Group Decision Making Based On Laplacian Energy Of An Intuitionistic Fuzzy Graph

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Abstract: Group decision-making shows an significant role when allocating with decision making problems with the fast growth of society. The foremost determination of this paper is to show the reasonableness of some group decision making on the laplacian energy of an intuitionistic fuzzy graphs. we present numerical examples, including Alliance partner selection of a software company ,_Partner selection in supply chain management and the estimation of the outlines of reservoir action to illuminate the presentations of our planned concepts in result making to rank the best one..

Index Terms: Intuitionistic fuzzy graphs (IFGs); Laplacian energy; Alliance partner selection of a software company, Partner selection in supply chain management; schemes of reservoir operation..

1 INTRODUCTION

Group decision–making is one of the recycled tools in mortal being accomplishments, which calculated the optimal alternative from a given limited set of substitutes using the evidence given by group of conclusion makers or experts. Group decision-making plays an important role when dealing with decision making problems with the rapid growth of society. Previous many scholars have examined the attitudes for group decision-making based on different methods. Though in order to replicate the associations among the alternatives we need to make pair wise comparisations for all allotments in the progression of decision-making. Favourite relation is a influential quantitative decision technique that maintenances professionals in expressing their preferences over the given replacements. For a set of replacements $Z = \{z_1, z_2, ..., z_n\}$, the experts compare each pair of replacements and construct preference relations of every element in the preference relations is intuitionistic fuzzy number, then the concept of an Intuitionistic preference relationship (IPR) can be defined as follows :

Definition:-A Intuitionistic fuzzy preference retain on the set $Z = \{z_1, z_2, ..., z_n\}$ is denoted by a matrix $R = [\gamma_{ik}]_{n\times n}$ where $\gamma_{ik} = \left\langle z_j z_k, T(z_j z_k), F(z_i z_k)\right\rangle$. for all j, k=1,2,…,n. For suitability, let $\gamma_{jk} = \left\langle T_{jk}, F_{jk}\right\rangle$. where T_{jk} indicates the point to which the objective z_j is prepared to the objective z_k and F_{jk} is prepared as membership point with the circumstances $T_{jk}, F_{jk} \in [0,1]\ T_{jk} = F_{jk}\ T_{jj} = F_{jj} = 0\ for\ all\ j,k = 1,2,...,\ n$

A group decision-making problem regarding the "Alliance partner selection of a software company" is illuminated to exhibit the applicability of the projected perceptions of laplacian energy of an intuitionistic fuzzy graph in accurate development.

2 MAIN RESULTS

2.1 Alliance Partner Selection of a Software Company

East soft is one of the topmost five software companies in china. It proposals a rich assortment of commercial counting product engineering solutions, and associated to software products and stage and facilities. To progress the operation and attractiveness proficiency in the broad market, East soft strategies to found a planned alliance with a global corporation. After plentiful discussions, five transnational company would like to found a planned association with East soft; they are HP a_1, PHILIPS a_2, EMC a_3 ,SAP a_4 and LK a_5. To select the wanted planned alliance partner, three experts e_i $(i = 1,2,3)$ are invited to subsidize in the decision analysis, who originate from engineering management department, the human resources department and the finance department of East soft respectively. Established on their involvements, the specialists compare each couple of replacements and give separate judgements using the following IFPRS $R_i = [\gamma_{jk}^{(i)}]_{5\times 5}$ $(i = 1,2,3)$.

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The IFGS D_i corresponding to IFPRS $R_i\ (i = 1,2,3)$ given in table 1-3, are shown in figure.

***Table 1**: IFPR of expert from engineering management department*

	$a_1 R_2$	a_2	a_3	a_4	a_5
	a_1 (0,0)	(0.4,0.3)	(0.2,0.6)	(0.7,0.3)	(0.3,0.6)
	a_2 (0.3,0.4)	(0,0)	(0.7,0.3)	(0.4,0.4)	(0.1,0.5)
	a_3 (0.6,0.2)	(0.8,0.2)	(0,0)	(0.3,0.4)	(0.2,0.4)
	a_4 (0.3,0.7)	(0.4,0.4)	(0.4,0.3)	(0,0)	(0.3,0.3)
	a_5 (0.6,0.3)	(0.5,0.1)	(0.4,0.2)	(0.3,0.3)	(0,0)

***Table 2:** IFPR of expert fromhuman resources department*

	R_2 a_1	a_2	a_3	a_4	a_5
	a_1 (0,0)	(0.5,0.1)	(0.1,0.5)	(0.3,0.5)	(0.2,0.8)
	a_2 (0.1,0.5)	(0,0)	(0.5,0.4)	(0.6,0.1)	(0.4,0.6)
	a_3 (0.5,0.1)	(0.6,0.4)	(0,0)	(0.9,0.1)	(0.1,0.4)
	a_4 (0.5,0.3)	(0.1,0.6)	(0.3,0.7)	(0,0)	(0.8,0.2)
	a_5 (0.8,0.1)	(0.8,0.2)	(0.1,0.1)	(0.2,0.8)	(0,0)

Table 3: IFPR of expert from finance department

	R_3 a_1	a_2	a_3	a_4	a_5
	(0,0) a_1	(0.9,0.1)	(0.1,0.2)	(0.4,0.1)	(0.6,0.3)
	(0.7,0.2) a_2	(0,0)	(0.4,0.6)	(0.6,0.3)	(0.7,0.2)
	(0.2,0.1) a_3	(0.6,0.4)	(0,0)	(0.1,0.4)	(0.6,0.2)
	a_4 (0.1,0.4)	(0.4,0.6)	(0.4,0.1)	(0,0)	(0.6,0.3)
	a_5 (0.6,0.2)	(0.9,0.1)	(0.3,0.6)	(0.3,0.6)	(0,0)

The elements of the laplacian matrices of the IFGS $L(D_i) = R_i^L\ (i = 1,2,3)$ revealed in figure 1 are delivered in tables 4-6

***Table 4**: Essentials of the laplacian matrix of the IFPR D_1*

	$R_1^L a_1$	a_2	a_3	a_4	a_5
	a_1 (1.8,1.6)	(-0.4,-0.3)	(-0.2,-0.6)	(-0.7,-0.3)	(-0.3,-0.6)
	a_2 (-0.3,-0.4)	(2.1,1.0)	(-0.7,-0.3)	(-0.4,-0.4)	(-0.1,-0.5)
	a_3 (-0.6,-0.2)	(-0.8,-0.2)	(1.7,1.4)	(-0.3,-0.4)	(-0.2,-0.4)
	a_4 (-0.3,-0.7)	(-0.4,-0.4)	(-0.4,-0.3)	(1.7,1.4)	(-0.3,-0.3)
	a_5 (-0.6,-0.3)	(-0.5,-0.1)	(-0.4,-0.2)	(-0.3,-0.3)	(0.9,1.8)

***Table 5:** Essentials of the laplacian matrix of the IFPR D_2*

	$R_2^L a_1$	a_2	a_3	a_4	a_5
	a_1 (1.9,1.0)	(-0.5,-0.1)	(-0.1,-0.5)	(-0.3,-0.5)	(-0.2,-0.8)
	a_2 (-0.1,-0.5)	(2.0,1.3)	(-0.5,-0.4)	(-0.6,-0.1)	(-0.4,-0.6)
	a_3 (-0.5,-0.1)	(-0.6,-0.4)	(1.0,1.7)	(-0.9,-0.1)	(-0.1,-0.4)
	a_4 (-0.5,-0.3)	(-0.1,-0.6)	(-0.3,-0.7)	(2.0,1.5)	(-0.8,-0.2)
	a_5 (-0.8,-0.1)	(-0.8,-0.2)	(-0.1,-0.1)	(-0.2,-0.8)	(1.5,2.0)

***Table 6:** Essentials of the laplacian matrix of the IFPR D_3*

	$R_3^L a_1$	a_2	a_3	a_4	a_5
	a_1 (1.6,0.9)	(-0.9,-0.1)	(-0.1,-0.2)	(-0.4,-0.1)	(-0.6,-0.3)

d_2	(-0.7,-0.2)	(2.8,1.2)	(-0.4,-0.6)	(-0.6,-0.3)	(-0.7,-0.2)
d_3	(-0.2,-0.1)	(-0.6,-0.4)	(1.2,1.5)	(-0.1,-0.4)	(-0.6,-0.2)
d_4	(-0.1,-0.4)	(-0.4,-0.6)	(-0.4,-0.1)	(1.4,1.4)	(-0.6,-0.2)
d_5	(-0.6,-0.2)	(-0.9,-0.1)	(-0.3,-0.6)	(-0.3,-0.6)	(2.5,1.0)

The laplacian energy of each IFG is calculated as:

$$Spectrum\ of\ R_1^L\left(\gamma\left(D_1\right)\right) = \begin{bmatrix} 0, 1.3909, 2.7571, 1.9409 + 0.2040i, \\ 1.9409 - 0.2040i, 1.9272 \end{bmatrix}$$

$$LE\left(\mu\left(D_1\right)\right) = \left[3.9670\ \ , 3.0581\ \ \right]$$

$$Spectrum\ of\ R_2^L\left(\mu\left(D_2\right)\right) =$$

$$\left[0,\ 2.4080 + 0.433i, 2.4080 - 0.433i, 2.1640, 1.4201\right]$$

$$Spectrum\ of\ R_2^L\left(\mu\left(D_2\right)\right) = \begin{bmatrix} 0, 1.6906 + 0.257i, 1.6906 - 0.257i, \\ 2.0594 + 0.4218i, 2.0594 - 0.4218i \end{bmatrix}$$

$$LE\left(\mu\left(D_2\right)\right) = \left[4.1185\ \ , 3.5420\ \ \right]$$

$$Spectrum\ of\ R_3^L\left(\mu\left(D_3\right)\right) =$$

$$\left[0,\ 3.4918, 2.8238, 1.5922 + 0.1827i, 1.5922 - 0.1827i\right]$$

$$Spectrum\ of\ R_3^L\left(\mu\left(D_3\right)\right) =$$

$$\left[0,, 1.8039 + 0.2009i, 1.8039 - 0.2009i, 1.1207, 1..2716\right]$$

$$LE\left(\mu\left(D_3\right)\right) = \left[5.1315\ \ , 2.6238\ \ \right]$$

The heaviness of each expert can calculated as :

$$W_i = \left(\left(w_\mu\right)_i, \left(w_\gamma\right)_i\right) = \left[\frac{LE\left(\left(D_\mu\right)_i\right)}{\sum_{l=1}^{m} LE\left(\left(D_\mu\right)_l\right)}, \frac{LE\left(\left(D_\gamma\right)_i\right)}{\sum_{l=1}^{m} LE\left(\left(D_\gamma\right)_l\right)}\right]\ for\ i = 1,2,3,\dots, m$$

$$W_1 = \left(0.4838\ \ , 0.4247\ \ \right),$$

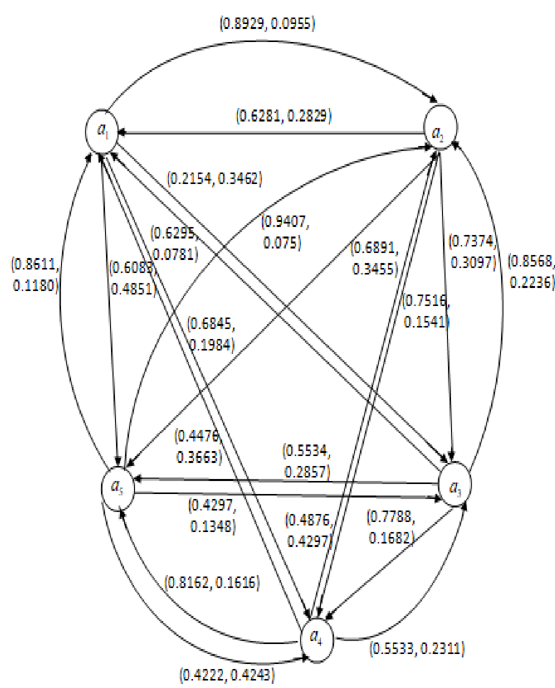
$$W_2 = \left(0.4903\ \ , 0.4723\ \ \right),\ W_3 = \left(0.5402\ \ , 0.4372\ \ \right)$$

based on which using IFWA operative, the fused IFPR is resolute as shown in table7.

Table 7: *The cooperative IFPR of all the above individual IFPRS*

R	a_1	a_2	a_3	a_5	a_5
a_1	(0,0)	(0.8929,0.0955)	(0.2154,0.3462)	(0.6845,0.1984)	(0.6083,0.4851)
a_2	(0.6581,0.2829)	(0,0)	(0.7374,0.3097)	(0.7516,0.1541)	(0.6891,0.3435)
a_3	(0.629,0.0781)	(0.8568,0.2236)	0,0)	(0.7788,0.1682)	(0.5534,0.2857)
a_4	(0.4676,0.3663)	(0.4876,0.4297)	(0.5533,0.2311)	0,0)	(0.8162,0.1616)
a_5	(0.8611,0.1180)	(0.9407,0.075)	(0.4297,0.1348)	(0.4222,0.4243)	0,0)

In the directed network compatible to a shared SVNPR overhead, we choice those intuitionistic statistics whose membership degrees $T_{jk} \geq 0.5\left(j,k = 1,2,3,4,5\right)$ and resultant incomplete illustration is exposed in figure.



Determine the out degrees $Out-d(a_j)(j=1,2,,3,4,5)$ of all criteria in a fractional absorbed network as follows:

$Out-d(a_1)=(2.1857,0.7790),$

$Out-d(a_2)=(2.8362,01.0921),$

$Out-d(a_3)=(2.818,0.7556)$

$Out-d(a_4)=(1.3695,0.3927),$

$Out-d(a_5)=(1.8018,0.193)$

Rendering to membership degrees of $Out-d(a_j)(j=1,2,,3,4,5)$, we have the position of the features $a_j\ (j=1,2,,3,4,5)$ as:

$$a_2>a_3>a_1>a_5>a_4$$

Thus the best choice is PHILIPS a_2.

2.2 In supply chain management ,Partner selection

Consider a problem regarding the variety of critical factors used to contact the potential partners of the company. Supply chain management depends on strategic relationship among establishments connected to supply chain. By actual organization, corporations help from lower cost, lower inventory levels, evidence distribution and thus stronger inexpensive edge. Many features may influence the management of companies. Between them the following is the list of four critical factors [ ].

Cf_1 : Response time and supply capacity

Cf_2 : Quality and technical skills

Cf_3 : Price and cost

Cf_4 : Service level

In order to rank the above four critical factors $Cf_i\,(i=1,2,3,4)$ we invited committee of three decision makers $e_k\,(k=1,2,3)$.These decision makers compare each pair of these factors and provide instuitionistic fuzzy preferences contained in the IFPRS $R_k=[\gamma_{ij}^k]_{4\times 4}(k=1,2,3)$ respectively.

$$R_1=\begin{bmatrix}(0,0)&(0.6,0.2)&(0.9,0)&(0.7,0.2)\\(0.5,0.4)&(0,0)&(0.3,0.7)&(0.8,0)\\(0,0.9)&(0.8,0.2)&(0,0)&(0.1,0.9)\\(0.4,0.6)&(0.5,0.4)&(0.9,0.1)&(0,0)\end{bmatrix}$$

$$R_2=\begin{bmatrix}(0,0)&(0.3,0.2)&(0.7,0.3)&(0.5,0.5)\\(0.9,0)&(0,0)&(0.7,0.2)&(0.1,0.7)\\(0.6,0.3)&(0.4,0.6)&(0,0)&(0.3,0.6)\\(0.8,0.2)&(0.7,0.1)&(0.6,0.3)&(0,0)\end{bmatrix}$$

$$R_3=\begin{bmatrix}(0,0)&(0.4,0.9)&(0.6,0.3)&(0.7,0.2)\\(0.9,0.1)&(0,0)&(0.7,0.3)&(0.1,0.2)\\(0.4,0.5)&(0.6,0.2)&(0,0)&(0.4,0.2)\\(0.5,0.2)&(0.6,0.2)&(0.4,0.5)&(0,0)\end{bmatrix}$$

$$R_1^L[\mu(G)]=\begin{bmatrix}0.9&-0.6&-0.9&-0.7\\-0.5&1.9&-0.3&-0.8\\0&-0.8&2.1&-0.1\\-0.4&-0.5&-0.9&1.6\end{bmatrix},$$

$$R_1^L[\gamma(G)]=\begin{bmatrix}0.9&-0.6&-0.9&-0.7\\-0.5&1.9&-0.3&-0.8\\0&-0.8&2.1&-0.1\\-0.4&-0.5&-0.9&1.6\end{bmatrix}$$

$$Spectrum\ of\ R_1^L\left[\mu(G)\right]=\{0,1.6562,2.4219+0.4818i,2.4219-0.4818i\}$$

$$Spectrum\ of\ R_1^L\left[\gamma(G)\right]=\{0,2.1113,1.4443+0.5244i,\ 1.4443-0.5244i\}$$

$$LE\left[R_1^L(\mu(G))\right]=\left|0-\frac{6.5}{4}\right|+\left|1.6562-\frac{6.5}{4}\right|+\left|2.4219+0.4818i-\frac{6.5}{4}\right|+\left|2,4219-0.4818i-\frac{6.5}{4}\right|$$

$$=1.625+0.0312+0.9312+0.9312=3.5186$$

$$LE\left[R_1^L\left(\gamma\left(G\right)\right)\right]=\left|0-\frac{5}{4}\right|+\left|2.1113-\frac{5}{4}\right|$$

$$+\left|1.4443+0.5244i-\frac{5}{4}\right|+\left|1.4443-0.5244i-\frac{5}{4}\right|$$

$$=1.25+0.8613+0.5592+0.5592=3.2297$$

$$LE\ [R_1(G)]=[3.5186\ ,3.2297\ ]$$

$$R_1^L[\mu(G)]=\begin{bmatrix}0.9 & -0.6 & -0.9 & -0.7\\ -0.5 & 1.9 & -0.3 & -0.8\\ 0 & -0.8 & 2.1 & -0.1\\ -0.4 & -0.5 & -0.9 & 1.6\end{bmatrix},$$

$$R_1^L[\gamma(G)]=\begin{bmatrix}0.9 & -0.6 & -0.9 & -0.7\\ -0.5 & 1.9 & -0.3 & -0.8\\ 0 & -0.8 & 2.1 & -0.1\\ -0.4 & -0.5 & -0.9 & 1.6\end{bmatrix}$$

$$Spectrum\ of\ R_1^L\left[\mu\left(G\right)\right]=$$
$$\{0,1.6562,2.4219+0.4818i,2.4219-0.4818i\}$$

$$Spectrum\ of\ R_1^L\left[\gamma\left(G\right)\right]=$$
$$\{0,2.1113,1.4443+0.5244i,\ 1.4443-0.5244i\}$$

$$LE\left[R_1^L\left(\mu\left(G\right)\right)\right]=\left|0-\frac{6.5}{4}\right|+\left|1.6562-\frac{6.5}{4}\right|$$

$$+\left|2.4219+0.4818i-\frac{6.5}{4}\right|+\left|2,4219-0.4818i-\frac{6.5}{4}\right|$$

$$=1.625+0.0312+0.9312+0.9312=3.5186$$

$$LE\left[R_1^L\left(\gamma\left(G\right)\right)\right]=\left|0-\frac{5}{4}\right|+\left|2.1113-\frac{5}{4}\right|$$

$$+\left|1.4443+0.5244i-\frac{5}{4}\right|+\left|1.4443-0.5244i-\frac{5}{4}\right|$$

$$=1.25+0.8613+0.5592+0.5592=3.2297$$

$$LE\ [R_1(G)]=[3.5186\ ,3.2297\ ]$$

$$R_2^L[\mu(G)]=\begin{bmatrix}2.3 & -0.3 & -0.7 & -0.5\\ -0.9 & 1.4 & -0.7 & -0.1\\ -0.6 & -0.4 & 2.0 & -0.3\\ -0.8 & -0.7 & -0.6 & 0.9\end{bmatrix},$$

$$R_2^L[\gamma(G)]=\begin{bmatrix}0.5 & -0.2 & -0.3 & -0.2\\ 0 & 0.9 & -0.2 & -0.7\\ -0.3 & -0.6 & 0.8 & -0.1\\ -0.2 & -0.1 & -0.3 & 1.8\end{bmatrix}$$

$$Spectrum\ \ of\ \ R_2^L[\mu(G)]=\{0,2.8245\ ,1.5755\ ,2.2\}$$

$$Spectrum\ \ of\ \ R_2^L[\gamma(G)]=\{0,0.7435\ ,1.3083\ ,\ 1.9482\ \}$$

$$LE\left[R_2^L\left(\mu\left(G\right)\right)\right]=\left|0-\frac{6.6}{4}\right|+\left|2.824-\frac{6.6}{4}\right|$$

$$+\left|1.5755-\frac{6.6}{4}\right|+\left|2.2-\frac{6.6}{4}\right|$$

$$=1.65+0.1745+0.0745+0.55=3.449$$

$$LE\left[R_2^L\left(\gamma\left(G\right)\right)\right]=\left|0-\frac{4}{4}\right|+\left|0.7435-\frac{4}{4}\right|$$

$$+\left|1.3083-\frac{4}{4}\right|+\left|1.9482-\frac{4}{4}\right|$$

$$=1.+0..2565+0.3083+0.9482=2.513$$

$$LE\ [R_2(G)]=[3.449\ ,2.513\ ]$$

$$R_3^L[\mu(G)]=\begin{bmatrix}1.8 & -0.4 & -0.6 & -0.7\\ -0.9 & 1.6 & -0.7 & -0.1\\ -0.4 & -0.6 & 1.7 & -0.4\\ -0.5 & -0.6 & -0.4 & 0.8\end{bmatrix},$$

$$R_3^L[\gamma(G)]=\begin{bmatrix}0.8 & -0.9 & -0.3 & -0.2\\ -0.1 & 1.3 & -0.3 & -0.9\\ -0.5 & -0.2 & 1.1 & -0.2\\ -0.2 & -0.2 & -0.5 & 1.3\end{bmatrix}$$

$$Spectrum\ of\ R_3^L\left[\mu\left(G\right)\right]=$$
$$\{-0.1395,1.4806,2.2795+0.2163i,2.2795-0.2163i\}$$

$Spectrum\ of\ R_3^L\left[\gamma(G)\right]=$

$\{0, 1.3516+0.4482i, 1.3576, -0.4482i, 1.7849\}$

$$LE\left[R_3^L(\mu(G))\right]=\left|-0.1395-\frac{5.9}{4}\right|+\left|1.4806-\frac{5.9}{4}\right|$$
$$+\left|2.2795+0.2163i-\frac{5.9}{4}\right|+\left|2.2795-0.2163i-\frac{5.9}{4}\right|$$
$$=1.6145+0.0056+0.8331+0.8331=3.2863$$

$$LE\left[R_3^L(\gamma(G))\right]=\left|0-\frac{4.5}{4}\right|+\left|1.3516+0.4482i-\frac{4.5}{4}\right|$$
$$+\left|1.3516-0.4482i-\frac{4.5}{4}\right|+\left|1.7849-\frac{4.5}{4}\right|$$
$$=1.125+0.5049+0.5349+0.5049=2.6697$$

$$LE\left[R_3(G)\right]=[3.2863\ ,2.6697\ ]$$

The weight of each decision maker $e_k\,(k=1,2,3)$ can be calculated as

$$W_i=\left(\frac{LE\left(R_1^L(e_1)\right)}{\sum_{i=1}^{n} LE\left(R_i(e_k)\right)_i},\frac{LE\left(R_2^L(e_1)\right)}{\sum_{i=1}^{n} LE\left(R_i(e_k)\right)_i},\frac{LE\left(R_3^L(e_1)\right)}{\sum_{i=1}^{n} LE\left(R_i(e_k)\right)_i}\right)$$

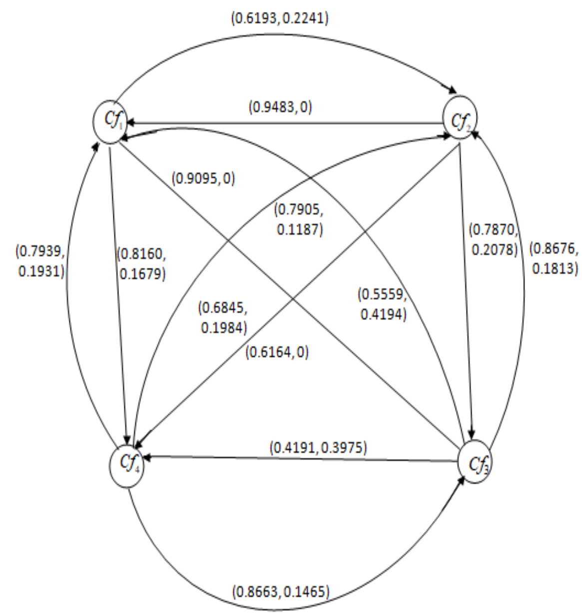
$W_1=(0.5214, 0.4786)$,

$W_2=(0.5784, 0.4216)$,

$W_3=(0.5517, 0.4483)$

Based on these weights, we determined the collective IFPR of all the above three individual IFPRs as shown in the table

	Cf1	Cf2	Cf3	Cf4
f1	(0,0)	(0.6193,0.2241)	(0.9095,0)	(0.8160,0.1679)
f2	(0.9483,0)	(0,0)	(0.7870,0.2078)	(0.6164,0)
f3	(0.5559,0.4194)	(0.8676,0.1813)	(0,0)	(0,4191,0.3975)
f4	(0.7939,0.1931)	(0.7905,0.1187)	(08663,0.1465)	(0,0)



In the directed network consistent to a combined IFPR above , we choice those intuitionistic fuzzy quantities whose relationship degrees $T_{jk}\geq 0.5(j,k=1,2,3,4)$ and subsequent limited diagram is exposed in the below figure

Figure: partial directed network of the fused IFPR

Estimate the out degrees $Out-d(Cf_j)(j=1,2,,3,4)$ of all standards in a limited directed network as follows:

$Out-d(Cf_1)=(2..3448\ ,0.392\ )$,

$Out-d(Cf_2)=(2..3517\ ,0.3078\ )$,

$Out-d(Cf_3)=(1.4235\ ,0.6007\ )$,

$Out-d(Cf_4)=(2.4505\ ,1.1781\ )$

By verifying the relationship degrees of $Out-d(Cf_j)(j=1,2,,3,4)$, we get the status of the factors $Cf_j\,(j=1,2,,3,4)$ as:

$$Cf_4 > Cf_2 > Cf_1 > Cf_3$$

Thus the best choice is Cf_4 (service level).

2.3 Calculation of the Outlines of Reservoir Operation

In this segment we focus on calculations the outline of reservoir operation. It is a water reserve system led by one of the reservoir with a complex situation and multiuse along with the anxious river basin and cascaded power stations in the river. The reservoir has been designated for many determinations such as power generation, irrigation total water supply for industry, residents, agriculture and environment etc. Due to dissimilar necessities for the divider of the amount of water, five reservoir process outlines x_1, x_2, x_3, x_4 and x_5 are recommended.

Scheme x_1: Supreme plant productivity, fulfilment of water usage in the river basin, lower and higher supply for society and the economy.

Scheme x_2: Supreme plant productivity, fulfilment of water usage in the river basin, upper and lower supply for society and the economy, lower supply for the ecosystem.

Scheme x_3: Supreme plant productivity, enough supply of water usage in the river basin, lower and higher supply for society and the economy, total supply for ecosystem and environment, 90% of which is passed down for flushing sands during low water periods.

Scheme x_4: Supreme plant productivity, enough supply of water usage in the river basin, lower and higher supply for society and the economy, total supply for ecosystem and environment, 50% of which is passed down for flushing sands during low water periods.

Scheme x_5: Supreme plant productivity, enough supply of water usage in the river basin, lower and higher supply for society and the economy, total supply for ecosystem and environment, during level and floods periods.

To choice the optimum outline, the administration selected four specialists $e_k(k=1,2,3,4)$ to calculate the five outlines. Based on their investigation, the specialists relate each couple of outlines and give distinct decisions using ensuing IFPRs $R_k = [\gamma_{ij}^{(k)}]_{5\times5}\ (k=1,2,3,4)$.

$$R_1 = \begin{bmatrix} (0,0) & (0.6,0.3) & (0.5,0.3) & (0.7,0.3) & (0.8,0.1) \\ (0.8,0.2) & (0,0) & (0.5,0.4) & (0.6,0.4) & (0.5,0.3) \\ (0.4,0.6) & (0.4,0.5) & (0,0) & (0.6,0.3) & (0.7,0.2) \\ (0.4,0.5) & (0.5,0.3) & (0.7,0.3) & (0,0) & (0.8,0.2) \\ (0.1,0.8) & (0.3,0.5) & (0.6,0.4) & (0.4,0.5) & (0,0) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} (0,0) & (0.7,0.2) & (0.8,0.2) & (0.7,0.3) & (0.7,0.1) \\ (0.2,0.7) & (0,0) & (0.5,0.5) & (0.6,0.4) & (0.5,0.3) \\ (0.4,0.5) & (0.7,0.3) & (0,0) & (0.4,0.6) & (0.7,0.3) \\ (0.6,0.4) & (0.2,0.8) & (0.6,0.4) & (0,0) & (0.5,0.3) \\ (0.3,0.7) & (0.4,0.5) & (0.6,0.3) & (0.3,0.5) & (0,0) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} (0,0) & (0.6,0.4) & (0.8,0.2) & (0.6,0.4) & (0.7,0.2) \\ (0.7,0.3) & (0,0) & (0.5,0.4) & (0.8,0.2) & (0.5,0.5) \\ (0.3,0.7) & (0.7,0.2) & (0,0) & (0.4,0.6) & (0.7,0.3) \\ (0.4,0.6) & (0.8,0.1) & (0.7,0.2) & (0,0) & (0.5,0.3) \\ (0.2,0.7) & (0.6,0.4) & (0.4,0.5) & (0.3,0.5) & (0,0) \end{bmatrix}$$

$$R_4 = \begin{bmatrix} (0,0) & (0.8,0.2) & (0.7,0.1) & (0.8,0.1) & (0.4,0.6) \\ (0.6,0.2) & (0,0) & (0.5,0.3) & (0.4,0.3) & (0.7,0.2) \\ (0.1,0.7) & (0.3,0.5) & (0,0) & (0.9,0.1) & (0.5,0.3) \\ (0.3,0.5) & (0.3,0.4) & (0.2,0.8) & (0,0) & (0.8,0.2) \\ (0.7,0.2) & (0.6,0.4) & (0.3,0.5) & (0.4,0.4) & (0,0) \end{bmatrix}$$

The laplacian matrices of IFDGs $L(D_k) = R_k^L\ (k=1,2,3,4)$

$$R_1^L[\mu(G)] = \begin{bmatrix} 1.7 & -0.6 & -0.5 & -0.7 & -0.8 \\ -0.8 & 1.8 & -0.5 & -0.6 & -0.5 \\ -0.4 & -0.4 & 2.3 & -0.6 & -0.7 \\ -0.4 & -0.5 & -0.7 & 2.3 & -0.8 \\ -0.1 & -0.3 & -0.6 & -0.4 & 2.8 \end{bmatrix},$$

$$R_1^L[\gamma(G)] = \begin{bmatrix} 2.1 & -0.3 & -0.3 & -0.3 & -0.1 \\ -0.2 & 1.6 & -0.4 & -0.4 & -0.3 \\ -0.6 & -0.5 & 1.4 & -0.3 & -0.2 \\ -0.5 & -0.3 & -0.3 & 1.5 & -0.2 \\ -0.8 & -0.5 & -0.4 & -0.5 & 0.8 \end{bmatrix}$$

$$Spectrum\ of\ R_1^L[\mu(G)] = \{0, 2.3609+0.0993i,\ 2.3609-0.0993i, 3.2250, 2.9532\}$$

$$Spectrum\ of\ R_1^L[\gamma(G)] = \{0, 2.3281, 1.3105, 2, 1.7613\}$$

$$LE\left[R_1^L\left(\mu(G)\right)\right]=\left|0-\frac{10.9}{5}\right|+\left|2.3609+0.0993i-\frac{10.9}{5}\right|+\left|2.3609-0.0993i-\frac{10.9}{5}\right|+\left|3.2250-\frac{10.9}{5}\right|+\left|2.9532-\frac{10.9}{5}\right|$$

$$=2.18+0.2063+0..2063+1.045+0.7732=4.3108$$

$$LE\left[R_1^L\left(\gamma(G)\right)\right]=\left|0-\frac{7.4}{5}\right|+\left|2.3281-\frac{7.4}{5}\right|+\left|1.3105-\frac{7.4}{5}\right|+\left|2-\frac{7.4}{5}\right|+\left|1.7613-\frac{7.4}{5}\right|$$

$$=1.48+0.8481+0.1695+0.52+0.2813=3.2989$$

$$LE\left[R_1(G)\right]=\left[4.4108\ ,3.2989\right]$$

$$R_2^L[\mu(G)]=\begin{bmatrix}1.5 & -0.7 & -0.8 & -0.7 & -0.7\\ -0.2 & 2.0 & -0.5 & -0.6 & -0.5\\ -0.4 & -0.7 & 2.5 & -0.4 & -0.7\\ -0.6 & -0.2 & -0.6 & 2.0 & -0.5\\ -0.3 & -0.4 & -0.6 & -0.3 & 2.4\end{bmatrix},$$

$$R_2^L[\gamma(G)]=\begin{bmatrix}2.3 & -0.2 & -0.2 & -0.3 & -0.1\\ -0.7 & 1.8 & -0.5 & -0.4 & -0.3\\ -0.5 & -0.3 & 1.4 & -0.6 & -0.3\\ -0.4 & -0.8 & -0.4 & 1.8 & -0.3\\ -0.7 & -0.5 & -0.3 & -0.5 & 1.0\end{bmatrix}$$

Spectrum of $R_2^L\left[\mu(G)\right]=$

$$\{0,2.27,2.5049+0.1375i,\ 2.5049-0.1375i,\ ,3.1203\}$$

Spectrum of $R_2^L\left[\gamma(G)\right]=$

$$\{0,2.4078+0.1615i,2.4078-0.1615i,2.0393,1.4450\}$$

$$LE\left[R_2^L\left(\mu(G)\right)\right]=\left|0-\frac{10.4}{5}\right|+\left|2.27-\frac{10.4}{5}\right|+\left|2.5049+0.1375i-\frac{10.4}{5}\right|+\left|2.5049-0.1375i-\frac{10.4}{5}\right|+\left|3.1203-\frac{10.4}{5}\right|=4.2034$$

.

$$LE\left[R_2^L\left(\gamma(G)\right)\right]=\left|0-\frac{8.3}{5}\right|+\left|2.4078+0.1615i-\frac{8.3}{5}\right|+\left\|2.4078-0.1615i-\frac{8.3}{5}\right\|+\left|2.0393-\frac{8.3}{5}\right|+\left|1.4450-\frac{8.3}{5}\right|=3.7844$$

$$LE\left[R_2(G)\right]=\left[4.2034\ ,3.7844\right]$$

$$R_3^L[\mu(G)]=\begin{bmatrix}1.6 & -0.6 & -0.8 & -0.6 & -0.7\\ -0.7 & 2.7 & -0.5 & -0.8 & -0.5\\ -0.3 & -0.7 & 2.4 & -0.4 & -0.7\\ -0.4 & -0.8 & -0.7 & 2.1 & -0.5\\ -0.2 & -0.6 & -0.4 & -0.3 & 2.4\end{bmatrix},$$

$$R_3^L[\gamma(G)]=\begin{bmatrix}2.3 & -0.4 & -0.2 & -0.4 & -0.2\\ -0.3 & 1.1 & -0.4 & -0.2 & -0.5\\ -0.7 & -0.2 & 1.3 & -0.6 & -0.3\\ -0.6 & -0.1 & -0.2 & 1.7 & -0.3\\ -0.7 & -0.4 & -0.5 & -0.5 & 1.3\end{bmatrix}$$

Spectrum of $R_3^L\left[\mu(G)\right]=$

$$\{0,2.2917,3.2788,\ 2.7375\ ,2.8920\}$$

Spectrum of $R_2^L\left[\gamma(G)\right]=$

$$\{0,2.6030,1.5130,1.8290,1.7551\}$$

$$LE\left[R_3^L\left(\mu(G)\right)\right]=\left|0-\frac{11.2}{5}\right|+\left|2.2917-\frac{11.2}{5}\right|+\left|3.2788.-\frac{11.2}{5}\right|+\left|2.7375-\frac{11.2}{5}\right|+\left|2.8920-\frac{11.2}{5}\right|$$

$$=4.48$$

$$LE\left[R_3^L\left(\gamma(G)\right)\right]=\left|0-\frac{7.7}{5}\right|+\left|2.6030-\frac{7.7}{5}\right|+\left|1.5130-\frac{7.7}{5}\right|+\left|1.8290-\frac{7.7}{5}\right|+\left|1.7551-\frac{7.7}{5}\right|$$

$$=3.1341$$

$$LE\left[R_3(G)\right]=\left[4.480\ ,3.1341\right]$$

$$R_4^L[\mu(G)] = \begin{bmatrix} 1.7 & -0.8 & -0.7 & -0.8 & -0.7 \\ -0.6 & 2.0 & -0.5 & -0.4 & -0.7 \\ -0.1 & -0.3 & 1.7 & -0.9 & -0.5 \\ -0.3 & -0.3 & -0.2 & 2.5 & -0.8 \\ -0.7 & -0.6 & -0.3 & -0.4 & 2.4 \end{bmatrix},$$

$$R_4^L[\gamma(G)] = \begin{bmatrix} 1.6 & -0.2 & -0.1 & -0.1 & -0.6 \\ -0.2 & 1.5 & -0.3 & -0.3 & -0.2 \\ -0.7 & -0.2 & 1.3 & -0.6 & -0.3 \\ -0. & -0.4 & -0.8 & 0.9 & -0.2 \\ -0.2 & -0.4 & -0.5 & -0.4 & 1.3 \end{bmatrix}$$

$$Spectrum\ of\ R_4^L\left[\mu(G)\right] =$$

$$\{0, 2.8884, 2.4310 + 0.1625i,\ 2.4310 - 0.1625i,\ 2.5495\}$$

$$Spectrum\ of\ R_4^L\left[\gamma(G)\right] =$$

$$\begin{Bmatrix} 0, 1.8393 + 0.3176i, 1.8393 - 0.3176, \\ 1.6607 + 0.0107i, 1.6607 - 0.0107i \end{Bmatrix}$$

$$LE\ \left[R_4^L(\mu(G))\right] = 41881$$

$$LE\ \left[R_3^L(\gamma(G))\right] = 3.0060$$

$$LE\ \left[R_3(G)\right] = \left[41881\ \ , 3.0060\ \ \right]$$

Then the mass of each proficient can designed as :

$$W_k = \left((W_\mu)_k, (W_\gamma)_k\right) = \left[\frac{LE\left((D_\mu)_k\right)}{\sum_{l=1}^{4} LE\left((D_\mu)_k\right)_l}, \frac{LE\left((D_\gamma)_k\right)}{\sum_{l=1}^{4} LE\left((D_\gamma)_k\right)_l}\right]$$

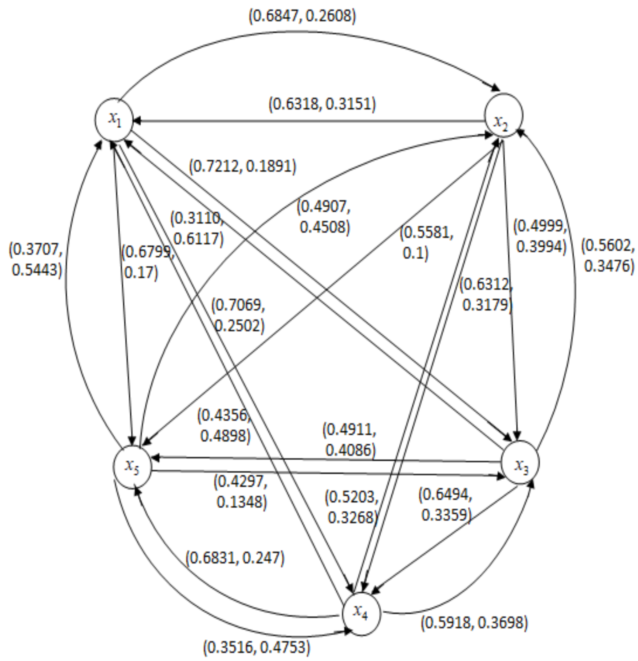
$$W_1 = (0.2552\ \ , 0.2494\ \ ), W_2 = (0.2432\ \ , 0.2862\ \ )$$

$$W_3 = (0.2592\ \ , 0.2370\ \ )\ W_4 = (0.2423\ \ , 0.2273\ \ )$$

Develop the amalgamation operative to use all the distinct IFPRs $R_k = \left(\gamma_{ij}^{(k)}\right)_{5\times5}(k = 1,2,3,4)$ into the combined IFPR $R = \left(\gamma_{ij}\right)_{5\times5}$.Here we relate the intuitionistic fuzzy subjective be around (IFWA) operator [ ] to use the distinct IFPR. Thus, we have

$$IFWA\ \left(\gamma_{ij}^{(1)}, \gamma_{ij}^{(2)}, ..., \gamma_{ij}^{(3)}\right) = \left(1 - \pi_{k=1}^{s}\left(1 - (\mu_{ij})^k\right)^{w_k}, \pi_{k=1}^{s}\left(\gamma_{ij}^{(k)}\right)^{w_k}\right)$$

$$R = \begin{bmatrix} (0,0) & \begin{pmatrix}0.6847, \\ 0.2608\end{pmatrix} & \begin{pmatrix}0.7212, \\ 0.1891\end{pmatrix} & \begin{pmatrix}0.7069, \\ 0.2502\end{pmatrix} & \begin{pmatrix}0.6799, \\ 0.17\end{pmatrix} \\ \begin{pmatrix}0.6318, \\ 0.3151\end{pmatrix} & (0,0) & \begin{pmatrix}0.4999, \\ 0.3944\end{pmatrix} & \begin{pmatrix}0.6312, \\ 0.3179\end{pmatrix} & \begin{pmatrix}0.5581, \\ 0.1\end{pmatrix} \\ \begin{pmatrix}0.3110, \\ 0.6117\end{pmatrix} & \begin{pmatrix}0.5602, \\ 0.3476\end{pmatrix} & (0,0) & \begin{pmatrix}0.6494, \\ 0.3359\end{pmatrix} & \begin{pmatrix}0.6604, \\ 0.27\end{pmatrix} \\ \begin{pmatrix}0.4356, \\ 0.4898\end{pmatrix} & \begin{pmatrix}0.5203, \\ 0.3268\end{pmatrix} & \begin{pmatrix}0.5918, \\ 0.3698\end{pmatrix} & (0.,0) & \begin{pmatrix}0.6831, \\ 0.247\end{pmatrix} \\ \begin{pmatrix}0.3707, \\ 0.5443\end{pmatrix} & \begin{pmatrix}0.4907, \\ 0.4508\end{pmatrix} & \begin{pmatrix}0.4911, \\ 0.4086\end{pmatrix} & \begin{pmatrix}0.3516, \\ 0.4753\end{pmatrix} & (0.,0) \end{bmatrix}$$



Calculate the out-degrees of all schemes of reservoir as :

$$out - d(x_1) = (2.7927\ \ , 0.8777\ \ ),$$

$$out - d(x_2) = (1.8211\ \ , 0.9418\ \ ),$$

$$out - d(x_3) = (1.87\ , 0.9746\ \ )$$

$$out - d(x_5) = (1.7952\ \ , 0.9439\ \ ),$$

$$out - d(x_1) = (0,0)$$

By verifying the membership degree of $out - d(x_j), (j = 1,2,3,4,5)$ as $x_1 > x_2 > x_3 > x_4 > x_5$

Thus the best choice is x_1 scheme in reservoir operation.

3. CONCLUSION

An Intuitionistic fuzzy model is cast-off in computer technology, communication, networking, when the thought of indeterminacy is current. In this paper, we have familiarized convinced novel ideas solicitation in group decision-making based on IFPRs is presented to illustrate the applicability of the proposed concepts of Intuitionistic Fuzzy Graphs. These discernments are also demonstrated with actual stage illustration. Also we recognize the status of the finest one.

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