Breakup Of A Dark Solitary Wave In Multiple Solitons Of Gray Nature In Axially Trapped Bose-Einstein Condensate

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Abstract : Formation of multiple gray solitons by the time evolution of a dark soliton in an axi-symmetrically trapped Bose-Einstein condensate considering the repulsive atom-atom interactions has been studied. By solving the 2D time-dependent Gross-Pitaevskii equation the in-trap column densities of the condensate have been calculated. The results exhibit that over very small time scale (~2-3 ms) multiple gray soiltons can be produced by breaking up a black solitary wave. Further increase in positive value of scattering length via magnetic Feshbach resonance decreases the time scale of of obtaining the multiple gray solitons. The properties like repulsion between neighboring gray solitons and change in grayness with axial distance from the center of the trap have been also analyzed.

Keywords: Atomic Bose-Einstein condensate, dark soliton, gray soliton.

1 INTRODUCTION

The solitonic feature is one of the fascinating dynamics of the nonlinear system such as Bose-Einstein condensate (BEC) of alkali atoms [1,2]. This arises due to the balance between nonlinearity and dispersion in the system. Extensive theoretical and experimental research works have been carried out in the field of bright and dark matter wave solitons in BECs [2-4]. Investigations of bright matter wave soliton trains in trapped BEC by tuning the atom-atom interactions have been discussed both experimentally and numerically [10-13]. Formation of matter wave soliton trains in BECs has been reported by Strecker [5] using a condensate of ^{7}Li atoms by magnetically tuning interactions from repulsive to attractive. Dynamics of bright soliton trains with attractive interactions and formation of dynamically generated soliton trains due to quantum mechanical phase fluctuations of a 1D BEC have been well analyzed [6]. Formation and the evolution of soliton trains produced by modulational instability (MI) has been investigated in the framework of time-dependent Gross-Pitaevskii equation [7]. MI is a phenomenon of decaying a large intensity hump into small pulse trains due to the effect ofnonlinearity [8,9]. Observation of MI in BECs has been a topic of interest in recent research on BEC. Real time observation of MI in the ^{85}Rb condensates has been reported [10]. Role of MI in formation of matter wave soliton trains from ^{7}Li BEC by exploiting Feshbach resonance is of great importance in this context [11]. Occurrence of MI in the BECs trapped in optical lattices has also been investigated in detail [12-14]. In the above references [2-8] the scattering length of the condensate is instantaneously changed to negative value from positive value and the initial wave function has not been imprinted with any fluctuating phase. The train of solitons appears in time scale ~10 ms and beyond.

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In this work I have made an attempt theoretically to generate soliton trains of gray nature by real time propagation of a single black soliton in an axi-symmetrically trapped BEC. Time-independent Gross-Pitaevskii equation for ^{23}Na atoms in a BEC with repulsive interatomic interaction considering a harmonic axisymmetric trap as used in [15] has been solved to get the stationary state solution. Then a phase step along axial direction has been imprinted at the center of the trap to generate a dark soliton. By the time propagation of this dark soliton based on the model of Gross-Pitaevskii equation, it transforms into multiple gray solitons within much smaller time scale compared to previously reported experimental results [10, 11]. Gray soliton formation is induced by MI which evolves with time. The scattering length of repulsive interatomic interaction can be controlled via Feshbach resonance [16]. Change in grayness with the axial distance from the center of the trap and the effect of increase in the positive value of scattering length on the dynamics of soliton train will also be investigated in this article. The paper is structured as follows. In section 2, the Gross-Pitaevskii equation and the Crank-Nicholoson discretization scheme to solve the GP equation taking initial black solitonic solution of BEC are given. Results and systematic analysis of the dynamics of gray multiple solitons have been provided in section 3 and the paper is concluded in section 4.

2 THEORY

We consider a cigar shaped BEC which can be described very well by the Gross-Pitaevslii (GP) equation in weakly interacting regime:

$$i\hbar\frac{\partial\psi(\boldsymbol{r},t)}{\partial t}=\left(-\frac{\hbar^2\nabla^2}{2m}+V(\boldsymbol{r})+g|\psi|^2\right)\psi(\boldsymbol{r},t) \qquad (1)$$

where 'm' is the mass of an atom, $g=4\pi\hbar^2a/m$ is the nonlinearity parameter with 'a' as the s-wave scattering length of atom-atom interaction, 'N' is the total number of trapped atoms and ψ is the complex wave function of the condensate. The external trapping potential V(r) is taken to be axially symmetric characterized by the radial ($\omega_\perp$) and axial (ω_z) frequencies:

$$V(\boldsymbol{r})=\frac{1}{2}m\omega_\perp^2\left(r^2+\lambda^2z^2\right) \qquad (2)$$

where$\lambda= \omega_{\perp}/\omega_z$ is the anisotropy parameter of the trap. The normalization of the wave function is given by

$$\int n(\boldsymbol{r})d\boldsymbol{r} = \int |\psi(\boldsymbol{r})|^2 d\boldsymbol{r} = 1 \qquad (3)$$

As we consider an axially symmetric BEC system the wave function ψ(r,t) depends on the radial position 'r', axial position z and time t. The wave function ψcan be written as

$$\psi(\boldsymbol{r},t) = \frac{\phi(r,t)}{r}$$

After transforming the variables r, z and t into dimensionless quantities x, z_1 and τ the rescaled GP equation is given by

$$i\frac{\partial\phi}{\partial\tau} = -\frac{1}{2}\left(\frac{d^2\phi}{dx^2} - \frac{1}{x}\frac{d\phi}{dx} + \frac{d^2\phi}{dz_1^2}\right) + \frac{1}{2}\left(x^2 - \frac{1}{x^2} + \lambda^2 z_1^2\right)\phi + \gamma\frac{|\phi|^2}{x^2}\phi \qquad (4)$$

Here γ (=4πaN) is a real constant, time and spatial coordinates (x and z) are measured in the units of $\omega_{\perp}$and $a_{\perp}$respectively; where $a_{\perp}=\sqrt{(\hbar/m\omega_{\perp})}$.

To follow the time evolution of the dark soliton in atomic condensate I have used the Crank-Nicholoson discretization scheme[17] in space and time to solve the time-dependent GP equation [Eq. (4)]. The iteration in time has been started with the normalized dark soliton like wave function

$$\varphi_{DS} = \tanh(z)\,\varphi_1 \qquad (5)$$

where$\varphi_1$is related to the normalized solution $\psi$of the time-independent GP equation of axially trapped condensate by $\varphi_1=r\psi$. The time-independent GP equation can be obtained by substituting $\psi(r,t)=\psi(r)\exp(-i\mu t/\hbar)$ (where $\mu$is the atomic chemical potential) as:

$$\mu\psi = \left(-\frac{\hbar^2\nabla^2}{2m} + V(\boldsymbol{r}) + g|\psi|^2\right)\psi(\boldsymbol{r},t) \qquad (6)$$

The column density of the condensate inside axially symmetric trap which is an experimentally measurable quantity, can be defined as

$$n_c(z) = \int dr|\psi(r,z)|^2 \qquad (7)$$

3 RESULT AND DISCUSSIONS

To realize the dynamics of gray solitons, the harmonic confinement of ultracold sodium ^{23}Na atoms is taken as axisymmetric with $\omega_{\perp}= 2\pi\times131$ Hz and $\omega_z= 2\pi\times13$ Hz as considered in the experiments of snake instability of gray solitons [15]. The radial direction of the harmonic trap is weak and the axial direction is strong. I first calculate the ground state wave function of the time-independent form of GP equation (Eq. (6)) with N= $10^4$and positive scattering length (a). The numerical algorithm to obtain ground state wave function is based on steepest descent method [18]. Using Eq. (5) (a dark solitary wave inside an axially trapped atomic BEC) as initial guess, the time evolution of the dark soliton formed inside the condensate of sodium atoms, has been analyzed in the light of enhancement of scattering length using magnetic Feshbach resonance [16]. Firstly the scattering length of interaction between atoms has been taken as a= 2.75 nm [16]. The stationary GP equation (Eq. (6)) for 10^4 atoms has been solved to get the normalized condensate wave function (φ_1). The normalized dark solitonic wave function(φ_{DS}) is obtained by multiplying a tan hyperbolic function to φ_1. This is equivalent to imprinting a phase step with the condensate wave function. Formation of multiple solitons has been obtained with time evolution of initial dark soliton after only few milliseconds. In Figs. 1(A), (B), (C) and (D) the contour plots of condensate densities as functions of radial and axial distances from the center of the trap have been plotted at times t=0, 1.2, 2.4 and 3.6 ms, respectively. The horizontal axis is the axial coordinate and the vertical axis is the radial coordinate. Fig. 2 displays the column density profiles n_c(z) of 10^4 atoms with a= 2.75 nm for time t= 0 (Fig. 2A), 0.607 ms (Fig. 2B), 1.2 ms (Fig. 2C), 2.43 ms (Fig. 2D) and 3.64 ms (Fig. 2E). It is evident from Figs. 1 and 2 that starting from an initial black solitary wave multiple solitons of gray nature (as the densities are not zero at the minima) have been formed with simple evolution in real time within only few milliseconds of propagation. The deviation in density of the time-evolved profile from the initial (t=0) profile is much prominent in the central region of the trap than the edges of the trap. That means the effect of MI is large in the center than the edges of the trap [11]. The number of peaks on either side of central peak up to z= 12 $a_{\perp}$ are 11, 5and 3 at t=1.2, 2.4 and 3.6ms, respectively. This result indicates that solitons are rarified as time propagates which suggests that there must be some repulsive interactions between the neighboring solitons inside the trap. As the magnitude of 'a' can be manipulated by using Feshbach Resonance, the effect of increase in 'a' on the MI i.e. on the dynamics of the formation of gray soliton trains from a single dark soliton has been studied next. Figs. 3A, 3B, 3C and 3D depict the contour plots of the densities of condensate (as functions of radial and axial coordinates) of 10^4 atoms with a= 27.5 nm at times t=0, 0.9, 1.2 and 1.8 ms, respectively. By making the scattering length 10 times larger than the previous case (Fig. 1 and Fig. 2), it has been found that the train of gray solitons are formed within much less time (Fig. 3). To get more clear picture about the number of solitons the column densities have been plotted as functions of axial distance (z) at times t=0 (A), 0.607 ms (B), 0.91 ms(C), 1.21 ms(D) and 1.52 ms (E). The number of peaks on either side of central peak up to z= 8 $a_{\perp}$are 10, 6, 4 and 3 for t= 0.607 ms, 0.91 ms, 1.21 ms and 1.52 ms, respectively. It can be inferred that the repulsive nature of interaction between neighboring solitons prevails for larger scattering length also.

4 CONCLUSION

In this work, I have investigated the formation of multiple gray solitons within BEC of 10^4 atoms loaded in anisotropic trap for repulsive interatomic interaction i.e. positive scattering length. Time evolution of a dark soliton inside a trapped BEC (based on Gross-Pitaevskii scheme) results in the occurrence of multiple solitons of gray nature. The multisoliton formation that has been obtained from a dark solitary wave can be considered as a result of MI as the single pulse of dark soliton transforms into multiple pulses. Previous experimental and theoretical observations of MI in BECs have been reported [5, 10, 11] for attractive interactions. This analysis shows that the multisoliton formation due to MI occurs also in case of repulsive interatomic interactions. This result additionally can be obtained within much smaller time (3 to 4 ms). If the scattering length is adjusted to a much larger value (which can be easily observed with the help of Feshbach resonance), the time limit of obtaining the multiple solitons is reduced to ~1 ms. The repulsions between neighboring solitons are in accordance with the repulsive nature of interactions.

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6 REFERENCES

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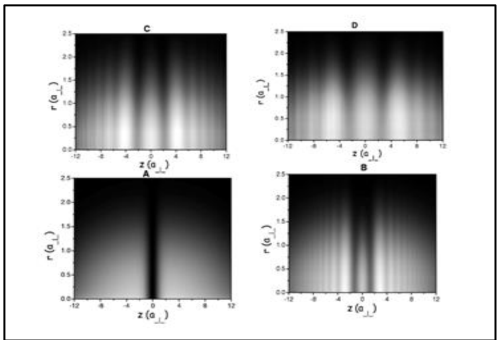

Fig. 1 *Time evolution of the densities of a dark solitary wave in a condensate confined in a cylindrically symmetric harmonic trap for a= 2.75 nm and N= 104. Fig. A corresponds to a dark soliton in BEC at t= 0. Figs. B, C and D display the evolution of the dark soliton and formation of multiple gray solitons at times t= 1.2 ms, 2.4 ms and 3.6 ms, respectively.*

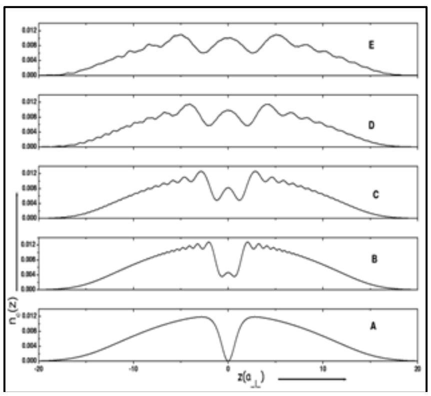

Fig. 2 Column density profile after propagation in time t= 0 (A), 0.607 ms (B), 1.2ms (C), 2.43 ms (D) and 3.64 ms (E) with a= 2.75nm.

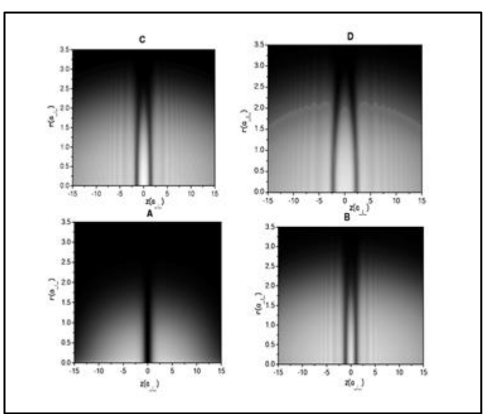

Fig. 3 *Time evolution of the densities of a dark solitary wave in a condensate confined in a cylindrically symmetric harmonic trap for a= 27.5 nm and N= 104. Fig. A corresponds to a dark soliton in BEC at t= 0. Figs. B, C and D display the evolution of the dark soliton and formation of multiple gray solitons at times t= 1.2 ms, 2.4 ms and 3.6 ms, respectively.*

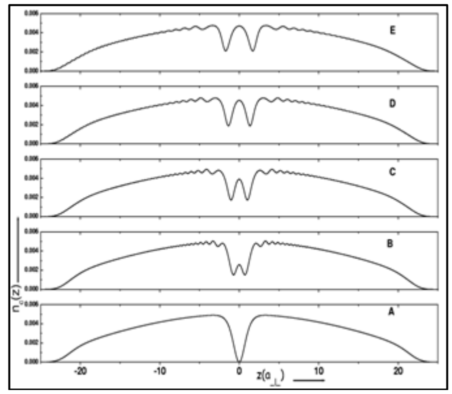

Fig. 4 *Column density profile after propagation in time t= 0 (A), 0.607 ms (B), 0.91 ms (C), 1.21 ms (D) and 1.52 ms (E) with a= 27.5nm.*