. Venkateswarlu et.al (2015),"Efficiency Evaluation of Total Manufacturing Sectors of India-DEA Approach", Global journal of Pure and Applied Mathematics, Vol. (11), Pp. 3145-3155

[17] B. Venkateswarlu, et.al (2009), "Assurance region efficiency of Total manufacturing sectors of Indian States", Asian journal of Economics and Econometrics, Vol. 9, pp.75-84.

[18] Venkateswarlu, B., Mahaboob, B., Subbarami Reddy, C., & Ravi Sankar, J. (2017). "A study on technical efficiency of a DMU" (review of literature). Paper presented at the IOP Conference Series: Materials Science and Engineering, 263(4) doi:10.1088/1757-899X/263/4/042124

[19] B. Venkateswarlu, B. Mahaboob , J. Ravi Sankar and B. Madhusudhana Rao (2018), "An Application of Goal Programming in Data Envelopment Analysis", International Journal of Engineering &Technology, Vol.(7), issue 4, Pp: 523-525.

[20] B Venkateswarlu, B Mahaboob, C Subbarami Reddy and J Ravi Sankar (2017), "Materials Science and Engineering Conference Series", Vol. (263), issue 4.

[21] B Venkateswarlu, M Mubashir Unnissa and B Mahaboob (2016), "Estimating Cost Analysis using Goal Programming", Indian Journal of Science and Technology, Vol. (9), Issue (4).

[22] B Venkateswarlu, B Mahaboob, C Subbarami Reddy and B Madhusudhana Rao (2017), "Fitting of full Cobb-Douglas and full VRTS cost frontiers by solving goal programming problem", Materials Science and Engineering Conference Series, Vol. (263), Issue (4)

[23] B Venkateswarlu, B Mahaboob, C Subbarami Reddy (2019), "New Results in Production Theory by Applying Goal Programming

[24] International Journal of Scientific & Technology Research.

[25] B Venkateswarlu, B Mahaboob, C Subbarami Reddy (2019), "Evaluation of Slack Based Efficiency of a Decision Making Unit", International Journal of Scientific & Technology Research, Vo. (8), Issue (11)