Nonlinear Feedback Control On Herd Behaviour Prey-Predator Model Affected By Toxic Substance

Vijayalakshmi. G.M , Shiva Reddy. K, Ranjith Kumar.G

Abstract: We have explored a mathematical predator-prey harvested model with a functional square root response in which both the species are infected by some environmental toxicants. The hypothesis of Catch-per-unit-effort was used on the both harvested species. In the predator system, prey species takes various survival mechanism to avoid predators. Because of this, the interaction concept is proportional to the square root of the prey population, which accurately models the process where the prey shows powerful herd structure showing that the predator commonly connects with the prey along the herd's outer corridor. The complex behaviour of the system is examined. We have considered the possible existence of equilibrium points. Several numerical examples have been discussed in order to reinforce the outcomes of the theoretical analysis.

Index Terms: Predator, prey, Herd behaviour, Toxicity, Catch per unit effort, Stability, Harvesting.

1 INTRODUCTION

The classical predator-prey model of interacting populations has been investigated extensively in this paper. Due to environmental toxicants such as electronic waste, industrial waste, bio-medical waste etc..., the investigation on the predator-prey interactions with harvesting becomes a significant role in modelling mathematics. So it must be understood to analyse the adequate measures on the controls of renewable resources, the energetic feature of it as well as the impacts of toxicants on it. For example the interaction of different species is critical to understand in the fisheries management. Hence one of important topic is how predators respond to changes in prey availability (functional response). The interaction between the level of consumption of the predator and the density of its prey is known as the predator's functional response. In the predator system, prey species take different defensive mechanisms to save themselves from predator. Because of this, the interaction concept is proportional to the square root of the prey population, which accurately models the process where the prey shows tight herd structure suggesting that the predator typically communicates with the prey along the herd's outer corridor. According to the square root concept, the solution behaviour near the origin is more subtle and fascinating than the standard models and makes ecological sense. Functional responses are categorized into three general types, for example Type I, Type II and Type III (Holling 1959, Hassell 1978). The Type I's functional response is initially independent of size as the consumption rates grow linearly with increased prey density until a plateau is reached at saturation. Saturation occurs when a predator is unable to handle prey faster, at which point, despite increasing prey size, ingestion remains constant and prey mortality is inverse density-dependent (Hassell 1978).

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The functional response of Type II is hyperbolic and inversely density-dependent as consumption rates grow to a upper asymptote at a decelerating rate, indicating higher costs or constraints associated with higher ingestion rates (Hassell 1978). Most multi-species models use functional responses of Holling type II or III [1, 9, 11, 12, 22]. The effects of diseased prey and predator model, harvesting efforts, food chain model, impulsive model, Bazykin model, immune response have been revisited [3-5, 7-9,12,14,15,18,19]. The prey predator model with time delays, by employing Lyapunov`s method, Allee effect, Square root functional response has been studied [2, 4, 6, 16, 17, 21]. Peter A. Braza [16] recently considered a prey-predator model in which the prey exhibits herd behaviour to interact with the predator along the herd's outer path. We regarded competition models and prey-predator method as a statistical result of herd behaviour, in which terms of communication use the prey population`s square root instead of the prey population. Use of square root correctly allows for the fact that the relations take place along the population boundary. Peter A. Braza[16] has shown that the source is either stable locally or unstable based on the location of the predator values and prey populations in the phase plane Based on the demonstrations given by authors, we have examine the dynamics of the harvested predator-prey system with square root functional response of prey and we stabilize our system asymptotically using non-linear feedback controllers by constructing suitable Lyapunov function. The models are extracted in section 2, using time budget arguments to obtain functions for Holling Type II response. We assumed average handling time is zero which transforms the response of Holling Type II into a simpler response of Lotka-Volterra. A steady state test will be done after re-scaling. In section 3, the equilibrium points of the proposed system are exists and stability criteria has been analysed. In section 4, by introducing non-linear feedback controllers, we have stabilize the system by constructing suitable Lyapunov function. In section 5, we analysed the dynamics of the system with time delay and have proved that the system with delay undergoes Hopf bifurcation. Section 6 and 7 are devoted to numerical simulations and conclusions.

2 DESCRIPTION OF THE MODEL

The basic predator model with logistic growth in the prey and a response feature of Holling type II response is given by

$$\frac{dX_1}{dT} = rX_1\left(1-\frac{X_1}{K}\right) - \frac{eX_1X_2}{1+t_heX_1}$$
$$\frac{dX_2}{dT} = -\gamma X_2 + \frac{beX_1X_2}{1+t_heX_1} \quad (1)$$

Where $X_1(T)$, $X_2(T)$ denotes the density of prey and predator species at time T, r - the prey population's internal growth rate, K - the prey population`s carrying capacity, $\gamma$ is the death rate of predator in the absence of prey, $e$ is the searching efficiency of $X_2(T)$ for $X_1(T)$, $b$ - the rate of conversion or consumption of biomass , $t_h$ is the $X_2$`s average handling time of X_1.

As argued by the authors in[16,17], it is easier to model the prey response function which exhibits herd behaviour in terms of the prey population's square root. This case derivation is very similar to the above derivation. In essence, the authors in [16,17] used only interaction terms $\sqrt{X_1X_2}$ proportional to this direction. We mainly focus on the dynamical behaviours of harvested prey-predator model in the presence of toxicity, the results are the equations below.

$$\frac{dX_1}{dT} = rX_1\left(1-\frac{X_1}{K}\right) - \frac{e\sqrt{X_1}X_2}{1+t_he\sqrt{X_1}} - c_1EX_1 - bX_1^3$$
$$\frac{dX_2}{dT} = -\gamma X_2 + \frac{be\sqrt{X_1}X_2}{1+t_he\sqrt{X_1}} - c_2EX_2 - dX_2^2 \quad (2)$$

In this c_1 and c_2 are the catchability coefficients of prey and predator populations. These populations are subjected to combined harvesting effort E, and the catch rate function of c_1EX_1 and c_2EX_2 follow the catch-per-unit-effort rules. In this system, the prey species is directly infected by environmental toxicants such as electronic waste, industrial waste, bio-medical waste etc.. and the predator species is indirectly infected by these toxicants through the feeding process on the infected prey species. Parameters b and d denote the coefficients of toxicity on prey and predator, respectively. All the parameters here we assumed are positive values.

This makes the equations a variation of a logistic prey growth Lotka-Volterra model The variables are scaled to study how the dynamics are affected by the parameters.

The mode (2) takes the following dimensionless form

$$\frac{dx_2}{dt} = -sx_2 + \frac{c\sqrt{x_1}x_2}{1+a\sqrt{x_1}} - g_2x_2 - h_2x_2^2$$
$$\frac{dx_2}{dt} = -sx_2 + \frac{c\sqrt{x_1}x_2}{1+a\sqrt{x_1}} - g_2x_2 - h_2x_2^2 \quad (3)$$

We also reduced the number of parameters for effective understanding and simulation with the rescaling variables

$$x_1 = \frac{X_1}{K},\quad x_2 = \frac{eX_2}{r\sqrt{K}},\quad \text{rT=t}$$

and the other parameters are

$$s = \frac{\gamma}{r},\quad a = t_he\sqrt{K},\quad c = \frac{be\sqrt{K}}{r}$$
$$g_1 = \frac{c_1E}{r},\quad h_1 = \frac{d_1k^2}{r},\quad h_2 = \frac{d_2\sqrt{k}}{e},\quad g_2 = \frac{c_2E}{r}$$

Now, we get a revised Lotka-Volterra model, which is equivalent to assuming the average handling time is zero that is $a = 0$.

Therefore our model is

$$\frac{dx_1}{dt} = x_1(1-x_1) - \sqrt{x_1}x_2 - g_1x_1 - h_1x_1^3$$
$$\frac{dx_2}{dt} = -sx_2 + c\sqrt{x_1}x_2 - g_2x_2 - h_2x_2^2 \quad (4)$$

3 ANASIS OF THE MODEL

3.1 Existence of Equilibrium

We got three equilibria for the system (4), namely

(a) The points under trivial state $E_T(0,0)$

(b) The points under axial state $E_A(1,0)$

(c) The Interior equilibrium state $E_I(x^*, y^*)$ where $x^* = q^2$ and $y^* = q(1-q^2-H)$ where $q = s-c$

3.2 Stability Analysis

We linearize our model in a small neighborhood of equilibrium solutions for the local stability study of various stable state solutions. This is achieved with the help of the model system (4) associated with the variational matrix. The parameters of Routh-Hurwitz criteria allow us to assess the reliability of these balancing solutions.

The analysis of the variational matrix $J(x_1, x_2)$ associated with the model system (4) evaluated at (x_1, x_2) is given by

$$J_{(x_1,x_2)} = \begin{bmatrix} 1-2x_1-\dfrac{x_2}{2\sqrt{x_1}}-g_1-3h_1x_1^2 & -\sqrt{x_1} \\ \dfrac{cx_2}{2\sqrt{x_1}} & -s+c\sqrt{x_1}-g_2-2h_2x_2 \end{bmatrix}$$

Lemma 3.2.1: $E_T(0,0)$ is asymptotically stable locally if $g_1 > 1$

Proof. The variational matrix for $E_T(0,0)$ is as follows

$$J_{(0,0)} = \begin{bmatrix} 1-g_1 & 0 \\ 0 & -(s+g_2) \end{bmatrix}$$

The Eigen values of $J_{(0,0)}$ are $\lambda_1 = 1-g_1,\ \lambda_2 = -(s+g_2)$. Since the negative Eigen value λ_2 imply $E_T(0,0)$ is Locally stable and asymptotic if $g_1 > 1$.

Lemma 3.2.2: $E_A(1,$ is locally asymptotically stable if $g_1+3h_1+1>0$ and $g_2+s>c_2$

Proof. The variational matrix for $E_A(1,0)$ is as follows

$$J_{(1,0)}=\begin{bmatrix}-(g_1+3h_1+1) & -1\\ 0 & -(g_2-c_2+s)\end{bmatrix}$$

The Eigen values of are $\lambda_1=-(g_1+3h_1+1)$, $\lambda_2=-(g_2-c_2+s)$. Since the negative Eigen value λ_1 and λ_2 imply $E_A(1,0)$ is Locally asymptotically stable. $E_A(1,0)$ is a stable node, if $g_2+s>c_2$.

Lemma 3.2.3: $E_I(x_1^*,x_2^*)$ is asymptotically stable if

$$s+2x_1^*+\frac{x_2^*}{2\sqrt{x_2^*}}+g_1+g_2+3h_1x_1^{*2}+2h_2x_2^*>1+c\sqrt{x_1^*}$$

and $$\left(1-2x_1^*-\frac{x_2^*}{2\sqrt{x_2^*}}-g_1-3h_1x_1^{*2}\right)$$

$$\left(-s+c\sqrt{x_1^*}-g_2-2h_2x_2^*\right)>cx_2^*$$

Proof. The variational matrix for $E_I(x_1^*,x_2^*)$ is as follows

$$J_{(x_1^*,x_2^*)}=\begin{bmatrix}1-2x_1^*-\frac{x_2^*}{2\sqrt{x_2^*}}-g_1-3h_1x_1^{*2} & -\sqrt{x_1^*}\\ \frac{cx_2^*}{2\sqrt{x_1^*}} & -s+c\sqrt{x_1^*}-g_2-2h_2x_2^*\end{bmatrix}$$

For the above matrix the characteristic equation is given by $\lambda^2-T\lambda+D=0$ where $T=trace$ and $D=\det erment$ of the matrix. The equilibrium point is asymptotically stable if $T<0$ and $D>0$

Here $$T\left(J_{(x_1^*,x_2^*)}\right)=1-s+c\sqrt{x_1^*}-2x_1^*-\frac{x_2^*}{2\sqrt{x_2^*}}$$

$$-g_1-g_2-3h_1x_1^{*2}-2h_2x_2^*$$

$$D\left(J_{(x_1^*,x_2^*)}\right)=\left(1-2x_1^*-\frac{x_2^*}{2\sqrt{x_2^*}}-g_1-3h_1x_1^{*2}\right)$$

$$\left(-s+c\sqrt{x_1^*}-g_2-2h_2x_2^*\right)-cx_2^*$$

Which implies $E_I(x_1^*,x_2^*)$ is asymptotically stable if and

This completes the proof.

4. NON LINEAR FEEDBACK CONTROL PROBLEM

This section includes the predator-prey harvested population model with non-linear feedback regulation. To address the controllers of prey-predator population model dynamics, we use non-linear feedback control method by assuming the system (4) with the following appropriate form

$$\begin{aligned}\frac{dx_1}{dt}&=x_1(1-x_1)-\sqrt{x_1}x_2-g_1x_1-h_1x_1^3+u_1\\ \frac{dx_2}{dt}&=-sx_2+c\sqrt{x_1}x_2-g_2x_2-h_2x_2^2+u_2\end{aligned}\qquad(5)$$

Here the controllers are u_1 *and* u_2.

Remember that the form (5) is more fitting for examining the system (4) with non-linear controls.

Theorem 4.1. Non-linear feedback controllers

$$\begin{aligned}u_1&=-x_1(1-x_1)+\sqrt{x_1}x_2+d_1x_1^3\\ u_2&=sx_2-c\sqrt{x_1}x_2+d_2x_2^2\end{aligned}\qquad(6)$$

The system (6) will be asymptotically stable in the Lyapunov sense.

Proof. The proof of this theorem gives the sufficient conditions for asymptotically stability by using Lyapunov stability theorem. Substituting (6) into (5), one can get the following non-linear system of differential equations.

$$\begin{aligned}\frac{dx_1}{dt}&=x_1(-c_1Ex_1)\\ \frac{dx_2}{dt}&=x_2(-c_2Ex_2)\end{aligned}\qquad(7)$$

Let us consider the equation (7) in the form of the Lyapunov function

$$2V(x_1,x_2)=x_1^2+x_2^2\qquad(8)$$

The function (8) is a positive definite form in relation to variables x_1,x_2 and its time derivative along the equation path (7) is given below.

$$\dot{V}(x_1,x_2)=-E\left[c_1x_1^2+c_2x_2^2\right]\qquad(9)$$

From the densities $x_1(t)$, $x_2(t)$ are usually positive and the parameters c_1,c_2 *and* E also take only positive values then

$$c_1x_1^2+c_2x_2^2>0\qquad(10)$$

$\forall\quad x_1(t),\ x_2(t)>0$. Using inequalities (10), we can verify that function (7) is a negative definite form which demonstrates the process of asymptotic stability of (6) in the case of Lyapunov function. Thus the system (4) with non-linear feedback controllers (6) is asymptotically stable, which completes the proof. We thus claim that the predator-prey harvested model with operational square root response and toxicity can be stabilized asymptotically using non-linear feedback controllers.

5. TIME DELAY ANALYSIS:

In this section, we are analysing the dynamics of the model (4) with time delay and without diffusion. In view of this, the model (3) can be rewritten as

$$\frac{dx_1}{dt} = x_1(1-x_1) - \sqrt{x_1}x_2(t-\tau) - g_1x_1 - h_1x_1^3$$
$$\frac{dx_2}{dt} = -sx_2 + c\sqrt{x_1}x_2(t-\tau) - g_2x_2 - h_2x_2^2 \qquad (11)$$

Now the Jacobian matrix J about the steady state $E(x^*, y^*)$ is

$$J = \begin{bmatrix} -x_1 + \frac{x_2}{2\sqrt{x_1}} - 2h_1x_1^2 & -\sqrt{x_1}e^{-\lambda\tau} \\ \frac{cx_2}{2\sqrt{x_1}} & c\sqrt{x_1}(1-e^{-\lambda\tau}) - h_2x_2 \end{bmatrix} \qquad (12)$$

The characteristic equation of (12) is given by

$$\lambda^2 + P_1\lambda + P_2 + e^{-\lambda\tau}(Q_1\lambda + Q_2) = 0 \qquad (13)$$

where

$$P_1 = x_1 - \frac{x_2}{2\sqrt{x_1}} - 2h_1x_1^2 - c\sqrt{x_1} + h_2x_2,$$
$$P_2 = -cx_1\sqrt{x_1} + h_2x_1x_2 - 2ch_1x_1^2\sqrt{x_1} + 2h_1h_2x_1^2x_2 + \frac{cx_2}{2} - \frac{h_2x_2^2}{2\sqrt{x_1}}$$
$$Q_1 = c\sqrt{x_1}, \quad Q_2 = c\sqrt{x_1} + 2ch_1x_1^2\sqrt{x_1}$$

If $\tau > 0$, supposed that there is a positive τ_0 such that equation (13) has pair of purely imaginary roots $\pm i\omega$, $\omega > 0$. Then ω satisfies

$$-\omega^2 + P_1\omega i + P_2 + (Q_1\omega i + Q_2)[\cos\omega t - i\sin\omega t] = 0 \qquad (14)$$

which is equivalent to

$$\omega^4 + (P_1^2 - 2P_2 - Q_1^2)\omega^2 + (P_2^2 - Q_2^2) = 0 \qquad (15)$$

If $P_1^2 - 2P_2 - Q_1^2 > 0, P_2^2 - Q_2^2 > 0$, then equation (16) has no real root. Thus, the real parts of all Eigen values of (13) are negative for all $\tau \geq 0$. If $P_2^2 - Q_2^2$ is negative, there is a unique positive ω_0 satisfying (15) and then there is a positive τ_0 such that equation (13) has pair of purely imaginary roots $\pm i\omega_0$ as $\tau = \tau_0$, and all eigen values with negative real parts as $0 < \tau < \tau_0$. From (14) τ_k corresponding to ω_0 can be obtained

$$\tau_k = \frac{1}{\omega_0}\arccos\left[\frac{(Q_2 - P_1Q_1)\omega^2 - P_2Q_2}{Q_1^2\omega^2 + Q_2^2}\right] + \frac{2n\pi}{\omega_0}, \; n = 0,1,2..... \qquad (16)$$

5.1. HOPF Bifurcation:

Based on the above results, we have the following.

Theorem 5.1. (i) If $0 < \tau < \tau_0$, equation (11) has steady state equilibrium which is locally asymptotically stable

(ii) Equation (11) can undergo a Hopf bifurcation if $\tau > \tau_0$, and a periodic orbit exists in the small neighbourhood of the equilibrium. Proof. To obtain the Hopf bifurcation, we need to check the transversal condition for the complex eigenvalues of the steady state equilibrium at $\tau = \tau_0$. Then, from equation (13), we have

$$\frac{d\lambda}{dt}\left[2\lambda + P_1 + Q_1e^{-\lambda\tau} - (Q_1\lambda + Q_2)\tau e^{-\lambda\tau}\right] = \lambda(Q_1\lambda + Q_2)e^{-\lambda\tau} \qquad (17)$$

$$\left(\frac{d\lambda}{dt}\right)^{-1} = \frac{2\lambda + P_1 + Q_1e^{-\lambda\tau} - (Q_1\lambda + Q_2)\tau e^{-\lambda\tau}}{\lambda(Q_1\lambda + Q_2)e^{-\lambda\tau}} \qquad (18)$$

$$= \operatorname{Re}\left[\frac{1}{\omega_0}\left(\frac{2i\omega_0 + P_1}{P_1\omega_0 + (\omega_0^2 - P_2)i} + \frac{Q_1}{(-Q_1\omega_0 + Q_2i)} + \tau i\right)\right]$$
$$= \frac{2\omega_0^2 + (P_1^2 - 2P_2 - Q_1^2)}{(Q_1^2\omega_0^2 + Q_2^2)} \qquad (19)$$

Under the condition $P_1^2 - 2P_2 - Q_1^2 > 0$, we have $\left.\frac{d\operatorname{Re}(\lambda)}{dt}\right|_{\lambda = i\omega_0} > 0$. Therefore, the transversality condition holds and Hopf bifurcation occurs at $\omega = \omega_0, \tau = \tau_0$.

To explain above analytical method by numerical example we consider the parameter values for the delay system are as given below.

6. NUMERICAL SIMULATION

In this section, we have given some numerical simulations and presented the time graph and phase portrait for system (4) which are supporting our theoretical predictions.

For the following figures, the initial values are $x_1(0) = 0.02,\; x_2(0) = 0.01$ and the parameter values are $s = 0.221,\; c = 0.98,\; h_1 = 0.094,\; h_2 = 0.814,\; g_1 = 0.668,\; g_2 = 0.059$

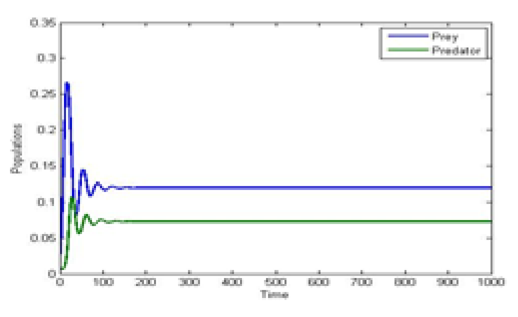

Fig. 1.

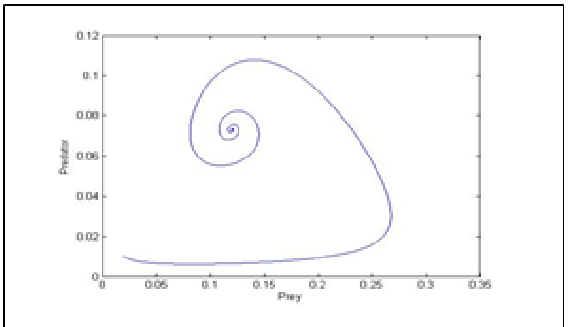

Fig. 2.

Figure 1 depicts the population trajectories of the system (4) and figure 2 is the respective phase portrait.

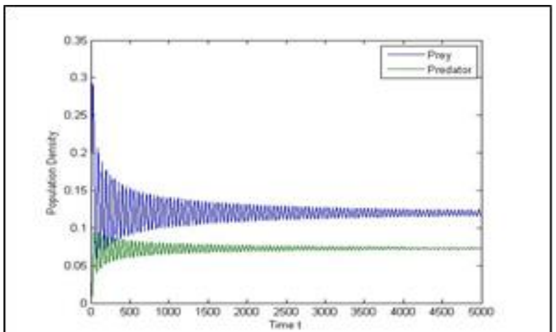

Fig. 3.

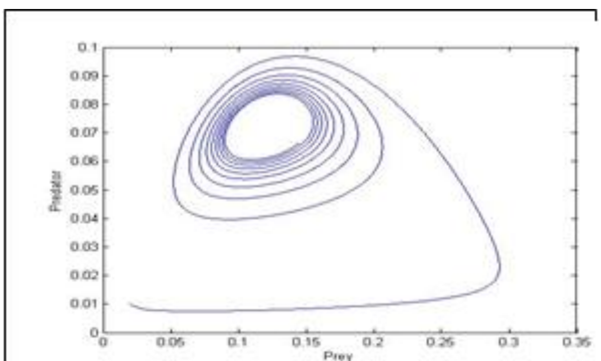

Fig. 4.

Figure 3&4 depicts for the system (11) with time delay $\tau = 3.5$ in predator with the same initial and parameter values.

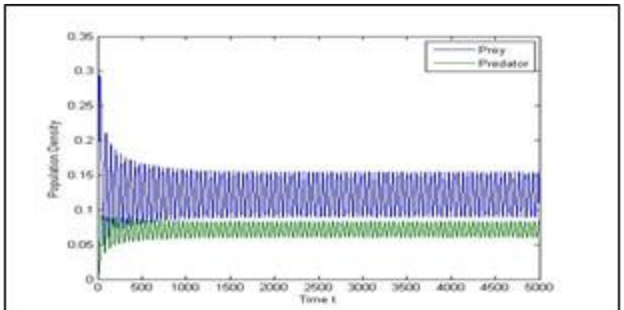

Fig. 5.

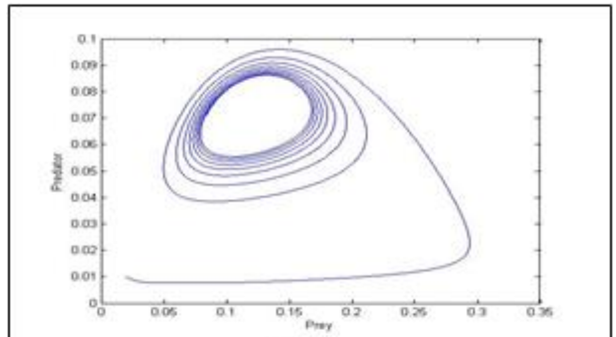

Fig. 6.

Figure 5&6 depicts for the system (11) with time delay $\tau = 3.9$ in predator with the same initial and parameter values.

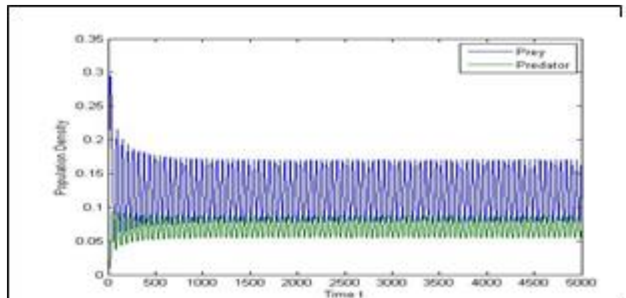

Fig. 7.

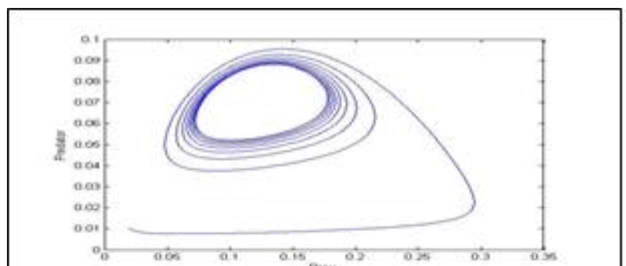

Fig. 8.

Figure 7&8 depicts for the system (11) with time delay $\tau = 4.2$ in predator with the same initial and parameter values.

7 CONCLUSION

We demonstrated a comprehensive and analysed on harvested prey-predator process with square root functional response and toxicity. In this work, $a = 0$ is considered to be the cumulative handling time equal to zero. We calculated the updated model's theoretical as well as quantitative results. The stability conditions of the modified harvested prey-predator system will be determined. The problem of toxic prey-predator system with non-linear feedback control is being studied. The regulated system's asymptotic stability is demonstrated by using the Lyapunov`s function. The required control inputs are obtained as non-linear feedback for this asymptotic stability. We analysed on harvested prey-predator process with time delay 0.4 in predator with the same initial values and parameter values. Also we have analysed if $0 < \tau < \tau_0$, the delay differential equation has steady state equilibrium which is locally asymptotically stable and if

$\tau > \tau_0$, the system with delay undergoes Hopf bifurcation. Several numerical examples have been discussed in order to reinforce the outcomes of the theoretical analysis.

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