Mathematical Thinking On Problem Solving And Self-Regulation Strategies Of Filipino Primary Grade Pupils

Lily G. Salangsang, Gener S.Subia

Abstract: This study explored the mathematical thinking on problem solving and self-regulation strategies of Filipino primary grade pupils in a school located in an urban area in Nueva Ecija, Philippines. The pupils solved arithmetic story problems in English. Results showed that the pupils were capable of solving story problems written in English and did much better when the problems were translated in Tagalog, their home language. They were able to solve multiplication and division problems prior to receiving formal instructions of these operations. Results also showed that the pupils employed self-regulated solution strategies like writing a number sentence, algorithm in subtraction, estimation, backward counting, trial and error, use of tally marks, making a drawing, skip counting, repeated addition, and invented algorithm. Because they were free to employ solution strategies of their own choice, invented algorithms and creativity came out from their works. The proficiency on the language in which the problem is written plays an important role in pupils' success in solving story problems in mathematics. The results of the study present relevant implications to problem solving pedagogies and mathematics instruction particularly in the primary level.

Index Terms: Algorithm, arithmetic, creativity, mathematical thinking, problem solving, proficiency, self-regulation strategies

1 INTRODUCTION

Problem solving is the most important aspect of learning mathematics. In the mathematics curriculum, early childhood mathematics is the basic foundation which paves the learner's foundation to a better and meaningful learning to solve problems in the upper grades and eventually in college. If schoolchildren learn to solve problems early, they will be more capable of solving a wide variety of complex and meaningful problems as they grow older. In a mathematics class, problem solving is a means to probe mathematical thinking of the learner. Mathematical thinking of children evolves from the different story problems that they can solve, and from the solution strategies that they can construct on their own. In the conventional way of teaching and learning problem solving, explanation is dominated by "teacher talk". The learners usually imitate the "teacher's way" of how to solve problems. The students spend their time copying definitions, mathematical formulas, problem solutions from chalk boards and recorded texts, and listening to the teacher's explanation until they "memorize" the information. Learners who have developed this conventional way of solving problems may not be a meaningful strategy to other and novel problem situations. The problem solving competence that the learner previously learned from memorized tasks would only be limited to problems of similar context and structure. Children think differently from adults. The children are naturally curious and have the intuitive ideas and skills to solve different types of problems. By looking into the capabilities of pupils to solve different problems even without direct instructions, their abilities may develop further and deeper.

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As a way of thinking, problem solving is not directly transmitted, but it rather develops progressively. If young learners are allowed to use or invent strategies that are meaningful to them and not forced to memorize procedures to follow, a problem solving schema is expected to emerge naturally. Problem solving is closely related to self-regulated learning [1]. According to [2], self-regulation is a skill that enables the learners to engage in learning activities and take charge of their thoughts, feelings, and actions that are planned, monitored, and reflected upon. Problem solving and self-regulation are both complex tasks that require the learners to be active and strategic problem solvers [3]. The learners tend to use self-regulated learning strategies while solving complex and challenging problems [4]. The author in [5] claimed that "meta-cognitive self-regulation can be central to success in problem solving because it enhances the selection and use of strategies for successful task completion". The study of [6] investigated elementary school learners' problem solving performances and their meta-cognitive self-regulatory behaviors. Findings of the study showed that while solving problems, the learners displayed different meta-cognitive behaviors such as suggesting a plan, assessing difficulty, reviewing progress, recognizing error, and self-questioning their thinking. Self-regulation may serve as a comprehensive framework for understanding how learners become active agents of their own learning process [7]. The use of self-regulation strategies is a crucial characteristic of, and significantly contributes to problem solving processes [8]. Zimmerman (2005) [2], added that motivation is also a core component of self-regulation. When a learner is motivated to achieve his/her goal, the greater is the ability to strive toward the achievement of that goal. When mathematical thinking in problem solving and self-regulation learning are determined early on among primary grade learners, these constructs develop progressively to adult life. This study explored the mathematical thinking in problem solving and the self-regulation strategies which Filipino primary grade learners used in the tasks given. Specifically, it answered the following questions:

1.What level of understanding do primary grade pupils have on different types of story problems?

2.What self-regulated solution strategies did they employ in solving the different story problems?
3.What problem solving schema emerges from the pupils' understanding and solution strategies displayed in the story problems?

2 METHODOLOGY

2.1 Research Design

Using a qualitative research design, case study type, the study was conducted at a primary grade school in an urban area in Nueva Ecija, Philippines.

2.2 Participants

Participants. Ten Grade 2 pupils, five boys and five girls, with above average, average, and below average academic performance were chosen purposively [9] as participants of the study. The participants were of the same age, and all speakers of Tagalog, their first language at home. Likewise the consent of the participants' parents was sought. Initially, there were 12 participants but two dropped out at the middle of the study when the parents had difficulty in fetching their children in school. In the study, the participants were coded to protect their identity.

2.3 Materials

Materials. The materials used were problem solving activity sheets, manipulatives, tape recorder, and a camera. Six story problems involving addition, subtraction, multiplication, and division problems were constructed. To determine the readability of the problems, these were tried out to five nonparticipant pupils enrolled during summer. The tape recorder was used to record responses of the participants when each was asked to clarify, explain and discuss his/her solution to each problem. A camera was also used to document some problem solving activities undertaken by the participants

2.4 Procedure

Procedure. Before actual data gathering, the researcher was allowed to observe during mathematics class. The researcher observed what concepts about addition, subtraction, multiplication, and division did the pupils hold and what problem types were they already exposed to. On some occasions, the researcher participated in some class activities like group games. The results of the observation helped the researcher in writing the story problems and in planning the data collection method. In the actual data gathering, and as agreed upon with the school principal, teacher, and the parents, the special class was conducted in the afternoon as their regular classes were held in the morning. On the first day of classes, the participants were oriented about the problem solving tasks. The story problems were all written in English. Everybody was excited about the problem solving tasks, the manipulative materials and the tape recorder. The pupils enjoyed doing the activities as each one of them was given the chance to record his/her voice while reading the problem, and when answering the questions asked by the researcher. On the first day, the data gathered got contaminated. The pupils who were first called to explain their work narrated to their classmates what transpired during the interview. The pupils who were last interviewed just repeated the given explanations of their peers. Because of what happened in the first day, the researcher decided to change the plan. The participants were given their new schedule and time slot, and each was individually met by the researcher. Different problems were given to each participant each day. This was purposely done so that the pupils outside the classroom waiting for their turn to be called could not get any lead or hint on the type of problem given as well as the answer to the problem. Data gathering was made this way. The researcher gave the activity sheet to the pupil and asked him/her to read the problem. After the pupil had finished reading, the researcher asked each pupil some questions to determine the pupil's level of understanding the problem. Some questions asked were: Can you tell me what the problem is all about? What is being asked for in the problem? On probing the pupils' thoughts on the problem solution and strategies employed, they were asked to clarify, explain or discuss the answer obtained. On the average, the researcher spent fifteen to thirty minutes with each pupil. Some pupils solved a problem using paper and pen, and others used manipulatives like sticks and beads. They also made tally marks and drawings. The pupils' understanding of the story problems, were described as very good, good, and poor. The pupils were rated with very good understanding if they understood the problem with no given translation in Tagalog, good understanding if the pupil asked help from the researcher in translating some English words to Tagalog; poor understanding, if the pupil asked the researcher to translate the problem from English to Tagalog. The solution strategies were described according to the solution made by the pupils to arrive at the correct answer to the problem.

3 RESULTS AND DISCUSSION

As demonstrated in this document, the numbering for sections upper case

1. Pupils' Level of Understanding on Different Story Problems

Three pupils had very good level of understanding the story problems written in English. They read the problems themselves. They translated the story problems correctly from English to Tagalog. They did not request for the translation of English words to Tagalog.Five pupils displayed good understanding of the story problems in English. They did not show complete understanding of the story problems especially the verbs with past tense like caught and bought. When these verbs were translated to Tagalog, they were able to proceed with the given tasks on their own and without the assistance of the teacher.Two pupils showed poor understanding of the story problems. They read the story problems slowly, and almost all problems were translated to them in Tagalog.Majority of the pupils revealed their difficulty in understanding the story problems written in English, and this eventually affected the pupils' ability to solve the story problems. Since their language at home was Tagalog, their access on the use of English language may be limited. Their limited proficiency in English may be the reason why they encountered difficulty in understanding the problem. The translation of some words in the problems, or the entire problem enabled them to understand better the context of the problems. Many problems were grasped completely by the pupils when these were translated to their home language. One of the possible reasons why the pupils have no full understanding of story problems in mathematics is because of poor reading comprehension. Unless a pupil has complete grasp of the

context and meaning of the problem, his/her succeeding efforts to solve the problem cannot be guaranteed. This finding agrees to the research findings of [10] and [11], that problem solving and reading disabilities affect their learning.The pupils' difficulty in developing their proficiency in English can also be traced from their home environment. At present, most pupils are seen engaged in playing games, using computers, cell phones, and other electronic devices once they get home, instead of sitting down and read a book in their spare time. A lot of families, specially, with parents working, have no so much time to sit down with their children to read with them or listen while reading.

2. Self- Regulated Solution Strategies Employed by the Pupils

The self-regulated strategies employed by the pupils in solving the different types of problem were the following:

2.1 Writing a number sentence

> Problem 1
> *Aling Nita sells fruits every day. One day she sold 86 bananas. The next day, she sold 25 apples. How many fruits did Aling Nita sell in two days?*

Problem 1. was solved by all pupils by writing a number sentence. Almost all of them translated the problem into a number sentence in vertical form. The concept of "carrying" was evident in their solutions.

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  1
 86
+25
111   fruits
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One of the pupils [Pupil D] explained her work this way.
R: What did you do?
P: Pinag-plus. (She added 6 and 5 with fingers, then wrote 1 on top of 8)
R: What is that 1 written on top of 8?
P: "Carry" 1.
R: Bakit ka nag-carry 1? (Why did you put carry 1?)
P: Pag pinag-add yung 5 and 6, 11 ang sagot dito. (When 5 and 6 are added, the sum is 11). May one and tens. (The sum has ones and tens digit). Pag hindi nag-carry sa itaas ng 8, magiging 1011. [If 1 is not written on top of 8, the number becomes 1011]. The addition algorithm given by the ten pupils was similar. Their solution showed their previous knowledge on the ones and tens digit of a number, and a procedural way of adding two-digit numbers. However, majority of them were not able to explain further, the meaning of the ones and tens digit. This seems to suggest that pupils may know how to perform the algorithm on addition without understanding the underlying concept. Other pupils explained that the addition algorithm was used because it is difficult to count large numbers with fingers (mahirap ibilang sa kamay yung 86 at 25) . In an interview with other pupils, it was found out that some can mentally get the sum of two ones-digit number, while others used their fingers in counting. When the numbers involved are large, they employed the addition algorithm using paper and pen to obtain the correct answer.

2.2 Algorithm on Subtraction

> Problem 2
> *There are 35 pupils who join in the Sports fest. Some of them win in the games. There were 15 pupils who did not win. How many pupils win in the games?*

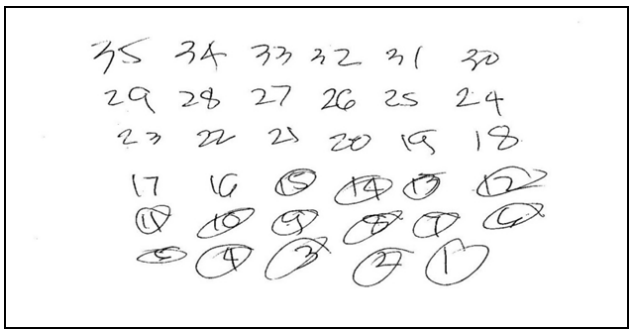
Problem 2 was also solved by eight pupils using the algorithm on subtraction. In this algorithm, the lesser number is taken from the greater. Many pupils subtracted 15 from 35 and the difference obtained was 20.

2.3 Estimation

Problem 2 was solved by Pupil H using estimation. The explanation given by the pupil was as follows: "15 plus 15 ay 30.[The pupil was able to figure out that there are two 15s in 30]. Dinagdagan ko ng 5 yung 15 para maging 20 (I added 5 to 15 to make it 20). Twenty yung nanalo". (Twenty pupils won). In this strategy, the pupil proposed an approximate answer within which the right answer might fall, and the assumed answer was check against the given conditions in the problem.

2.4. Backward Counting

Pupil E solved Problem 2 by using backward counting. In her solution, she wrote down the numbers from 35 to 1. Then she circled the numbers from 15 to 1. She obtained the answer 20 by counting the numbers not circled.



2.5 Trial and Error

> Problem 3
> *Mang Nitoy went fishing one day. He caught 25 hito [cat fish and some tilapia[another fish variety]. He caught 70 fish in all. How many tilapia did he catch that day?*

Pupil F solved Problem 3 by trial and error. When he first read the problem, he said "ang hirap yata" [The problem seems difficult]. He read the problem three times until he finally got the answer of 45 tilapia. He explained his work this way.

P: Pag 50 and sagot sobra ng lima. (If 50 is the answer, there is an excess of 5)
R: Why do you say so?
P: Kasi 50 plus 25 ay 75 (50 added to 25 is 75)
R: And so?
P: Babawasan yung 50 ng 5 para maging 75 . Ayun 45.(Subtract 5 from 50 to get 45)

R: Ano ang sagot sa tanong? (What is the answer to the question?)

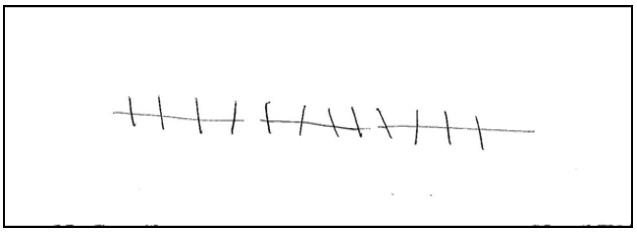
P: 45 na tilapia. (45 pieces of tilapia)

In his strategy, he approximated the range within which the correct answer might fall. He adjusted this number, until the desired answer is obtained when checked in relation to the problem.

2.6 Partitioning and Use of Tally Marks

Problem 4

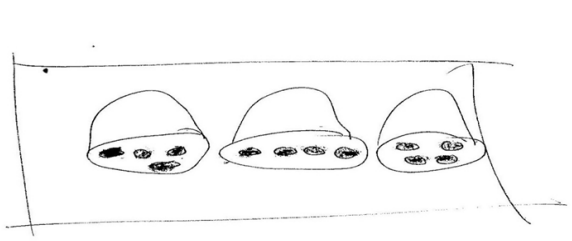
Shaira has 12 oranges. She puts 4 oranges in each bag. How many bags did she fill?



In the work of Pupil B, she drew 12 tally marks. She counted four tally marks. She counted another four tally marks and did the same. She did the same thing for the third time. Then she answered "three bags". Eight pupils used pop sticks to model the problem. They counted 12 sticks, and made three groups of 4 sticks. They based their answer on the number of groups formed.

2.7 Making a Drawing

Pupil D solved Problem 4 through the use of a drawing. Before she did this, she asked the researcher if she can draw and color the drawing. Her work is as follows.



While the pupil was making a drawing, the researcher observed that she was doing this with enjoyment. When she was coloring the bags that she drew, smiles were seen on her lips. After finishing her work, she said that she drew three baskets each filled with four oranges. This strategy employed by Pupil D shows her artistic inclination.

2.8 Skip Counting

Problem 5

Shan has 4 boxes of crayons. There are 10 crayons in each box. How many crayons does he have in all?

Problem 5 was solved by five pupils using skip counting. They used their fingers to do this. They represented the four boxes of crayons with four fingers. For the first finger, they counted 10, 20 for the second finger, 30 for the third finger, and 40 for the fourth finger.

2.9 Repeated Addition

Pupil B solved Problem 5 by repeated addition. She wrote four tens on her answer sheet this way.

10 + 10 + 10 + 10

She explained that she added the first two tens (10 + 10) to get 20. From this sum (20), she added the third ten (20+ 10), and got 30. From 30 she added 10 (30 + 10) and got 40. Five of the pupils used the same strategy in solving Problem 5.

2.10 Modeling

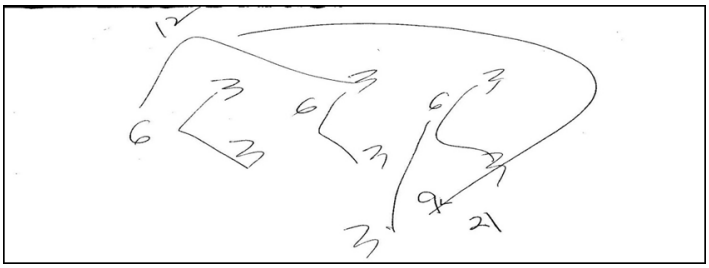
Problem 6

Grace has a project in art. She has 7 envelopes of art paper. Each envelope contains 3 art papers. How many art papers does she have in all?

Problem 6 was solved by nine pupils by modeling. This strategy is similar to acting out the problem. They used pop sticks to make seven groups of three. After doing this, they counted all the pop sticks used and got a total of 21.

2.11 Invented Algorithm

Pupil I solved Problem 6 with an invented algorithm. His work is shown in the figure.



In his work sheet, he wrote seven 3's. He first combined the three pairs of 3's, and each pair has a sum of 6. The two 6's were added and the sum is 12. The remaining 6 was added to 3 and wrote a sum of 9. The sum of 9 was added to 12 to get 21. The work of the pupil manifests his previous knowledge of adding two one digit numbers, and one and two digit numbers. The solution strategies employed by the pupils show the absence of the conventional and formal procedure of multiplying two one –digit numbers, but this fact did not hinder the pupils to arrive at the correct answer to the story problem.

The different solution strategies that the pupils constructed in solving different types of story problems show that these came out naturally based on their existing knowledge, and prior experiences. The solution strategies which they constructed on their own revealed a continuous process of self-regulation, self-motivation, and autonomy. These results agree to the results of the studies conducted by [5], [6], [7] and [8]. Despite

the absence of formal instruction on how to solve multiplication and division story problems, this did not hinder the pupils' inherent cognition and self –construction and regulated solution strategies that make sense to them. The pupils' lack of reading comprehension influences their ability to construct and complete the viable solution the problem. This only proves that a pupil cannot proceed and arrive at the correct answer to the question asked for in the story problem if he/she does not understand what the problem is all about.

3. Problem Solving Schema Emerging from the Pupils' Understanding and Solution Strategies on the Story Problems

The pupils' understanding on the story problems and the solution strategies that they employed revealed their problem solving schema. Results of the study show that the language used in stating the story problem has influence the pupils' ability to grasp the meaning and the context of the story problems. The findings also indicate that early language experiences are important in developing mathematical concepts and representations of the story problems. In other words, proficiency in the language in which the story problem is written matters in word problem solving. In the primary grade level, linguistic scaffolding had to be done to help pupils in solving word problems. In the solution strategies constructed by the pupils, the conventional strategy of writing number sentences was found evident in addition and subtraction problems. The problem solving schema of the pupils on addition and subtraction problems were also determined from the other solution strategies constructed by the pupils like estimation, drawing and backward counting, trial and error. In multiplication and multiplication story problems, the pupils solved the problems creatively by making drawing, skip counting, repeated addition, modeling, and inventing algorithm even if no formal instruction was given to them. These actions demonstrated by the pupils support the theory of Piaget (cited in [12]) which tells that in the concrete operational stage (6 to 12 years) there is a need for physical manipulation in the study of mathematics. These meaningful actions that they performed in this stage will later transform into mental structures. Henceforth, pupils will be capable of understanding abstract concepts without the use of physical representations. Their problem solving schema was clearly defined and organized despite of the fact that they have not yet received prior formal instruction. In the constructed strategies that they made, the explanations that they gave clearly show the essence of why and how they did them. The use of drawing and invented algorithms shown by some pupils explains the level of motivation, creativity and maturity in solving story problems. Based on these results, primary pupils have already existing problem solving schema which they develop on their own.

4 CONCLUSION

Reading proficiency, comprehension and self-regulated strategies play important roles in pupils' mathematical thinking on problem solving. Learning how to read and comprehend a problem in a language (English), not only their native dialect, is equally important as learning mathematics. Knowing the vocabulary and understanding the problem context help the learners learn problem solving and self-regulate the strategies most meaningful and interesting to them. Emphasis on the reading abilities of the learners has to be given focus in the mathematics curriculum in the primary level which to help the learners reduce their difficulties (Subia, 2018) in the subject.

5 ACKNOWLEDGMENT

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