Evaluating Different Types Of Efficiency Stability Regions And Their Infeasibility In DEA

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Abstract: DEA is a decision making tool based on LPP for estimating the relative efficiencies of a group of comparable units. Data Envelopment Analysis has strong connection with theory of production in Econometrics and it is often applied for benchmarking in operational management problems. In this research article a classification of efficiency stability regions has been proposed and their infeasibilities are presented. The main purpose of this article is to investigate how robust is the efficiency of an extremely efficient Decision Making Unit (DMU). Input oriented super efficiency problems of DMU have been solved in this article in order to examine how robust an extremely efficient DMU is for increases of inputs

Index Terms: Efficiency stability region, Super efficiency DMU (Decision Making Unit), Infeasibility, DEA (Data Envelopment Analysis), LPP (Linear Programming Problem)

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1 INTRODUCTION

It is desired not only to calculate the super efficiency of an extremely efficient DMU, but to know how robust the efficiency of this DMU for increases of inputs and decreases in outputs. The following member examines how robust the efficiency of these DMUs is under (a) Increases inputs only (b) decreases of outputs only (c) simultaneous increases of inputs and decreases of outputs. DEA is a mathematical programming method to compute relative frequency of DMUs with multiple input-output. A main essence of this Data Envelopment Analysis technique which has been deliberated by many researchers is Sensitivity Analysis. Lawrence M. et.al [1] developed a method for performing a sensitivity analysis of the efficient DMUs within the CCR model of DEA. The method provides an exact input region and output stability region within which the efficiency of a specific efficient DMU remains unchanged. P. Zamani et.al [2] in their paper mentioned the issue of sensitivity of efficiency of classification of VRTS technology for increasing the trustworthiness of DEA results in practical applications when an additional DMU needs to be added to the set being considered. In 2017, B. Venkateswarlu et.al [3] in their research paper proposed an innovative discussion on TE of a Decision Making Unit.

2 EFFICIENCY STABILITY REGIONS EXTREMELY EFFICIENT DMUS:

(i) Efficiency Stability region – Type –I

To examine how robust an extremely efficient DMU is for increases of inputs, we solve super efficiency problems of DMU, which are input oriented.

$$Min\ \beta_k$$

$$subject\ to \sum_{\substack{j=1\\ j\neq 0}}^{n} \lambda_j\, x_{kj} \le \beta_k\, x_{k0}\ ,\ \sum_{\substack{j=1\\ j\neq 0}}^{n} \lambda_j\, x_{kj} \le x_{i0}\ ,\ \ i\neq k$$

$$\lambda_1 y_{r1} + \lambda_2 y_{r2} + ... + \lambda_n y_{rn} \ge y_{r0}\ ,\ \ r\in\{1,2,....s\} \qquad (2.1)$$

Let the optimal value of β_k be β_k^*.We solve as many super efficiency problems as there are inputs. Thrall.et.al [ ] proved that, $\beta_k > 1$, k=1, 2 …m. If k^{th} input alone is increased, the extremely efficient DMU_0 remains efficient, iff, $\beta_k \in (1, \beta_k^*)$, k is an integer from 1 to m. Let $\beta_k = \beta_k^*$. $\beta_k^* x_{k0}$ is an efficient input. $\beta_k \in (1, \beta_k^*)$, DMU remains to be efficient. Conversely, Let DMU_0 be efficient. But $\beta_k > \beta_k^*$ and β_k^* x_{k0} is efficient k^{th} output.

Consider the envelopment problem

$$Min\ \ \phi$$

$$subject\ to \sum_{j\neq 0} \lambda_j\ \ x_{kj} + \lambda_0\, x_{k0}\ \ \le \phi\, (\beta_k\, x_{k0})$$

$$\sum_{j\neq 0} \lambda_j\ \ x_{ij} + \lambda_0\, x_{i0} \le\ \phi\ x_{k0}\ ,\ i\neq k \qquad (2.2)$$

$$\sum_{j\neq 0} \lambda_j\ \ y_{rj} + \lambda_0\, y_{k0} \le y_{r0}\ ,\ \ r\in\{1,2...s\}$$

Since DMU_0 still remains to be extremely efficient,

$\lambda_j^* = 0\ , \forall\ j\neq 0$ and $\lambda_0^* = 1, \phi^* = 1$ and all slacks vanish.

From the first constraint, we have,

$$x_{k0} = \varphi^*\, \beta_k\, x_{k0}$$

$$\varphi^*\, \beta_k\ = 1 \qquad (2.3)$$

$$1 \le \beta_k^* < \beta_k\ \ \Rightarrow \beta_k > 1 \qquad (2.4)$$

From (2.3) and (2.4) it follows that $\theta^* < 1$. A contradiction If DMU_0 is efficient, $\beta_k \in (1, \beta_k^*)$.

Consider the unit output isoquant as shown below:

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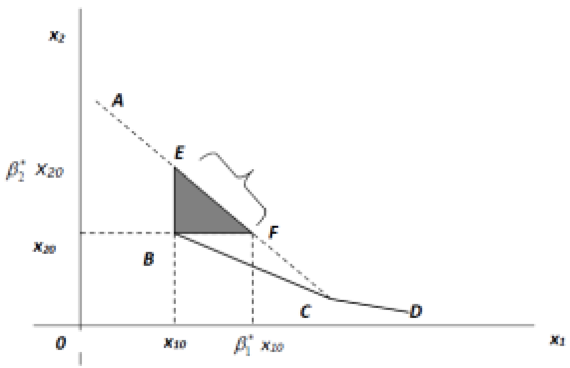

Fig: (2.1)

Decision making units A, B, C and D constitute unit output isoquant. Decision making unit B is removed from the reference set. The reduced isoquant is formed by the line segments AC and CD. For variation in the inputs such that,

$x_{10} \le \beta_1 x_{10} \le \beta_1^* x_{10}$,and $x_{20} \le \beta_2 x_{20} \le \beta_2^* x_{20}$

The extremely efficient DMU B (DMU_0) remains to be efficient. The shaded region is the input stability region.

(ii) Efficiency Stability Region – Type-II

Let $0 < \alpha_k \le 1$. We solve the following CCR model:

$$Max\ \alpha_k$$

Subject to $\sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j\, y_{kj} \ge \alpha_k\, y_{k0}$, $\sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j\, y_{rj} \ge y_{r0}$, $r \ne k$

$$\sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j\, x_{kj} \le x_{i0},\ i \in \{1,2,\ldots,n\} \text{ and } \lambda_j\ ,\ \alpha_k\ \ge 0 \qquad (2.5)$$

This optimization problem is solved for k=1, 2… n. Consequently we obtain α_r^*. If k^{th} input is decreased, the extremely efficient DMU continues to be efficient if and only if, $\alpha_k \in \{\alpha_k^* \le \alpha_k \le 1\}$

The proof is similar to the Type-I: Efficiency Stability region.

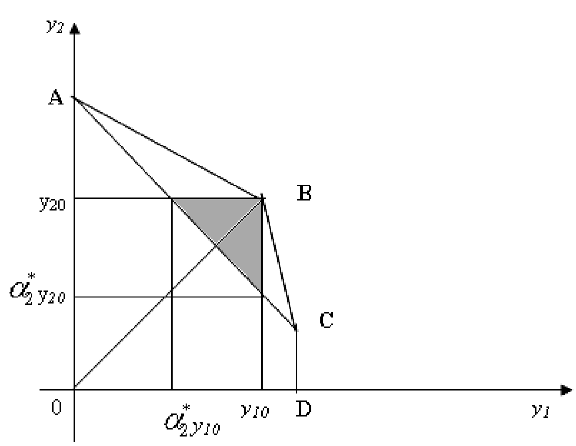

Fig-(2.2)

The shaded region is the region of efficiency stability for an extremely efficient DMU_0 operating at the point B. Thus, we have the following:

$$\Lambda = \{(\alpha_1, \alpha_2) : \alpha_1^* \le \alpha_1 \le 1\ , \alpha_2^* \le \alpha_n \le 1\ ,\ A_1\alpha_1 + A_2\alpha_2 \ge 1\}$$

More generally, if there are s outputs produced by an extreme efficient DMU_0, then we have

$$\Lambda = \{(\alpha_1, \ldots\ldots, \alpha_s) : \alpha_r^* \le \alpha_r \le 1,\ r = 1,2,\ldots\ldots, s,\ \ A_1\alpha_1 + A_2\alpha_2 + \ldots\ldots\ldots . + A_s\alpha_s \ge 1\}$$

(iii) Efficiency stability region – Type-III:

All outputs are reduced simultaneously, the extremely efficient DMU_0 stands efficient iff

$$(\alpha_1, \alpha_2, \ldots . \alpha_s) \in \Lambda = \{(\alpha_1, \ldots\ldots, \alpha_s) : \alpha_r^* \le \alpha_r \le 1,\ A_1\alpha_1 + A_2\alpha_2 + \ldots . + A_s\alpha_s \ge 1\}$$

$\alpha_1^* \le \alpha_1 \le 1$ and $\alpha_2^* \le \alpha_2 \le 1$

In the case of s outputs we have, the bounding hyper plane, given by

$A_1\alpha_1 + A_2\alpha_2 + \ldots\ldots\ldots A_s\alpha_s = 1$ (2.7)

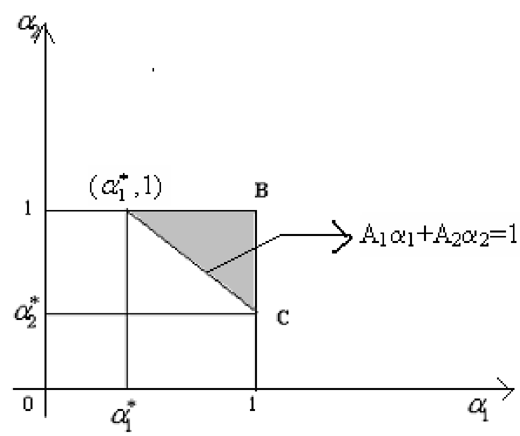

Fig: (2.3)

In the output space, we have the bounding hyper plane given by $G_1y_1 + G_2y_2 + \ldots\ldots\ldots + G_sy_s = 1$ (2.8)

Consider the following s output points which are hypothetical

α_1	α_2	…………….	α_s
α_1^*	1	………………	1
1	α_2^*	………………..	1
…	…	…………….	..
1	1	…………………..	α_s^*

Since the hyper plane (2.7) passes through the s points we have,

$$\left.\begin{array}{l} A_1\alpha_1^* + \ldots\ldots\ldots\ldots\ldots\ldots + A_s = 1 \\ - - - - - - - - - - - - - - - - - - \\ A_1 + \ldots\ldots\ldots\ldots\ldots\ldots + \alpha_s^* = 1 \end{array}\right\}$$

$$0 \le \hat{\beta}_i \le \hat{\beta}_i^* \Leftrightarrow 0 \le \frac{\hat{\beta}_i}{\hat{\beta}_i^*} \le 1 and\ 0 \le \hat{\alpha}_r \le \hat{\alpha}_r^* \Leftrightarrow 0 \le \frac{\hat{\alpha}_r}{\hat{\alpha}_r^*}$$

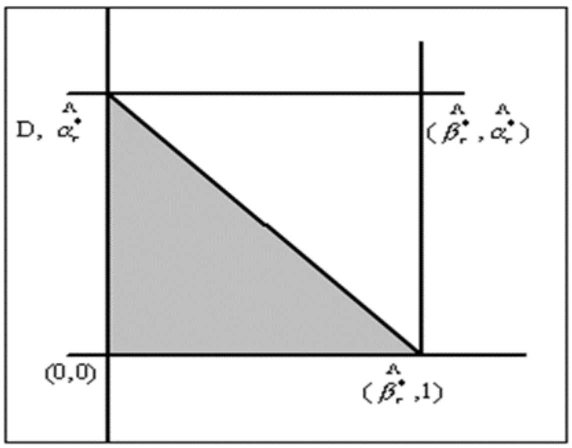

Fig: (2.4)

The shaded region is the region of efficiency stability of DMU_0. In two dimensions the bounding (hyper plane) straight line has the equation,

$D_i \hat{\beta}_i + E_i \hat{\alpha}_i = 1$. Since this passes through $\left(\hat{\beta}^*_i, 1\right)$ and $\left(1, \hat{\alpha}^*_i\right)$

$$D_i \hat{\beta}^*_i = 1 \Rightarrow D_i = \left(\hat{\beta}^*_i\right)^{-1}, E_i \hat{\alpha}^*_i = 1 \Rightarrow E_i = \left(\hat{\alpha}^*_i\right)^{-1}$$

Consequently, the bounding straight line takes the form,

$\frac{\hat{\beta}_i}{\hat{\beta}^*_i} + \frac{\hat{\alpha}_i}{\hat{\alpha}^*_i} = 1$. More generally, in $m+s$ dimensional space the bounding hyper plane can be expressed as follows:

$$\frac{\hat{\beta}_1}{\hat{\beta}^*_1} + \dots + \frac{\hat{\beta}_m}{\hat{\beta}^*_m} + \frac{\hat{\alpha}_1}{\hat{\alpha}^*_1} + \dots + \frac{\hat{\alpha}_s}{\hat{\alpha}^*_s} = 1 \quad (2.11)$$

The case that involves $\hat{\beta}_i$ and $\hat{\alpha}_i$ is same as the one that involves β_i, since the former case arises by shifting origin from (1, 1) to (0, 0), we have the efficient stability region for DMU_0, given by,

$$S = \left\{ \left(\hat{\beta}_1, \dots, \hat{\beta}_m, \hat{\alpha}_1 \dots \hat{\alpha}_s\right) : 0 \le \hat{\beta}_i \le \hat{\beta}^*_i,\ 0 \le \hat{\alpha}_r \le \alpha^*_r, \frac{\hat{\beta}_1}{\hat{\beta}^*_1} + \dots + \frac{\hat{\beta}_m}{\hat{\beta}^*_m} + \frac{\hat{\alpha}_1}{\hat{\alpha}^*_1} + \dots + \frac{\hat{\alpha}_s}{\hat{\alpha}^*_s} \le 1 \right\}$$

$\left(\hat{\beta}_1, \dots, \hat{\beta}_m, \hat{\alpha}_1, \dots \hat{\alpha}_s\right) \in S \quad \Rightarrow$ DMU_0 is efficient

We also have,

$$\pi = \left\{ (\beta_1, \dots, \beta_m, \alpha_1, \dots, \alpha_s) : 1 \le \beta_i \le \beta^*_i,\ \alpha^*_r \le \alpha_r \le 1, C_1\beta_1 + \dots + C_m\beta_m + C_{m+1}\alpha_{m+1}, \dots + C_{m+s}\beta_{m+s} \le 1 \right\}$$

$$(\beta_1, \dots, \beta_m, \alpha_1, \dots, \alpha_s) \in \pi \quad \Leftrightarrow \quad \left(\hat{\beta}_1, \dots, \hat{\beta}_m, \hat{\alpha}_1, \dots \hat{\alpha}_s\right) \in S$$

Where $C_i = \hat{\beta}^*_i$, $i \in \{1, 2, \dots, m\}$ and $C_j = \hat{\alpha}^*_i$, j=m+1…m+s

The Hyperplane (2.8) and the efficient outputs, which are s in number give the following system of conditions:

$$G_1 \alpha^*_1 y_{10} + G_2 y_{20} + \dots + G_s y_{s0} = 1$$
$$G_1 y_{10} + G_2 \alpha^*_2 y_{20} + \dots + G_s y_{s0} = 1 \quad (2.12)$$
$$G_1 y_{10} + G_2 y_{20} + \dots + G_s \alpha^*_s y_{s0} = 1$$

From the systems (2.10) and (2.12) we obtain,

$G_r = \frac{A_r}{y_{r0}}$, r =1, 2… s. The hyper plane (2.6) takes the form,

$$\frac{A_1}{y_{10}} y_1 + \frac{A_2}{y_{20}} y_2 + \dots + \frac{A_s}{y_{s0}} y_s = 1$$

Thus, we have found two sets Γ and Λ for which
$(\beta_1, \beta_2 \dots \beta_m) \in \Gamma \Leftrightarrow DMU_0$ preserves its input efficiency
$(\alpha_1, \alpha_2 \dots \alpha_s) \in \Gamma \Leftrightarrow DMU_0$ preserves its output efficiency
(iv)The condition $(\beta_1, \beta_2 \dots \beta_m, \alpha_1, \alpha_2 \dots \alpha_s) \in \pi$,
Where $\pi = \{(\beta_1, \beta_2 \dots \beta_m, \alpha_1, \alpha_2 \dots \alpha_s)$: $1 \le \beta_r \le \beta^*_i$, i =1, 2….m; $\alpha^*_r \le \alpha_r \le 1$, r=1,2,….s; $C_1\beta_1 + C_2\beta_2 + \dots + C_m\beta_m + C_{m+1}\beta_{m+1} + \dots + C_{m+s}\beta_{m+s} \le 1\}$ is sufficient for extremely efficient DMU_0 to preserve efficiency.

Define $\hat{\beta}_i = \beta_i - 1$, $i \in \{1, 2, \dots, m\}$ and $\hat{\alpha}_r = 1 - \alpha_r$ r = 1, 2,….,s, $1 \le \beta_I$ is less or equal to β^*_i $0 \le \beta_i - 1 \le \beta^*_i - 1$, $0 \le \hat{\beta}_i \le \hat{\beta}^*_i$, i = 1, 2…m

$\alpha^*_r \le \alpha_r \le 1$, ⇔ $-1 \le -\alpha_r \le -\alpha^*_r$ and $0 \le 1 - \alpha_r \le 1 - \alpha^*_r$

$0 \le \hat{\alpha}_r \le \hat{\alpha}^*_r$, r = 1, 2… s

Sub vectors of efficient units: Sometimes one may have interest in sub vectors of inputs. Efficiency stability has to be examined for a sub vector of inputs for an extremely efficient DMU, Viz., and DMU0

Define $\hat{x}_{i0} = \begin{cases} d\ x_{i0}, & d > 1, \quad i \in I \\ x_{i0}, & \quad i \notin I \end{cases}$

Consider the following CCR modified linear programming problem:
Min ρ
Subject to $\sum_{j \ne 0} \lambda_j x_{ij} \le \rho\ x_{i0}$

$\sum_{j \ne 0} \lambda_j x_{ij} \le x_{i0}$, $i \notin I$, $\sum_{j \ne 0} \lambda_j y_{rj} \ge y_{r0}$ and $\rho, \lambda_j \ge 0$

is infeasible in the presence of certain pattern of zero inputs and outputs, without loss of generality.

Consider
Min ρ
Subject to $\sum_{j\neq 0}\lambda_j x_{ij} \le \rho\, x_{i0}$, $i \in \{1,2,\ldots,m\}$ (3.2)

$\sum_{j\neq 0}\lambda_j y_{rj} \le y_{r0}$, $r \in \{1,2,\ldots,s\}$

ρ, λ_j are non-negative

Infeasibility: Let I_0= {i: x_{i0} = 0}, I_o^c = {i: x_{i0} ≠ 0}, π ={r: y_{r0} ≠ 0} and π_0^c ={r: y_{r0} = 0}

If the input oriented CCR problem is infeasible, there will be no k satisfying $I_0 \cap I_k = I_0$, $\pi_0 \cap \pi_k = \pi_0$. Not all inputs.

PROOF: Consider the CCR Problem,

Min ρ

Subject to $\sum_{j\neq 0}\lambda_j x_{ij} + \lambda_k x_{ik} \le \rho\, x_{i0}$, $i \in I$

$$\sum_{j\neq 0}\lambda_j y_{rj} + \lambda_k y_{rk} \ge y_{r0} \ ,r = 1,2,\ldots,s \quad (3.3)$$

Let the CCR Problem be infeasible. Let if possible $I_0 \cap I_k = I_0$, and $\pi_0 \cap \pi_k = \pi_0$.

$\sum_{\substack{j\neq 0\\ j\neq k}}\lambda_j x_{ij} \le 0$, $\forall i \in I_0$ and $\sum_{\substack{j\neq 0\\ j\neq k}}\lambda_j x_{ij} + \lambda_k x_{ik} \le \rho\, x_{i0}$, $\forall i \in I_0^c$

$\sum_{\substack{j\neq 0\\ j\neq k}}\lambda_j y_{rj} + \lambda_k y_{rj} \ge y_{r0}, \forall i \in \pi_0$ and $\sum_{\substack{j\neq 0\\ j\neq k}}\lambda_j y_{rj} \ge y_{r0}, \forall i \in \pi_0^c$

Let λ_j=0, j ≠ 0, k. Then the above system reduces to,

$\lambda_k x_{ik} \le \rho\, x_{i0}$, ∀ i∈ I_o^c and

$\lambda_k y_{rj} \ge y_{r0}$, ∀ r∈π_0 (3.4)

These constraints are feasible, so that (λ_k^c, ρ_0), be a feasible solutions to (3.4). Hence (λ_j = 0, $\lambda_k = \lambda_o^c$, ρ = ρ_0, j ≠ 0, k) is a feasible solution to (3.3). A contradiction, no k satisfies $I_0 \cap I_k = I_0$, and $\pi_0 \cap \pi_k = \pi_0$.

4. Infeasibility of type-(2):

Let DMU be such that, I_0= {i: x_{i0}=0}, and DMU_j be such that I_j= {i: x_{ij}=0}. Similarly, for outputs we have, H_0={r: $y_{r0 \neq 0}$} and H_j={r: y_{rj} ≠ 0}. If $I_0 \cap I_j \neq I_0$ for each j ≠ 0 or $H_0 \cap H_j \neq H_0$. For each j≠0 then the modified CCR input efficiency problem is infeasible.

Consider Min ρ

Such that $\sum_{j\neq 0}\lambda_j x_{ij} \le \rho\, x_{i0}$

$\sum_{j\neq 0}\lambda_j y_{rj} \ge y_{r0}$ and ρ, λ_j

Let $I_0 \cap I_j \neq I_0$ for each j ≠ 0.

Consider the constraint $\sum_{j\neq 0}\lambda_j x_{ij} + s_i = \rho\, x_{i0}$

where $s_j \ge 0$ is slack variable.

Let i ∈I_0 → x_{i0} = 0, consequently, we obtain,

$$\sum_{j\neq 0}\lambda_j x_{ij} + s_i = 0 \quad (4.1)$$

Consider the following data setup for i[th] constraint,

x_{i0}, x_{i1}, x_{i2}… x_{ij}, ….. x_{in}, j ≠ 0
(0), (>0) (>0) …..(>0)…. (>0)
x_{i0}=0→x_{ij} >0, ∀ j ≠ 0 (4.2)

From (4.1) and (4.2), it follows that s_i=0, λ_j equals zero, for all nonzero j. λ_j=0, ∀ j ≠ 0⇒ the LP problem is infeasible. In the light of the principle of CRTS, the new sensitivity analysis approach fails to work only when a certain pattern of zero inputs/outputs occurs

VRTS: $\sum_{j\neq 0}\lambda_j\ equals\ 1$ and NIRS: $\sum_{j\neq 0}\lambda_j \le 1$,

NDRS: $\sum_{j\neq 0}\lambda_j \ge 1$

Output based BCC problem with equal inputs
Max g
Such that $\sum_{j\neq 0}\lambda_j y_{rj} \ge g\ y_{r0}$, $r \in \{1,2,\ldots,s\}$

$\sum_{j\neq 0}\lambda_j = 1$ and λ_j = 0, g free

5. Infeasibility – type-(3):

Suppose g* be the extreme value of the above output problem with equal inputs. If g* <1, then the input based modified DEA measures under the constraints of VRS and NIRS are infeasible respectively.

Proof: Consider the following LP Problems:
Max g
Subject to $\sum_{j\neq 0}\lambda_j y_{rj} \ge g\ y_{r0}$ and $\sum_{j\neq 0}\lambda_j = 1$ (5.1)

Min ρ
Subject to $\sum_{j\neq 0}\lambda_j x_{ij} \le \rho\, x_{i0}$, $\sum_{j\neq 0}\lambda_j y_{rj} \ge y_{r0}$
and $\sum_{j\neq 0}\lambda_j = 1$ (5.2)

For each set of λ_j for which $\sum_{j\neq 0}\lambda_j = 1$ and $\sum_{j\neq 0}\lambda_j y_{rj} \ge y_{r0}$

We can always find a ρ such that, $\sum_{j\neq 0}\lambda_j x_{ij} \le \rho\, x_{i0}$,

Since the left hand side expression is finite in value and ρ is unrestricted.

Let λ_j^*, j ≠ 0 and ρ* be such a set of values. Thus we have,

$\sum_{j\neq 0}\lambda_j^* x_{ij} \le \rho\, x_{i0}$, $\sum_{j\neq 0}\lambda_j^* y_{rj} \ge y_{r0}$ and $\sum_{j\neq 0}\lambda_j^* = 1$, λ_j^*, j≠ 0, g=1 is a feasible solution to the constraints of (5.1). Then, g* = max g>1.But, by hypothesis g*<1. We cannot find λ_j for which $\sum_{j\neq 0}\lambda_j^* y_{rj} \ge y_{r0}$ .The input oriented LP Problem (5.2) is infeasible. The output constraint cannot be satisfied for any set of λ_j, j is not equal 0, for which $\sum_{j\neq 0}\lambda_j \le 1$ .g* <1 implies the input problem is infeasible under VRS ($\sum_{j\neq 0}\lambda_j = 1$), NIRS

($\sum_{j\neq 0}\lambda_j \leq 1$)

6. Infeasibility – type (4):

Consider the following linear programming problems:

h* = Min h

Subject to $\sum_{j\neq 0}\lambda_j \; x_{ij} \leq h \; x_{i0}$, i = 1, 2….n

$$\sum_{j\neq 0}\lambda_j = 1 \qquad (6.1)$$

g* =Max g

Subject to $\sum_{j\neq 0}\lambda_j^* \; x_{ij} \leq x_{i0}, \; i\in\{1,2,...,n\}$

$$\sum_{j\neq 0}\lambda_j^* y_{rj} \geq g \; y_{r0} \quad r\in\{1,2,...,s\}$$

$$\sum_{j\neq 0}\lambda_j = 1 \qquad (6.2)$$

Let h* >1. For each set of λ_j (j≠0) values for which $\sum_{j\neq 0}\lambda_j = 1$ and $\sum_{j\neq 0}\lambda_j^* \; x_{ij} \leq x_{i0}$, i = 1, 2… m, we can find g such that $\sum_{j\neq 0}\lambda_j^* y_{rj} \geq g \; y_{r0}$

Let λ_j^* (j≠ 0) and g* values such that

$\sum_{j\neq 0}\lambda_j^* \; x_{ij} \leq x_{i0}, i\in\{1,2,...,m\}$,

$\sum_{j\neq 0}\lambda_j^* y_{rj} \geq g^* \; y_{r0} \quad r\in\{1,2,..,s\}$ $\sum_{j\neq 0}\lambda_j = 1$ and λ_j^* (j≠ 0), h=1 satisfy the constraints of (6.1)

Consequently h*=Min h<1. But, by hypothesis h*>1. Therefore, we cannot find λ_j (j≠0) for which the constraint $\sum_{j\neq 0}\lambda_j^* \; x_{ij} \leq x_{i0}$ is satisfied. Hence under VRS the output oriented problem is infeasible.

If possible, suppose $\sum_{j\neq 0}\lambda_j^* \; x_{ij} \leq x_{i0}$, where

$$\sum_{j\neq 0}\lambda_j = \theta > 1$$

$$\frac{\lambda_j}{\theta}x_{ij} \leq \frac{1}{\theta}x_{i0}$$

$\sum_{j\neq 0}\frac{\lambda_j}{\theta}x_{ij} \leq \frac{1}{\theta}x_{i0} \leq x_{i0}$, where $\sum_{j\neq 0}\hat{\lambda}_j = 1$, Thus, there exists $\hat{\lambda}_j$, such that $\sum_{j\neq 0}\hat{\lambda}_j = 1$, for which the input constraint is satisfied, a contradiction. The problem is infeasible under NDRS condition. h* >1 ⇨ the output problem is infeasible under VRS and NDRS restrictions.

4 CONCLUSION

The above discussion proposes super efficiency problems of DMU which are input oriented to examine how robust an extremely efficient DMUs for increases of inputs. Different types of efficiency stability regions with their infeasibility have been presented exhaustively. In the context of future research one can propose an innovative method for the evaluation of slack based efficiency of a DMU in which Fractional Programming Problem (FPP) can be applied. Moreover some envelopment problems can be evaluated in order to identify extremely efficient DMUS

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