M^X/G/1 Retrial Queueing Model With Pre-Emptive Resume Priority, Multi Optional Second Phase, Feedback And Orbital Search

M. Nila, Dr. D. Sumitha

Abstract: This paper is concerned with M^X/G/1 retrial queueing model with pre-emptive resume priority, multi optional second phase and feedback. Priority customers are considered while the server is in service. Two stages of heterogeneous service, first essential service (FES) and second multi optional service (SMOS) are rendered. After termination of each service, with certain probability the customer may go to the retrial queue to repeat his service again, or may leave the system. After finishing service, with certain probability the server check whether there is any customers in the retrial queue. Steady state solutions and various performance measures are derived.

Index Terms: Retrial queue, Batch arrival, Steady state, Pre-emptive resume priority, Multi optional second phase, Feedback, Orbital search.

1. INTRODUCTION

Retrial queueing models are widely applied in telecommunication networks, call centers, switching systems and packet switch networks. Many researchers have studied queueing networks with the concept of pre-emptive resume priority and feedback. Krishna Kumar et. al. (2002) investigated two phases of retrial queue model with pre-emptive priority. Rajadurai et. al. (2016) analyzed pre-emptive priority retrial queue with immediate Bernoulli feedback under working vacations. Pankaj sharma (2018) considered the retrial queueing system with Bernoulli feedback and modified vacation. Chen et. al. (2015) dealt retrial queue with Bernoulli vacation, priority and customers feedback. Gao (2015) discussed a pre-emptive retrial queue with two types of customers. Many researchers have discussed the queueing models with two phases of service. Kirupa and Udaya Chandrika (2015) studied bulk arrival retrial queue with negative customers, optional service and feedback. Madhan (2018) illustrated single server retrial queue providing different types of essential service and optional service. Arrar et. al. (2018) examined retrial queue with priority customers. Ayyappan and Tamizhselvi (2017) evaluated the transient analysis of bulk arrival general service retrial queueing system with priority, feedback, orbital search and modified vacation. Sumitha and Udaya Chandrika (2012) analyzed retrial queue with vacation and orbital search. In this present paper, we consider M^X/G/1 retrial queueing model with pre-emptive resume priority, multi optional second phase, feedback and orbital search. The model is described in section 2. The steady state solution and performance measures are obtained in section 3.

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2 MODEL DESCRIPTION

We discuss M/G/1 retrial queue with pre-emptive resume priority and feedback. Consider batch arrival of customers according to Poisson distribution of rate λ. Let Y be the random variable, batch size of the system with P{Y=k} = Ck, k = 0, 1, 2, 3.... $\sum_{i=1}^{\infty} C_k = 1$, the probability generating function (PGF) C(z) having moments m_1 and m_2.

If the incoming batch finds the server idle, then the primary customer in the batch receives the service immediately and rest of them join the retrial queue to make their retrials later. Retrial time is assumed to follow a general distribution with distribution function (CDF) A(x), probability density function(PDF) a(x), Laplace Stieltjes transform (LST) A*(s) and hazard rate function

$\eta(x) = a(x) / 1-A(x)$. The server gives FES to all customers. During the essential service, the arriving priority customer either sends the existing customer in service to the retrial queue with probability τ to get his service or may go to the retrial queue with complementary probability. FES times are assumed to follow general distribution with CDF $B_0(x)$, PDF $b_0(x)$, LST $B_0^*(s)$, having two moments μ_{01} and μ_{02} and hazard rate function $\mu_0(x) = b_0(x) / 1 - B_0(x)$. After completion of FES, the customer may depart the system with probability r_0, opt anyone of the SMOS with probability r_i (for i= 1 to M) or move to the retrial queue as a feedback customer with probability $\beta (1 - r_0 - \sum_{i=1}^{M} r_i)$.

At the completion of SMOS, the customer may rejoin to the retrial queue to repeat his service with prob δ or may depart the system. Multi optional second phase service times are assumed to follow general distribution with CDF $B_i(x)$, PDF $b_i(x)$, LST $B_i^*(s)$, having two moments μ_{i1} and μ_{i2} and hazard rate function $\mu_i(x) = b_i(x) / 1 - B_i(x)$, for i= 1 to M. After executing each service, the server quests for the customers in the retrial queue with probability θ or remains idle.

Define the Markov process:

$\{N(t); t \geq 0\} = \{S(t), C(t), \varepsilon_0(t), \varepsilon_1(t), \varepsilon_2(t); t \geq 0\}$

S(t) = Number of customers in the orbit at time t, C(t) = Number of servers .

C(t) = 0 idle server at time t.
= 1 busy server in FES at time t.
= 2 busy server in SMOS at time t.

Define probability densities

$R_n(x, t)dx = P\{C(t)=0, S(t)=n, x \le \varepsilon_0(t) < x + dx\};$

$t \ge 0, x \ge 0, n \ge 1$

$W_n(x, t)dx = P\{C(t)=1, S(t)=n, x \le \varepsilon_1(t) < x + dx\};$

$t \ge 0, x \ge 0, n \ge 0$

$P^i{}_n(x,t)dx = P\{C(t)=2, S(t)=n, x \le \varepsilon_2(t) < x + dx\};$

$t \ge 0, x \ge 0, n \ge 0, 1 \le i \le M$

3 STEADY STATE DISTRIBUTION

Steady state equations governing the model is given below

$$\lambda R_0 = r_0 \int_0^\infty W_0(x)\mu_0(x)\,dx + \bar{\delta} \int_0^\infty \sum_{i=1}^{M} P_0^i(x)\mu_i(x)\,dx \quad (1)$$

$$\frac{d}{dx} R_n(x) = -[\lambda + \eta(x)]R_n(x), \; n \ge 1 \quad (2)$$

$$\frac{d}{dx} W_n(x) = -[\lambda + \mu_0(x)]W_n(x) + (1-\tau)\lambda \sum_{k=1}^{n} C_k W_{n-k}(x), \quad n \ge 0 \quad (3)$$

$$\frac{d}{dx} P_0^i(x) = -[\lambda + \mu_i(x)]P_0^i(x), \quad \text{for } i = 1 \text{ to } M \quad (4)$$

$$\frac{d}{dx} P_n^i(x) = -[\lambda + \mu_i(x)]P_n^i(x) + \lambda \sum_{k=1}^{n} C_K P_{n-k}^i(x), \quad n \ge 1, \text{for } i = 1 \text{ to } M \quad (5)$$

with boundary conditions

$$R_n(0) = r_0\bar{\theta} \int_0^\infty W_n(x)\mu_0(x)\,dx + \beta \int_0^\infty W_{n-1}(x)\mu_0(x)dx + \bar{\theta}\bar{\delta} \int_0^\infty \sum_{i=1}^{M} P_n^i(x)\mu_i(x)\,dx + \delta \int_0^\infty \sum_{i=1}^{M} P_{n-1}^i(x)\mu_i(x)\,dx, \quad n \ge 1 \quad (6)$$

$$W_o(0) = \lambda c_1 R_0 + \int_0^\infty R_1(x)\eta(x)dx + r_0\theta \int_0^\infty W_1(x)\mu_0(x)\,dx + \theta\bar{\delta} \int_0^\infty \sum_{i=1}^{M} P_1^i(x)\mu_i(x)\,dx \quad (7)$$

$$W_n(0) = \lambda \int_0^\infty \sum_{k=1}^{n} C_K R_{n-k+1}(x)dx + \int_0^\infty R_{n+1}(x)\,\eta(x)\,dx + \lambda c_{n+1} R_0 + r_0\theta \int_0^\infty W_{n+1}(x)\,\mu_0(x)\,dx + \tau\lambda \int_0^\infty \sum_{i=1}^{M} C_k W_{n-k}(x)\,dx + \theta\bar{\delta} \int_0^\infty \sum_{i=1}^{M} P_{n+1}^i(x)\mu_i(x)dx, \; n \ge 1 \quad (8)$$

$$P_n^i(0) = r_i \int_0^\infty W_n(x)\mu_0(x)\,dx, \; n \ge 0, \; \text{for } i = 1 \text{ to } M \quad (9)$$

The normalizing condition is

$$R_0 + \sum_{n=1}^{\infty} \int_0^\infty R_n(x)dx + \sum_{n=0}^{\infty} \int_0^\infty W_n(x)dx + \sum_{n=0}^{\infty} \int_0^\infty \sum_{i=1}^{M} P_n^i(x)dx = 1 \quad (10)$$

Define the PGF

$$R(x, z) = \sum_{n=1}^{\infty} R_n(x)z^n, \quad W(x,z) = \sum_{n=0}^{\infty} W_n(x)z^n,$$

$$P_i(x,z) = \sum_{n=0}^{\infty} P_n^i(x)z^n, \quad \text{for } i = 1 \text{ to } M$$

Multiply by z^n to the equations (1) – (9) and taking summation over n, n=0, 1, 2, 3 …we obtain the differential equations.

$$\left(\frac{d}{dx} + \lambda + \eta(x)\right) R(x,z) = 0 \quad (11)$$

$$\left(\frac{d}{dx} + \lambda(1 - \bar{\tau}c(z)) + \mu_0(x)\right) W(x,z) = 0 \quad (12)$$

$$\left(\frac{d}{dx} + \lambda(1 - c(z)) + \mu_i(x)\right) P_i(x,z) = 0, \; 1 \le i \le M \quad (13)$$

$$R(0,z) = r_0\bar{\theta} \int_0^\infty W(x,z)\mu_0(x)\,dx + \beta z \int_0^\infty W(x,z)\mu_0(x)\,dx + \bar{\theta}\,\bar{\delta} \int_0^\infty \sum_{i=1}^{M} P_i(x,z)\mu_i(x)\,dx + \delta\, z \int_0^\infty \sum_{i=1}^{M} P_i(x,z)\mu_i(x)\,dx - \lambda R_0 \quad (14)$$

$$W(0,z) = \frac{c(z)\lambda R_0}{z} + \frac{1}{z} \int_0^\infty R(x,z)\eta(x)\,dx + \frac{\lambda}{z} \int_0^\infty R(x,z)c(z)dx + \frac{\theta r_0}{z} \int_0^\infty W(x,z)\mu_0(x)\,dx + \frac{\theta\bar{\delta}}{z} \int_0^\infty \sum_{i=1}^{M} P_i(x,z)\mu_i(x)dx + \tau\lambda \int_0^\infty W(x,z)c(z)\,dx \quad (15)$$

$$P_i(0,z) = r_i \int_0^\infty W(x,z)\mu_0(x)\,dx, \; \text{for } i=1 \text{ to } M \quad (16)$$

Solving the equations (11) – (16), we get

$$R(x,z) = R(0,z)e^{-\lambda x}(1 - A(x)) \quad (17)$$

$$W(x,z) = W(0,z)e^{-\lambda[1-\bar{\tau}c(z)]x}(1 - B_0(x)) \quad (18)$$

$$P_i(x,z) = P_i(0,z)e^{-\lambda[1-c(z)]x}(1 - B_i(x)), \quad i = 1 \text{ to } M \quad (19)$$

$$R(0,z) = \lambda R_0\{(1 - \bar{\tau}c(z))[z - c(z)K_1(z) - \theta K_2(z)] - \tau z c(z)(1 - B_0^*(\lambda - \bar{\tau}\lambda c(z)))\} / D(z) \quad (20)$$

$$W(0,z) = \lambda R_0 \left(1 - c(z)\right)A^*(\lambda)\left(1 - \bar{\tau}c(z)\right)/D(z) \qquad (21)$$

$$P_i(0, z) = r_i \lambda R_0 A^*(\lambda)B_0^*\left(\lambda - \bar{\tau}\lambda c(z)\right)\left(1 - \bar{\tau}c(z)\right)(1 - c(z)) / D(z), \; i = 1 \text{ to } M \qquad (22)$$

Where

$$D(z) = \left(1 - \bar{\tau}c(z)\right)\{[c(z) + \left(1 - c(z)\right)A^*(\lambda)]K_1(z) + \theta K_2(z) - z\} + \tau z c(z)(1 - B_0^*(\lambda - \bar{\tau}\lambda c(z)))$$

$$K_1(z) = B_0^*\left(\lambda - \bar{\tau}\lambda c(z)\right)[\bar{\theta}\, r_0 + \beta z + (\delta z + \bar{\theta}\bar{\delta}) \sum_{i=1}^{M} r_i B_i^*\left(\lambda - \lambda c(z)\right)]$$

$$K_2(z) = B_0^*\left(\lambda - \lambda\bar{\tau}c(z)\right)[r_0 + \bar{\delta} \sum_{i=1}^{M} r_i B_i^*\left(\lambda - \lambda c(z)\right)]$$

Putting R(0, z), W(0, z) and $P_i(0, z)$ and integrate with respect to x , we have

$$R(z) = R_0\left(1 - A^*(\lambda)\right)\{\left(1 - \bar{\tau}c(z)\right)[z - c(z)K_1(z) - \theta K_2(z)] - \tau z c(z)(1 - B_0^*(\lambda - \bar{\tau}\lambda c(z)))\} / D(z)$$

$$W(z) = R_0\left(1 - c(z)\right)A^*(\lambda) [1 - B_0^*\left(\lambda - \bar{\tau}\lambda c(z)\right)] / D(z)$$

$$P_i(z) = R_0 r_i [1 - B_i^*\left(\lambda - \lambda c(z)\right)] \left(1 - \bar{\tau}c(z)\right)A^*(\lambda) B_0^*\left(\lambda - \bar{\tau}\lambda c(z)\right) / D(z), \quad \text{for i=1 to M}$$

Using the normalizing condition in equation (10) we get R_0 as

$$R_0 = T_1 / \tau A^*(\lambda)B_0^* (\tau\lambda)[1 - \beta - \delta\sum_{i=1}^{M} r_i]$$

where

$$T_1 = B_0^*(\tau\lambda) \{\tau + m_1 - \tau(\beta + \delta\sum_{i=1}^{M} r_i + \sum_{i=1}^{M} r_i\lambda m_1\mu_{i1}) - [1 - A^*(\lambda)][\tau m_1 - \theta\tau m_1(r_0 + \bar{\delta}\sum_{i=1}^{M} r_i)]\} - m_1$$

Performance measures:

Server idle probability during the retrial time is

$$R = (1 - A^*(\lambda))\{\tau\{m_1 B_0^* (\tau\lambda)[1 - \theta r_0 - \theta\bar{\delta}\sum_{i=1}^{M} r_i] + B_0^*(\tau\lambda) [\beta + \delta \sum_{i=1}^{M} r_i + \sum_{i=1}^{M} r_i\lambda m_1\mu_{i1} - 1]\} + m_1(1 - B_0^* (\tau\lambda))\} / \tau A^*(\lambda)B_0^* (\tau\lambda)[1 - \beta - \delta\sum_{i=1}^{M} r_i]$$

Server busy in FES probability is given by

$$W = m_1[1 - B_0^* (\tau\lambda)] / \tau B_0^* (\tau\lambda)[1 - \beta - \delta\sum_{i=1}^{M} r_i]$$

Server busy in i^{th} SMOS probability is given by

$$P_i = \sum_{i=1}^{M} r_i\lambda m_1\mu_{i1} / 1 - \beta - \delta\sum_{i=1}^{M} r_i, \quad \text{for } i = 1 \text{ to } M$$

The PGF of the number of customers in the retrial queue is

$$P_q(z) = R_0 + R(z) + W(z) + \sum_{i=1}^{M} P_i(z)$$

$$= R_0 A^*(\lambda)\{(1 - B_0^*(\lambda - \bar{\tau}\lambda c(z)))(1 - c(z) + \tau z c(z)) + (1 - \bar{\tau}c(z)) B_0^*\left(\lambda - \bar{\tau}\lambda c(z)\right)(r_0 + \beta z + \sum_{i=1}^{M} r_i - \delta(1 - z)\sum_{i=1}^{M} r_i B_i^*(\lambda - \lambda c(z))) - (1 - \bar{\tau}c(z))z\} / D(z)$$

The PGF of the number of customers in the system is

$$P_s(z) = R_0 + R(z) + zW(z) + z\sum_{i=1}^{M} P_i(z)$$

$$= R_0 A^*(\lambda)\{(1 - B_0^*(\lambda - \bar{\tau}\lambda c(z)))(z - \bar{\tau} z c(z)) + (1 - \bar{\tau}c(z)) B_0^*\left(\lambda - \bar{\tau}\lambda c(z)\right)(r_0 + \beta z + z\sum_{i=1}^{M} r_i + \bar{\delta}(1 - z)\sum_{i=1}^{M} r_i B_i^*(\lambda - \lambda c(z))) - (1 - \bar{\tau}c(z))z\} / D(z)$$

The expected number of customer in the retrial queue is

$$L_q = \lim_{z\to 1} \frac{d}{dz} P_q(z) = \frac{N_2 D_1 - D_2 N_1}{2(D_1)^2}$$

$$N_1 = \tau R_0 A^*(\lambda) B_0^* (\tau\lambda) [\beta + \delta\sum_{i=1}^{M} r_i - 1]$$

$$N_2 = 2R_0 A^*(\lambda)[\tau(\beta - 1)h_1 + m_1(1 - \tau B_0^* (\tau\lambda)) + \tau\delta B_0^* (\tau\lambda) \sum_{i=1}^{M} r_i\lambda m_1\mu_{i1} + \tau\delta h_1\sum_{i=1}^{M} r_i - \bar{\tau} m_1 B_0^*(\tau\lambda) (\beta + \delta\sum_{i=1}^{M} r_i)$$

$$D_1 = -B_0^*(\tau\lambda)\{\tau + m_1 - \tau(\beta + \delta\sum_{i=1}^{M} r_i + \sum_{i=1}^{M} r_i\lambda m_1\mu_{i1}) -$$

$$[1 - A^*(\lambda)][\tau m_1 - \theta\tau m_1(r_0 + \bar{\delta}\sum_{i=1}^{M} r_i)]\} + m_1$$

$$D_2 = 2\{h_1(\bar{\theta}r_0 + \beta) + B_0^*(\tau\lambda)[\beta + \delta\sum_{i=1}^{M} r_i] + (\bar{\theta}\,\bar{\delta} + \delta)\sum_{i=1}^{M}$$

$$r_i[B_0^*(\tau\lambda)\lambda m_1\mu_{i1} + h_1]\}[\tau m_1(1 - A^*(\lambda))] + B_0^*(\tau\lambda)$$

$$\{(\bar{\theta}r_0 + \beta) + \left(\bar{\theta}\,\bar{\delta} + \delta\right)\sum_{i=1}^{M} r_i\}[\tau m_2(1 - A^*(\lambda)) -$$

$$2m_1^2\bar{\tau}(1 - A^*(\lambda))] + h_1\{2\tau(\beta - 1) + 2\tau\delta\sum_{i=1}^{M} r_i$$

$$+2\tau\sum_{i=1}^{M} r_i\lambda m_1\mu_{i1} - 2m_1\} + B_0^*(\tau\lambda)\{2(\delta\tau - m_1\bar{\tau})$$

$$\sum_{i=1}^{M} r_i\lambda m_1\mu_{i1} - 2m_1\bar{\tau}(\beta + \delta\sum_{i=1}^{M} r_i) - m_2 - 2\tau m_1$$

$$+\tau\sum_{i=1}^{M} r_i(\lambda m_2\mu_{i1} + \lambda^2 m_1^2\mu_{i2})\} + 2m_1 + m_2$$

$$h_1 = \lim_{z\to 1}\frac{d}{dz}B_0^*\left(\lambda - \bar{\tau}\lambda c(z)\right)$$

The expected number of customers in the system is

$$L_s = \lim_{z\to 1}\frac{d}{dz}P_s(z)$$

$$= L_q + W + \sum_{i=1}^{M} P_i$$

4 CONCLUSION

In this paper, we analyzed batch arrival retrial queueing model with pre-emptive resume priority, Multi optional second phase, feedback and orbital search. Various performance measures like the probability of the idle server, probability of the busy server and expected value of the orbit size, expected system size are derived. The research on the present investigation can be further extended by including the concepts of working breakdown, collisions, and vacation.

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