# Solving Two Stage Fully Interval Integer Transportation Problems

**V. E Sobana, D. Anuradha, Kaspar S**

**Abstract:** A split and bind method has been proposed in this article to find all the possible solutions for two-stage fully interval integer transportation problem (TSFIITP). It is supported with a numerical example in medication logistics to determine the relevance of the proposed method.

**Index Terms**: Interval integer transportation problem, Optimal solution, Two-stage interval integer transportation problem.

## 1. INTRODUCTION

The transportation problem (TP) is one of the best optimization methods that can be applied to different areas of human being activity. TP handles for optimizing the cost of goods shipped from a number of origins to various destinations. Destinations are incapable to obtain the amount in surplus of their smallest requirement, due to storage restrictions in some circumstances. They are prepared to obtain the surplus in the second stage, after they have used up part of the entire first shipment. According to Pandian and Natarajan [15], the manufactured goods shipped to the destination have two stages in such circumstances. Only the sufficient quantity of the manufactured goods is transported in the first stage, so that the smallest demands of the destinations are met and, once this has been done, the remaining quantities in the origins transport to the destinations according to the cost consideration. In these two stages the transportation of the product from the origins to the destinations is carried out in simultaneously. The goal is to minimize the sum of the costs of transportation in the two stages. Many researchers [8, 9, 11, 17, 2] have proposed two-stage time minimization problem and fuzzy two-stage TPs for single and multi-objective problems for solving them. A bi-criteria multi-stage TP was solved by Ellaimony et al. [6] without any transportation restrictions on the intermediate stages by using a heuristic procedure. Pandian and Natarajan [13] obtained an optimal solution for a TP based on the zeropoint method (ZPM). Several effective algorithms have beendeveloped for solving TPs with the hypothesis of accurate supply, demand, and penalty factors. When problems arise in actual life, these situations cannot always be met. To treat with inaccurate coefficients in TPs, various approaches have been used to solve interval programming approach and imprecise data such as [3, 4, 5, 20, 12, 14, 18]. Akilbasha et al. [1] have developed an advanced method for pharmaceutical sciences called the mid-width method for obtaining an optimal solution to fully interval integer TPs. Prabhjot Kaur et al. [16] and Sharma et al. [19] presented two-stage interval time minimization TPs. In this paper, we proposed split and bind method for finding all possible solutions for the TSFIITP. This algorithm helps decision-making people to select a suitable solution based on their present circumstances and the same is illustrated by a numerical example in medication logistics to determine the relevance of the proposed method.


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## 2 FULLY INTERVAL INTEGER TRANSPORTATION PROBLEM (FIITP)

Consider a FIITP is shown as below:

(G1) $\text{Minimize } [z_1, z_2] = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [t_{ij}, s_{ij}]$

Subject to

$$\sum_{j=1}^{n}[t_{ij}, s_{ij}] = [a_i, p_i], \quad i = 1,2,...,m$$

(1)

$$\sum_{i=1}^{m}[t_{ij}, s_{ij}] = [b_j, q_j], \quad j = 1,2,...,n$$

(2)

$$t_{ij} \geq 0, s_{ij} \geq 0, \text{ for all i and j are integers}$$

(3)

Where $c_{ij}$ and $d_{ij}$ are positive real numbers for all i and j, $a_i$ and $p_i$ are positive real numbers for all i and $b_j$ and $q_j$ are positive real numbers for all j.

Upper and Lower bound ITP of the problem (G1) is given below:

| Upper bound ITP (UBITP) of FIITP : | Lower bound ITP (LBITP) of FIITP : |
| --- | --- |
| (G3) $\text{Minimize } z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} s_{ij}$ | (G2) $\text{Minimize } z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} t_{ij}$ |
| Subject to | Subject to |
| $\sum_{j=1}^{n} s_{ij} = p_i, \quad i = 1,2,...,m$ | $\sum_{j=1}^{n} t_{ij} = a_i, \quad i = 1,2,...,m$ |
| $\sum_{i=1}^{m} s_{ij} = q_j, \quad j = 1,2,...,n$ | $\sum_{i=1}^{m} t_{ij} = b_j, \quad j = 1,2,...,n$ |
| $s_{ij} \geq 0, \ \forall\ i$ and $j$ are integers | $t_{ij} \geq 0, \ \forall\ i$ and $j$ are integers |
| The optimal objective of (G3) | The optimal objective of |

| | |
|---|---|
| is $z_2^0 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}s_{ij}^0$ (4) | (G2)is $z_1^0 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}t_{ij}^0$ (5) |

The definitions of the arithmetic operators, partial ordering of closed bounded intervals, feasible and optimal solutions of the interval can be found in [7, 10, 14].

## 3 TWO STAGE FULLY INTERVAL INTEGER TRANSPORTATION PROBLEM

When there is a storage problem at the destinations due to maximum storage conditions the problem (G1) can be converted as TSFIITP.

We define the same as follows

(P1) $\text{Minimize}\left[z_1, z_2\right] = \left[z_3 + z_4, z_5 + z_6\right]$

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij}e_{ij}, d_{ij}g_{ij}\right] \le \left[z_3, z_5\right]$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij}f_{ij}, d_{ij}h_{ij}\right] \le \left[z_4, z_6\right]$$

$$\sum_{j=1}^{n}\left[e_{ij}, g_{ij}\right] \le \left[a_i, p_i\right],\ i = 1,2,...,m$$

$$\sum_{i=1}^{m}\left[e_{ij}, g_{ij}\right] = \left[l_j, k_j\right],\ j = 1,2,...,n$$

$$\sum_{j=1}^{n}\left[f_{ij}, h_{ij}\right] = \left[a_i - \sum_{j=1}^{n} e_{ij}, p_i - \sum_{j=1}^{n} g_{ij}\right]$$

$$\sum_{i=1}^{m}\left[f_{ij}, h_{ij}\right] = \left[b_j - l_j, q_j - k_j\right],\ j = 1,2,...,n$$

$$e_{ij}, f_{ij}, g_{ij}, h_{ij} \ge 0,\ \text{for all } i \text{ and } j \text{ are integers}$$

Where $c_{ij}$ and $d_{ij}$ is the unit transportation cost from origin i to destination j, $a_i$ and $p_i$ are the source at the ith origin, $b_j$ and $q_j$ are the demand at jth destination, $l_j$ and $k_j$ are the maximum storage capacity of the j^th destinations, $e_{ij}$, $g_{ij}$ and $f_{ij}$, $h_{ij}$ are the amount transported from ith origin to j^th destinations in first stage and second stage.

Upper and Lower bound two stage ITP of the problem (P1) is given below:

| Upper bound two-stage ITP (TSUBITP): | Lower bound two-stage ITP (TSLBITP): |
|---|---|
| (P3) $\text{Minimize } z_2 = z_5 + z_6$<br>Subject to<br>$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}g_{ij} \le z_5$<br>$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}h_{ij} \le z_6$<br>$\sum_{j=1}^{n} g_{ij} \le p_i,\ i = 1,2,...,m$<br>$\sum_{i=1}^{m} g_{ij} = k_j,\ j = 1,2,...,n$<br>$\sum_{j=1}^{n} h_{ij} = p_i - \sum_{j=1}^{n} g_{ij}, i = 1,2,...,m$<br>$\sum_{i=1}^{m} h_{ij} = q_j - k_j,\ j = 1,2,...,n$<br>$g_{ij}, h_{ij} \ge 0,\ \forall\ i \text{ and } j \text{ are integers.}$<br>where $k_j$ is the positive real number for all j. | (P2) Minimize $z_1 = z_3 + z_4$<br>Subject to<br>$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}e_{ij} \le z_3$<br>$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}f_{ij} \le z_4$<br>$\sum_{j=1}^{n} e_{ij} \le a_i,\ i = 1,2,...,m$<br>$\sum_{i=1}^{m} e_{ij} = l_j,\ j = 1,2,...,n$<br>$\sum_{j=1}^{n} f_{ij} = a_i - \sum_{j=1}^{n} e_{ij}, i = 1,2,...,m$<br>$\sum_{i=1}^{m} f_{ij} = b_j - l_j,\ j = 1,2,...,n$<br>$e_{ij}, f_{ij} \ge 0, \forall\ i \text{ and } j \text{ are integers.}$<br>where $l_j$ is the positive real number for all j. |

## 4 SPLIT AND BIND METHOD

We now propose a split and bind method for obtaining all the possible solutions to TSFIITP (P1). Pandian and Natarajan have proved the existence of the optimal solutions to the FIITP and two-stage TPs in [14, 15]. In a similar manner, we prove the existence of the optimal solution to the TSFIITP in the following theorem which will be used in the proposed method.

### 4.1 Theorem

An optimal solution of the problem (P1) = [(P2), (P3)] can be obtained from an optimal solution of the problem (G1) = [(G2), (G3)]. Further the respective optimal objective values of the problem (P1) = [(P2), (P3)] and (G1) = [(G2), (G3)] are the same.

Proof

Let (G1) = [(G2), (G3)] be the given FIITP.

Let $\left\{t_{ij}^0, \forall\ i \text{ and } j\right\}$ and $\left\{s_{ij}^0, \forall\ i \text{ and } j\right\}$ are the optimal solutions of the problems (G2) and (G3) with objective value $v^0$ and $w^0$ respectively.

This implies $\left\{\left[t_{ij}^0, s_{ij}^0\right], \forall\ i \text{ and } j\right\}$ is an optimal solution of the problem (G1) with objective value $\left[v^0, w^0\right]$.

Let (P1) = [(P2), (P3)] be the TSFIITP constructed from (G1) = [(G2), (G3)].

In the problem (P1), let us consider the TSLBITP problem (P2).

Let $\left\{e_{ij}^0, \forall\ i \text{ and } j\right\}$ and $\left\{f_{ij}^0, \forall\ i \text{ and } j\right\}$ be two sets of positive real numbers, obtained from the set $\left\{t_{ij}^0, \forall\ i \text{ and } j\right\}$ such that $\sum_{j=1}^{n} e_{ij}^0 \le a_i,\ i = 1,2,...,m$

$$\sum_{i=1}^{m} e_{ij}^0 = l_j,\ j = 1,2,...,n$$

and $f_{ij}^0 = t_{ij}^0 - e_{ij}^0, \forall\ i \text{ and } j$.

Let $z_3^0 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}e_{ij}^0$, $z_4^0 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}f_{ij}^0$ and $v^0 = z_3^0 + z_4^0$.

Clearly, $\{z_3^0, z_4^0, e_{ij}^0 + f_{ij}^0, \forall\ i \text{ and } j\}$ is a feasible solution to the problem (P2).

Now consider $v^0 = z_3^0 + z_4^0$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}e_{ij}^0 + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}f_{ij}^0 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}(e_{ij}^0 + f_{ij}^0)$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}(t_{ij}^0) = z_1^0 \quad \text{by (5).}$$

This implies $v^0 = z_1^0$. (6)

Thus, $\{e_{ij}^0 + f_{ij}^0, \forall\, i \text{ and } j\}$ is an optimal solution to the problem (P2) with objective value $v^0 = z_3^0 + z_4^0$.

In a similar way starting with the TSUBITP problem one can prove that $w^0 = z_2^0 = z_5^0 + z_6^0$ (7)

Combining equations (6) and (7) we get $[v^0, w^0] = [z_3^0 + z_4^0, z_5^0 + z_6^0]$ which proves the statement of theorem.

The split and bind method proceeds as follows:

Step 1: Construct the problem (P3) from the problem (P1).

Step 2: Obtain an optimal solution for the problem (P3) according to any transportation algorithm without taking into account the maximum capacity of the destinations. Let $\{s_{ij}^0, \forall\, i \text{ and } j\}$ be an optimal solution for the UBITP.

Step 3: Obtain various possible split ups, for each allocated cell column wise taking into account the feasibility of capacity and optimal solutions present in that column.

Step 4: In stage I, consider same possible solution combinations obtained from the earlier split ups say $g_{ij}^0$.

Step 5: In stage 2,

a) Modify each supply as i[th] row supply is equal to (old supply) - $\left(\sum_{i\ fixed} g_{ij}^0\right)$

b) Similarly modify each demand as j[th] column demand is equal to $(q_j) - (k_j)$

c) Modify each allotted cell as $h_{ij}^0$ is equal to $(s_{ij}^0) - (g_{ij}^0)$

Step 6: Repeat the Step 4 and Step 5 to obtain all possible optimal solutions to the problem (P3).

Step 7: Construct the problem (P2) from the problem (P1).

Step 8: Obtain an optimal solution for the problem (P2) according to any transportation algorithm without taking into account the maximum capacity of the destinations. Let $\{t_{ij}^0, \forall\, i \text{ and } j\}$ be an optimal solution for the LBITP with $t_{ij}^0 \le s_{ij}^0,\ \forall\, i \text{ and } j$.

Step 9: Repeat Step 3 to Step 5 to obtain all possible optimal solutions to the problem (P2).

Step 10: The optimal solution of the problem (P1) is obtained from the optimal solutions of the problem [(P2), (P3)] (by theorem 4.1)

The split and bind method for solving a TSFIITP is shown below using an example.

Consider a medication- manufacturing company consisting three factories and four warehouses. The maximum capacity of the destinations of the medication- manufacturing company isgivenas $[l_1, k_1] = [5,5]$, $[l_2, k_2] = [2,4]$, $[l_3, k_3] = [10,10]$, $[l_4, k_4] = [12,14]$ respectively. The below table shows the cost of transportation, supplies and demands are in the form of intervals.

| | Warehouses | | | | |
|---|---|---|---|---|---|
| | W1 | W2 | W3 | W4 | Supply |
| Factory 1 | [2,4] | [2,6] | [10,18] | [8,16] | [7,9] |
| Factory 2 | [1,2] | [7,10] | [2,6] | [3,5] | [17,21] |
| Factory 3 | [7,9] | [7,11] | [3,5] | [5,7] | [16,18] |
| Demand | [10,12] | [2,4] | [13,15] | [15,17] | |

Now, using step 1 the TSUBITP to the given TSFIITP is given below

| | Warehouses | | | | |
|---|---|---|---|---|---|
| | W1 | W2 | W3 | W4 | Supply |
| Factory 1 | 4 | 6 | 18 | 16 | 9 |
| Factory 2 | 2 | 10 | 6 | 5 | 21 |
| Factory 3 | 9 | 11 | 5 | 7 | 18 |
| Demand | 12 | 4 | 15 | 17 | |

with maximum capacity of the upper bound destinations are $k_1 = 5$, $k_2 = 4$, $k_3 = 10$, $k_4 = 14$ respectively.

Now, as in Step 2, using the ZPM [13], the optimal solution to the UBITP is $s_{11}^0 = 5$, $s_{12}^0 = 4$, $s_{21}^0 = 7$, $s_{24}^0 = 14$, $s_{33}^0 = 15$, $s_{34}^0 = 3$ with the total minimum transportation cost as 224.

Now, as in Step 3, we consider the split ups of the above problem is given below

| | Warehouses | | | | |
|---|---|---|---|---|---|
| | W1 | W2 | W3 | W4 | Supply |
| Factory 1 | 5(0,1,2,3,4,5) | 4(4) | | | 9 |
| Factory 2 | 7(5,4,3,2,1,0) | | | 14(14,13,12,11) | 21 |
| Factory 3 | | | 15(10) | 3(0,1,2,3) | 18 |
| Demand | 12(5) | 4(4) | 15(10) | 17(14) | |

Now, using Step 4 to Step 6, we obtain all possible optimal solutions with minimum total transportation costs at 224 of the TSUBITP which is shown below.

| S. No | Stage-I UB optimal solutions | Stage-II UB optimal solutions |
|---|---|---|
| 1 | $g_{11}^{01} = 0$, $g_{12}^{01} = 4$, $g_{21}^{01} = 5$, $g_{24}^{01} = 14$, $g_{33}^{01} = 10$, $g_{34}^{01} = 0$ with total transportation cost is 154. | $h_{11}^{01} = 5$, $h_{12}^{01} = 0$, $h_{21}^{01} = 2$, $h_{24}^{01} = 0$, $h_{33}^{01} = 5$, $h_{34}^{01} = 3$ with total transportation cost is 70. |
| 2 | $g_{11}^{02} = 1$, $g_{12}^{02} = 4$, $g_{21}^{02} = 4$, $g_{24}^{02} = 14$, $g_{33}^{02} = 10$, $g_{34}^{02} = 0$ with total transportation cost is 156. | $h_{11}^{02} = 4$, $h_{12}^{02} = 0$, $h_{21}^{02} = 3$, $h_{24}^{02} = 0$, $h_{33}^{02} = 5$, $h_{34}^{02} = 3$ with total transportation cost is 68. |

| | | |
|---|---|---|
| 3 | $g_{11}^{03}=2$ , $g_{12}^{03}=4$ , $g_{21}^{03}=3$ , $g_{24}^{03}=14$ , $g_{33}^{03}=10$ , $g_{34}^{03}=0$ with total transportation cost is 158. | $h_{11}^{03}=3$ , $h_{12}^{03}=0$ , $h_{21}^{03}=4$ , $h_{24}^{03}=0$ , $h_{33}^{03}=5$ , $h_{34}^{03}=3$ with total transportation cost is 66. |
| 4 | $g_{11}^{04}=0$ , $g_{12}^{04}=4$ , $g_{21}^{04}=5$ , $g_{24}^{04}=13$ , $g_{33}^{04}=10$ , $g_{34}^{04}=1$ with total transportation cost is 156. | $h_{11}^{04}=5$ , $h_{12}^{04}=0$ , $h_{21}^{04}=2$ , $h_{24}^{04}=1$ , $h_{33}^{04}=5$ , $h_{34}^{04}=2$ with total transportation cost is 68. |
| 5 | $g_{11}^{05}=1$ , $g_{12}^{05}=4$ , $g_{21}^{05}=4$ , $g_{24}^{05}=13$ , $g_{33}^{05}=10$ , $g_{34}^{05}=1$ with total transportation cost is 158. | $h_{11}^{05}=4$ , $h_{12}^{05}=0$ , $h_{21}^{05}=3$ , $h_{24}^{05}=1$ , $h_{33}^{05}=5$ , $h_{34}^{05}=2$ with total transportation cost is 66. |
| 6 | $g_{11}^{06}=2$ , $g_{12}^{06}=4$ , $g_{21}^{06}=3$ , $g_{24}^{06}=13$ , $g_{33}^{06}=10$ , $g_{34}^{06}=1$ with total transportation cost is 160. | $h_{11}^{06}=3$ , $h_{12}^{06}=0$ , $h_{21}^{06}=4$ , $h_{24}^{06}=1$ , $h_{33}^{06}=5$ , $h_{34}^{06}=2$ with total transportation cost is 64. |
| 7 | $g_{11}^{07}=0$ , $g_{12}^{07}=4$ , $g_{21}^{07}=5$ , $g_{24}^{07}=12$ , $g_{33}^{07}=10$ , $g_{34}^{07}=2$ with total transportation cost is 158. | $h_{11}^{07}=5$ , $h_{12}^{07}=0$ , $h_{21}^{07}=2$ , $h_{24}^{07}=2$ , $h_{33}^{07}=5$ , $h_{34}^{07}=1$ with total transportation cost is 66. |
| 8 | $g_{11}^{08}=1$ , $g_{12}^{08}=4$ , $g_{21}^{08}=4$ , $g_{24}^{08}=12$ , $g_{33}^{08}=10$ , $g_{34}^{08}=2$ with total transportation cost is 160. | $h_{11}^{08}=4$ , $h_{12}^{08}=0$ , $h_{21}^{08}=3$ , $h_{24}^{08}=2$ , $h_{33}^{08}=5$ , $h_{34}^{08}=1$ with total transportation cost is 64. |
| 9 | $g_{11}^{09}=2$ , $g_{12}^{09}=4$ , $g_{21}^{09}=3$ , $g_{24}^{09}=12$ , $g_{33}^{09}=10$ , $g_{34}^{09}=2$ with total transportation cost is 162. | $h_{11}^{09}=3$ , $h_{12}^{09}=0$ , $h_{21}^{09}=4$ , $h_{24}^{09}=2$ , $h_{33}^{09}=5$ , $h_{34}^{09}=1$ with total transportation cost is 62. |
| 10 | $g_{11}^{010}=0$ , $g_{12}^{010}=4$ , $g_{21}^{010}=5$ , $g_{24}^{010}=11$ , $g_{33}^{010}=10$ , $g_{34}^{010}=3$ with total transportation cost is 160. | $h_{11}^{010}=5$ , $h_{12}^{010}=0$ , $h_{21}^{010}=2$ , $h_{24}^{010}=3$ , $h_{33}^{010}=5$ , $h_{34}^{010}=0$ with total transportation cost is 64. |
| 11 | $g_{11}^{011}=1$ , $g_{12}^{011}=4$ , $g_{21}^{011}=4$ , $g_{24}^{011}=11$ , $g_{33}^{011}=10$ , $g_{34}^{011}=3$ with total transportation cost is 162. | $h_{11}^{011}=4$ , $h_{12}^{011}=0$ , $h_{21}^{011}=3$ , $h_{24}^{011}=3$ , $h_{33}^{011}=5$ , $h_{34}^{011}=0$ with total transportation cost is 62. |
| 12 | $g_{11}^{012}=2$ , $g_{12}^{012}=4$ , $g_{21}^{012}=3$ , $g_{24}^{012}=11$ , $g_{33}^{012}=10$ , $g_{34}^{012}=3$ with total transportation cost is 164. | $h_{11}^{012}=3$ , $h_{12}^{012}=0$ , $h_{21}^{012}=4$ , $h_{24}^{012}=3$ , $h_{33}^{012}=5$ , $h_{34}^{012}=0$ with total transportation cost is 60. |
| 13 | $g_{11}^{013}=5$ , $g_{12}^{013}=4$ , $g_{21}^{013}=0$ , $g_{24}^{013}=14$ , $g_{33}^{013}=10$ , $g_{34}^{013}=0$ with total transportation cost is 164. | $h_{11}^{013}=0$ , $h_{12}^{013}=0$ , $h_{21}^{013}=7$ , $h_{24}^{013}=0$ , $h_{33}^{013}=5$ , $h_{34}^{013}=3$ with total transportation cost is 60. |
| 14 | $g_{11}^{014}=4$ , $g_{12}^{014}=4$ , $g_{21}^{014}=1$ , $g_{24}^{014}=14$ , $g_{33}^{014}=10$ , $g_{34}^{014}=0$ with total transportation cost is 162. | $h_{11}^{014}=1$ , $h_{12}^{014}=0$ , $h_{21}^{014}=6$ , $h_{24}^{014}=0$ , $h_{33}^{014}=5$ , $h_{34}^{014}=3$ with total transportation cost is 62. |
| 15 | $g_{11}^{015}=3$ , $g_{12}^{015}=4$ , $g_{21}^{015}=2$ , $g_{24}^{015}=14$ , $g_{33}^{015}=10$ , $g_{34}^{015}=0$ with total transportation cost is 160. | $h_{11}^{015}=2$ , $h_{12}^{015}=0$ , $h_{21}^{015}=5$ , $h_{24}^{015}=0$ , $h_{33}^{015}=5$ , $h_{34}^{015}=3$ with total transportation cost is 64. |
| 16 | $g_{11}^{016}=5$ , $g_{12}^{016}=4$ , $g_{21}^{016}=0$ , $g_{24}^{016}=13$ , $g_{33}^{016}=10$ , $g_{34}^{016}=1$ with total transportation cost is 166. | $h_{11}^{016}=0$ , $h_{12}^{016}=0$ , $h_{21}^{016}=7$ , $h_{24}^{016}=1$ , $h_{33}^{016}=5$ , $h_{34}^{016}=2$ with total transportation cost is 58. |
| 17 | $g_{11}^{017}=4$ , $g_{12}^{017}=4$ , $g_{21}^{017}=1$ , $g_{24}^{017}=13$ , $g_{33}^{017}=10$ , $g_{34}^{017}=1$ with total transportation cost is 164. | $h_{11}^{017}=1$ , $h_{12}^{017}=0$ , $h_{21}^{017}=6$ , $h_{24}^{017}=1$ , $h_{33}^{017}=5$ , $h_{34}^{017}=2$ with total transportation cost is 60. |
| 18 | $g_{11}^{018}=3$ , $g_{12}^{018}=4$ , $g_{21}^{018}=2$ , $g_{24}^{018}=13$ , $g_{33}^{018}=10$ , $g_{34}^{018}=1$ with total transportation cost is 162. | $h_{11}^{018}=2$ , $h_{12}^{018}=0$ , $h_{21}^{018}=5$ , $h_{24}^{018}=1$ , $h_{33}^{018}=5$ , $h_{34}^{018}=2$ with total transportation cost is 62. |
| 19 | $g_{11}^{019}=5$ , $g_{12}^{019}=4$ , $g_{21}^{019}=0$ , $g_{24}^{019}=12$ , $g_{33}^{019}=10$ , $g_{34}^{019}=2$ with total transportation cost is 168. | $h_{11}^{019}=0$ , $h_{12}^{019}=0$ , $h_{21}^{019}=7$ , $h_{24}^{019}=2$ , $h_{33}^{019}=5$ , $h_{34}^{019}=1$ with total transportation cost is 56. |
| 20 | $g_{11}^{020}=4$ , $g_{12}^{020}=4$ , $g_{21}^{020}=1$ , $g_{24}^{020}=12$ , $g_{33}^{020}=10$ , $g_{34}^{020}=2$ with total transportation cost is 166. | $h_{11}^{020}=1$ , $h_{12}^{020}=0$ , $h_{21}^{020}=6$ , $h_{24}^{020}=2$ , $h_{33}^{020}=5$ , $h_{34}^{020}=1$ with total transportation cost is 58. |
| 21 | $g_{11}^{021}=3$ , $g_{12}^{021}=4$ , $g_{21}^{021}=2$ , $g_{24}^{021}=12$ , $g_{33}^{021}=10$ , $g_{34}^{021}=2$ with total transportation cost is 164. | $h_{11}^{021}=2$ , $h_{12}^{021}=0$ , $h_{21}^{021}=5$ , $h_{24}^{021}=2$ , $h_{33}^{021}=5$ , $h_{34}^{021}=1$ with total transportation cost is 60. |
| 22 | $g_{11}^{022}=5$ , $g_{12}^{022}=4$ , $g_{21}^{022}=0$ , $g_{24}^{022}=11$ , $g_{33}^{022}=10$ , $g_{34}^{022}=3$ with total transportation cost is 170. | $h_{11}^{022}=0$ , $h_{12}^{022}=0$ , $h_{21}^{022}=7$ , $h_{24}^{022}=3$ , $h_{33}^{022}=5$ , $h_{34}^{022}=0$ with total transportation cost is 54. |
| 23 | $g_{11}^{023}=4$ , $g_{12}^{023}=4$ , $g_{21}^{023}=1$ , $g_{24}^{023}=11$ , $g_{33}^{023}=10$ , $g_{34}^{023}=3$ with total transportation cost is 168. | $h_{11}^{023}=1$ , $h_{12}^{023}=0$ , $h_{21}^{023}=6$ , $h_{24}^{023}=3$ , $h_{33}^{023}=5$ , $h_{34}^{023}=0$ with total transportation cost is 56. |
| 24 | $g_{11}^{024}=3$ , $g_{12}^{024}=4$ , $g_{21}^{024}=2$ , $g_{24}^{024}=11$ , $g_{33}^{024}=10$ , $g_{34}^{024}=3$ with total transportation cost is 166. | $h_{11}^{024}=2$ , $h_{12}^{024}=0$ , $h_{21}^{024}=5$ , $h_{24}^{024}=3$ , $h_{33}^{024}=5$ , $h_{34}^{024}=0$ with total transportation cost is 58. |

Now, using Step 7 the TSLBITP to the given TSFIITP is given below

| | Warehouses | | | | |
|---|---|---|---|---|---|
| | W1 | W2 | W3 | W4 | Supply |
| Factory 1 | 2 | 2 | 10 | 8 | 7 |
| Factory 2 | 1 | 7 | 2 | 3 | 17 |
| Factory 3 | 7 | 7 | 3 | 5 | 16 |
| Demand | 10 | 2 | 13 | 15 | |

with maximum capacity of the lower bound destinations are $l_1=5$ , $l_2=2$ , $l_3=10$ and $l_4=12$ respectively.

Now, as in Step 8, using the ZPM [13], the optimal solution to the LBITP is $t_{11}^{0}=5$ , $t_{12}^{0}=2$ , $t_{21}^{0}=5$ , $t_{24}^{0}=12$ , $t_{33}^{0}=13$ and $t_{34}^{0}=3$ with the total minimum transportation costs as 109.

Now, using Step 9, we obtain all possible optimal solutions with minimum total transportation costs at 109 of the TSLBITP which is shown below.

| S.No | Stage-I LB optimal solutions | Stage-II LB optimal solutions |
|---|---|---|
| 1 | $e_{11}^{01}=0$, $e_{12}^{01}=2$, $e_{21}^{01}=5$, $e_{24}^{01}=12$, $e_{33}^{01}=10$, $e_{34}^{01}=0$<br>with total transportation cost is 75. | $f_{11}^{01}=5$, $f_{12}^{01}=0$, $f_{21}^{01}=0$, $f_{24}^{01}=0$, $f_{33}^{01}=3$, $f_{34}^{01}=3$<br>with total transportation cost is 34. |
| 2 | $e_{11}^{02}=1$, $e_{12}^{02}=2$, $e_{21}^{02}=4$, $e_{24}^{02}=12$, $e_{33}^{02}=10$, $e_{34}^{02}=0$<br>with total transportation cost is 76. | $f_{11}^{02}=4$, $f_{12}^{02}=0$, $f_{21}^{02}=1$, $f_{24}^{02}=0$, $f_{33}^{02}=3$, $f_{34}^{02}=3$<br>with total transportation cost is 33. |
| 3 | $e_{11}^{03}=2$, $e_{12}^{03}=2$, $e_{21}^{03}=3$, $e_{24}^{03}=12$, $e_{33}^{03}=10$, $e_{34}^{03}=0$<br>with total transportation cost is 77. | $f_{11}^{03}=3$, $f_{12}^{03}=0$, $f_{21}^{03}=2$, $f_{24}^{03}=0$, $f_{33}^{03}=3$, $f_{34}^{03}=3$<br>with total transportation cost is 32 |
| 4 | $e_{11}^{04}=0$, $e_{12}^{04}=2$, $e_{21}^{04}=5$, $e_{24}^{04}=11$, $e_{33}^{04}=10$, $e_{34}^{04}=1$<br>with total transportation cost is 77. | $f_{11}^{04}=5$, $f_{12}^{04}=0$, $f_{21}^{04}=0$, $f_{24}^{04}=1$, $f_{33}^{04}=3$, $f_{34}^{04}=2$<br>with total transportation cost is 32. |
| 5 | $e_{11}^{05}=1$, $e_{12}^{05}=2$, $e_{21}^{05}=4$, $e_{24}^{05}=11$, $e_{33}^{05}=10$, $e_{34}^{05}=1$<br>with total transportation cost is 78. | $f_{11}^{05}=4$, $f_{12}^{05}=0$, $f_{21}^{05}=1$, $f_{24}^{05}=1$, $f_{33}^{05}=3$, $f_{34}^{05}=2$<br>with total transportation cost is 31. |
| 6 | $e_{11}^{06}=2$, $e_{12}^{06}=2$, $e_{21}^{06}=3$, $e_{24}^{06}=11$, $e_{33}^{06}=10$, $e_{34}^{06}=1$<br>with total transportation cost is 79. | $f_{11}^{06}=3$, $f_{12}^{06}=0$, $f_{21}^{06}=2$, $f_{24}^{06}=1$, $f_{33}^{06}=3$, $f_{34}^{06}=2$<br>with total transportation cost is 30. |
| 7 | $e_{11}^{07}=0$, $e_{12}^{07}=2$, $e_{21}^{07}=5$, $e_{24}^{07}=10$, $e_{33}^{07}=10$, $e_{34}^{07}=2$<br>with total transportation cost is 79 | $f_{11}^{07}=5$, $f_{12}^{07}=0$, $f_{21}^{07}=0$, $f_{24}^{07}=2$, $f_{33}^{07}=3$, $f_{34}^{07}=1$<br>with total transportation cost is 30 |
| 8 | $e_{11}^{08}=1$, $e_{12}^{08}=2$, $e_{21}^{08}=4$, $e_{24}^{08}=10$, $e_{33}^{08}=10$, $e_{34}^{08}=2$<br>with total transportation cost is 80. | $f_{11}^{08}=4$, $f_{12}^{08}=0$, $f_{21}^{08}=1$, $f_{24}^{08}=2$, $f_{33}^{08}=3$, $f_{34}^{08}=1$<br>with total transportation cost is 29. |
| 9 | $e_{11}^{09}=2$, $e_{12}^{09}=2$, $e_{21}^{09}=3$, $e_{24}^{09}=10$, $e_{33}^{09}=10$, $e_{34}^{09}=2$<br>with total transportation cost is 81. | $f_{11}^{09}=3$, $f_{12}^{09}=0$, $f_{21}^{09}=2$, $f_{24}^{09}=2$, $f_{33}^{09}=3$, $f_{34}^{09}=1$<br>with total transportation cost is 28. |
| 10 | $e_{11}^{010}=0$, $e_{12}^{010}=2$, $e_{21}^{010}=5$, $e_{24}^{010}=9$, $e_{33}^{010}=10$, $e_{34}^{010}=3$<br>with total transportation cost is 81. | $f_{11}^{010}=5$, $f_{12}^{010}=0$, $f_{21}^{010}=0$, $f_{24}^{010}=3$, $f_{33}^{010}=3$, $f_{34}^{010}=0$<br>with total transportation cost is 28. |
| 11 | $e_{11}^{011}=1$, $e_{12}^{011}=2$, $e_{21}^{011}=4$, $e_{24}^{011}=9$, $e_{33}^{011}=10$, $e_{34}^{011}=3$<br>with total transportation cost is 82. | $f_{11}^{011}=4$, $f_{12}^{011}=0$, $f_{21}^{011}=1$, $f_{24}^{011}=3$, $f_{33}^{011}=3$, $f_{34}^{011}=0$<br>with total transportation cost is 27. |
| 12 | $e_{11}^{012}=2$, $e_{12}^{012}=2$, $e_{21}^{012}=3$, $e_{24}^{012}=9$, $e_{33}^{012}=10$, $e_{34}^{012}=3$<br>with total transportation cost is 83. | $f_{11}^{012}=3$, $f_{12}^{012}=0$, $f_{21}^{012}=2$, $f_{24}^{012}=3$, $f_{33}^{012}=3$, $f_{34}^{012}=0$<br>with total transportation cost is 26. |
| 13 | $e_{11}^{013}=5$, $e_{12}^{013}=2$, $e_{21}^{013}=0$, $e_{21}^{013}=12$, $e_{33}^{013}=10$, $e_{34}^{013}=0$<br>with total transportation cost is 80. | $f_{11}^{013}=0$, $f_{12}^{013}=0$, $f_{21}^{013}=5$, $f_{24}^{013}=0$, $f_{33}^{013}=3$, $f_{34}^{013}=3$<br>with total transportation cost is 29. |
| 14 | $e_{11}^{014}=4$, $e_{12}^{014}=2$, $e_{21}^{014}=1$, $e_{24}^{014}=12$, $e_{33}^{014}=10$, $e_{34}^{014}=0$<br>with total transportation cost is 79. | $f_{11}^{014}=1$, $f_{12}^{014}=0$, $f_{21}^{014}=4$, $f_{24}^{014}=0$, $f_{33}^{014}=3$, $f_{34}^{014}=3$<br>with total transportation cost is 30. |
| 15 | $e_{11}^{015}=3$, $e_{12}^{015}=2$, $e_{21}^{015}=2$, $e_{24}^{015}=12$, $e_{33}^{015}=10$, $e_{34}^{015}=0$<br>with total transportation cost is 78. | $f_{11}^{015}=2$, $f_{12}^{015}=0$, $f_{21}^{015}=3$, $f_{24}^{015}=0$, $f_{33}^{015}=3$, $f_{34}^{015}=3$<br>with total transportation cost is 31. |
| 16 | $e_{11}^{016}=5$, $e_{12}^{016}=2$, $e_{21}^{016}=0$, $e_{24}^{016}=11$, $e_{33}^{016}=10$, $e_{34}^{016}=1$<br>with total transportation cost is 82. | $f_{11}^{016}=0$, $f_{12}^{016}=0$, $f_{21}^{016}=5$, $f_{24}^{016}=1$, $f_{33}^{016}=3$, $f_{34}^{016}=2$<br>with total transportation cost is 27. |
| 17 | $e_{11}^{017}=4$, $e_{12}^{017}=2$, $e_{21}^{017}=1$, $e_{24}^{017}=11$, $e_{33}^{017}=10$, $e_{34}^{017}=1$<br>with total transportation cost is 81. | $f_{11}^{017}=1$, $f_{12}^{017}=0$, $f_{21}^{017}=4$, $f_{24}^{017}=1$, $f_{33}^{017}=3$, $f_{34}^{017}=2$<br>with total transportation cost is 28. |
| 18 | $e_{11}^{018}=3$, $e_{12}^{018}=2$, $e_{21}^{018}=2$, $e_{24}^{018}=11$, $e_{33}^{018}=10$, $e_{34}^{018}=1$<br>with total transportation cost is 80. | $f_{11}^{018}=2$, $f_{12}^{018}=0$, $f_{21}^{018}=3$, $f_{24}^{018}=1$, $f_{33}^{018}=3$, $f_{34}^{018}=2$<br>with total transportation cost is 29. |
| 19 | $e_{11}^{019}=5$, $e_{12}^{019}=2$, $e_{21}^{019}=0$, $e_{24}^{019}=10$, $e_{33}^{019}=10$, $e_{34}^{019}=2$<br>with total transportation cost is 84. | $f_{11}^{019}=0$, $f_{12}^{019}=0$, $f_{21}^{019}=5$, $f_{24}^{019}=2$, $f_{33}^{019}=3$, $f_{34}^{019}=1$<br>with total transportation cost is 25. |
| 20 | $e_{11}^{020}=4$, $e_{12}^{020}=2$, $e_{21}^{020}=1$, $e_{24}^{020}=10$, $e_{33}^{020}=10$, $e_{34}^{020}=2$<br>with total transportation cost is 83. | $f_{11}^{020}=1$, $f_{12}^{020}=0$, $f_{21}^{020}=4$, $f_{24}^{020}=2$, $f_{33}^{020}=3$, $f_{34}^{020}=1$<br>with total transportation cost is 26. |
| 21 | $e_{11}^{021}=3$, $e_{12}^{021}=2$, $e_{21}^{021}=2$, $e_{24}^{021}=10$, $e_{33}^{021}=10$, $e_{34}^{021}=2$<br>with total transportation cost is 82. | $f_{11}^{021}=2$, $f_{12}^{021}=0$, $f_{21}^{021}=3$, $f_{24}^{021}=2$, $f_{33}^{021}=3$, $f_{34}^{021}=1$<br>with total transportation cost is 27. |
| 22 | $e_{11}^{022}=5$, $e_{12}^{022}=2$, $e_{21}^{022}=0$, $e_{24}^{022}=9$, $e_{33}^{022}=10$, $e_{34}^{022}=3$<br>with total transportation cost is 86. | $f_{11}^{022}=0$, $f_{12}^{022}=0$, $f_{21}^{022}=5$, $f_{24}^{022}=3$, $f_{33}^{022}=3$, $f_{34}^{022}=0$<br>with total transportation cost is 23. |
| 23 | $e_{11}^{023}=4$, $e_{12}^{023}=2$, $e_{21}^{023}=1$, $e_{24}^{023}=9$, $e_{33}^{023}=10$, $e_{34}^{023}=3$<br>with total transportation cost is 85. | $f_{11}^{023}=1$, $f_{12}^{023}=0$, $f_{21}^{023}=4$, $f_{24}^{023}=3$, $f_{33}^{023}=3$, $f_{34}^{023}=0$<br>with total transportation cost is 24. |
| 24 | $e_{11}^{024}=3$, $e_{12}^{024}=2$, $e_{21}^{024}=2$, $e_{24}^{024}=9$, $e_{33}^{024}=10$, $e_{34}^{024}=3$<br>with total transportation cost is 84. | $f_{11}^{024}=2$, $f_{12}^{024}=0$, $f_{21}^{024}=3$, $f_{24}^{024}=3$, $f_{33}^{024}=3$, $f_{34}^{024}=0$<br>with total transportation cost is 25. |

By Step 10, the twenty four optimal solutions of TSFIITP are obtained from the optimal solutions of the problem [(P2), (P3)] with minimum total transportation costs at [109,224] and is shown below

| S.No | Stage-I optimal solutions | Stage-II optimal solutions |
|---|---|---|
| 1 | $[e_{11}^{01},g_{11}^{01}]=[0,0]$, $[e_{12}^{01},g_{12}^{01}]=[2,4]$, $[e_{21}^{01},g_{21}^{01}]=[5,5]$, $[e_{24}^{01},g_{24}^{01}]=[12,14]$, $[e_{33}^{01},g_{33}^{01}]=[10,10]$, $[e_{34}^{01},g_{34}^{01}]=[0,0]$ with total transportation cost is [75,154]. | $[f_{11}^{01},h_{11}^{01}]=[5,5]$, $[f_{12}^{01},h_{12}^{01}]=[0,0]$, $[f_{21}^{01},h_{21}^{01}]=[0,2]$, $[f_{24}^{01},h_{24}^{01}]=[0,0]$, $[f_{33}^{01},h_{33}^{01}]=[3,5]$, $[f_{34}^{01},h_{34}^{01}]=[3,3]$ with total transportation cost is [34,70]. |
| 2 | $[e_{11}^{02},g_{11}^{02}]=[1,1]$, $[e_{12}^{02},g_{12}^{02}]=[2,4]$, $[e_{21}^{02},g_{21}^{02}]=[4,4]$, $[e_{24}^{02},g_{24}^{02}]=[12,14]$, $[e_{33}^{02},g_{33}^{02}]=[10,10]$, $[e_{34}^{02},g_{34}^{02}]=[0,0]$ with total transportation cost is [76,156]. | $[f_{11}^{02},h_{11}^{02}]=[4,4]$, $[f_{12}^{02},h_{12}^{02}]=[0,0]$, $[f_{21}^{02},h_{21}^{02}]=[1,3]$, $[f_{24}^{02},h_{24}^{02}]=[0,0]$, $[f_{33}^{02},h_{33}^{02}]=[3,5]$, $[f_{34}^{02},h_{34}^{02}]=[3,3]$ with total transportation cost is [33,68]. |
| 3 | $[e_{11}^{03},g_{11}^{03}]=[2,2]$, $[e_{12}^{03},g_{12}^{03}]=[2,4]$, $[e_{21}^{03},g_{21}^{03}]=[3,3]$, $[e_{24}^{03},g_{24}^{03}]=[12,14]$, $[e_{33}^{03},g_{33}^{03}]=[10,10]$, $[e_{34}^{03},g_{34}^{03}]=[0,0]$ with total transportation cost is [77,158]. | $[f_{11}^{03},h_{11}^{03}]=[3,3]$, $[f_{12}^{03},h_{12}^{03}]=[0,0]$, $[f_{21}^{03},h_{21}^{03}]=[2,4]$, $[f_{24}^{03},h_{24}^{03}]=[0,0]$, $[f_{33}^{03},h_{33}^{03}]=[3,5]$, $[f_{34}^{03},h_{34}^{03}]=[3,3]$ with total transportation cost is [32,66]. |
| 4 | $[e_{11}^{04},g_{11}^{04}]=[0,0]$, $[e_{12}^{04},g_{12}^{04}]=[2,4]$, $[e_{21}^{04},g_{21}^{04}]=[5,5]$, $[e_{24}^{04},g_{24}^{04}]=[11,13]$, $[e_{33}^{04},g_{33}^{04}]=[10,10]$, $[e_{34}^{04},g_{34}^{04}]=[1,1]$ with total transportation cost is [77,156]. | $[f_{11}^{04},h_{11}^{04}]=[5,5]$, $[f_{12}^{04},h_{12}^{04}]=[0,0]$, $[f_{21}^{04},h_{21}^{04}]=[0,2]$ $[f_{24}^{04},h_{24}^{04}]=[1,1]$, $[f_{33}^{04},h_{33}^{04}]=[3,5]$, $[f_{34}^{04},h_{34}^{04}]=[2,2]$ with total transportation cost is [32,68]. |
| 5 | $[e_{11}^{05},g_{11}^{05}]=[1,1]$, $[e_{12}^{05},g_{12}^{05}]=[2,4]$, $[e_{21}^{05},g_{21}^{05}]=[4,4]$, $[e_{24}^{05},g_{24}^{05}]=[11,13]$, $[e_{33}^{05},g_{33}^{05}]=[10,10]$, $[e_{34}^{05},g_{34}^{05}]=[1,1]$ with total transportation cost is [78,158]. | $[f_{11}^{05},h_{11}^{05}]=[4,4]$, $[f_{12}^{05},h_{12}^{05}]=[0,0]$, $[f_{21}^{05},h_{21}^{05}]=[1,3]$, $[f_{24}^{05},h_{24}^{05}]=[1,1]$, $[f_{33}^{05},h_{33}^{05}]=[3,5]$, $[f_{34}^{05},h_{34}^{05}]=[2,2]$, with total transportation cost is [31,66]. |
| 6 | $[e_{11}^{06},g_{11}^{06}]=[2,2]$, $[e_{12}^{06},g_{12}^{06}]=[2,4]$, $[e_{21}^{06},g_{21}^{06}]=[3,3]$, $[e_{24}^{06},g_{24}^{06}]=[11,13]$, $[e_{33}^{06},g_{33}^{06}]=[10,10]$, $[e_{34}^{06},g_{34}^{06}]=[1,1]$ with total transportation cost is [79,160]. | $[f_{11}^{06},h_{11}^{06}]=[3,3]$, $[f_{12}^{06},h_{12}^{06}]=[0,0]$, $[f_{21}^{06},h_{21}^{06}]=[2,4]$ $[f_{24}^{06},h_{24}^{06}]=[1,1]$, $[f_{33}^{06},h_{33}^{06}]=[3,5]$, $[f_{34}^{06},h_{34}^{06}]=[2,2]$ with total transportation cost is [30,64]. |
| 7 | $[e_{11}^{07},g_{11}^{07}]=[0,0]$, $[e_{12}^{07},g_{12}^{07}]=[2,4]$, $[e_{21}^{07},g_{21}^{07}]=[5,5]$, $[e_{24}^{07},g_{24}^{07}]=[10,12]$, $[e_{33}^{07},g_{33}^{07}]=[10,10]$, $[e_{34}^{07},g_{34}^{07}]=[2,2]$ with total transportation cost is [79,158]. | $[f_{11}^{07},h_{11}^{07}]=[5,5]$, $[f_{12}^{07},h_{12}^{07}]=[0,0]$, $[f_{21}^{07},h_{21}^{07}]=[0,2]$ $[f_{24}^{07},h_{24}^{07}]=[2,2]$, $[f_{33}^{07},h_{33}^{07}]=[3,5]$, $[f_{34}^{07},h_{34}^{07}]=[1,1]$ with total transportation cost is [30,66]. |
| 8 | $[e_{11}^{08},g_{11}^{08}]=[1,1]$, $[e_{12}^{08},g_{12}^{08}]=[2,4]$, $[e_{21}^{08},g_{21}^{08}]=[4,4]$, $[e_{24}^{08},g_{24}^{08}]=[10,12]$, $[e_{33}^{08},g_{33}^{08}]=[10,10]$, $[e_{34}^{08},g_{34}^{08}]=[2,2]$ with total transportation cost is [80,160]. | $[f_{11}^{08},h_{11}^{08}]=[4,4]$, $[f_{12}^{08},h_{12}^{08}]=[0,0]$, $[f_{21}^{08},h_{21}^{08}]=[1,3]$, $[f_{24}^{08},h_{24}^{08}]=[2,2]$, $[f_{33}^{08},h_{33}^{08}]=[3,5]$, $[f_{34}^{08},h_{34}^{08}]=[1,1]$ with total transportation cost is [29,64]. |
| 9 | $[e_{11}^{09},g_{11}^{09}]=[2,2]$, $[e_{12}^{09},g_{12}^{09}]=[2,4]$, $[e_{21}^{09},g_{21}^{09}]=[3,3]$, $[e_{24}^{09},g_{24}^{09}]=[10,12]$, $[e_{33}^{09},g_{33}^{09}]=[10,10]$, $[e_{34}^{09},g_{34}^{09}]=[2,2]$ with total transportation cost is [81,162]. | $[f_{11}^{09},h_{11}^{09}]=[3,3]$, $[f_{12}^{09},h_{12}^{09}]=[0,0]$, $[f_{21}^{09},h_{21}^{09}]=[2,4]$ $[f_{24}^{09},h_{24}^{09}]=[2,2]$, $[f_{33}^{09},h_{33}^{09}]=[3,5]$, $[f_{34}^{09},h_{34}^{09}]=[1,1]$ with total transportation cost is [28,62]. |
| 10 | $[e_{11}^{010},g_{11}^{010}]=[0,0]$, $[e_{12}^{010},g_{12}^{010}]=[2,4]$, $[e_{21}^{010},g_{21}^{010}]=[5,5]$ $[e_{24}^{010},g_{24}^{010}]=[9,11]$, $[e_{33}^{010},g_{33}^{010}]=[10,10]$, $[e_{34}^{010},g_{34}^{010}]=$ [3, 3]with total transportation cost is [81,160]. | $[f_{11}^{010},h_{11}^{010}]=[5,5]$, $[f_{12}^{010},h_{12}^{010}]=[0,0]$, $[f_{21}^{010},h_{21}^{010}]=[0,2]$, $[f_{24}^{010},h_{24}^{010}]=[3,3]$, $[f_{33}^{010},h_{33}^{010}]=[3,5]$, $[f_{34}^{010},h_{34}^{010}]=[0,0]$ with total transportation cost is [28,64]. |
| 11 | $[e_{11}^{011},g_{11}^{011}]=[1,1]$, $[e_{12}^{011},g_{12}^{011}]=[2,4]$, $[e_{21}^{011},g_{21}^{011}]=[4,4]$ $[e_{24}^{011},g_{24}^{011}]=[9,11]$, $[e_{33}^{011},g_{33}^{011}]=[10,10]$, $[e_{34}^{011},g_{34}^{011}]=[3,3]$ with total transportation cost is [82,162]. | $[f_{11}^{011},h_{11}^{011}]=[4,4]$, $[f_{12}^{011},h_{12}^{011}]=[0,0]$, $[f_{21}^{011},h_{21}^{011}]=[1,3]$, $[f_{24}^{011},h_{24}^{011}]=[3,3]$, $[f_{33}^{011},h_{33}^{011}]=[3,5]$, $[f_{34}^{011},h_{34}^{011}]=[0,0]$ with total transportation cost is [27,62]. |

| | | |
|---|---|---|
| 12 | $[e_{11}^{012}, g_{11}^{012}] = [2,2]$, $[e_{12}^{012}, g_{12}^{012}] = [2,4]$,<br>$[e_{21}^{012}, g_{21}^{012}] = [3,3]$ $[e_{24}^{012}, g_{24}^{012}] = [9,11]$,<br>$[e_{33}^{012}, g_{33}^{012}] = [6,10]$, $[e_{34}^{012}, g_{34}^{012}] = [3,3]$<br>with total transportation cost is [83,164]. | $[f_{11}^{012}, h_{11}^{012}] = [3,3]$, $[f_{12}^{012}, h_{12}^{012}] = [0,0]$,<br>$[f_{21}^{012}, h_{21}^{012}] = [2,4]$, $[f_{24}^{012}, h_{24}^{012}] = [3,3]$,<br>$[f_{33}^{012}, h_{33}^{012}] = [3,5]$, $[f_{34}^{012}, h_{34}^{012}] = [0,0]$<br>with total transportation cost is [26,60]. |
| 13 | $[e_{11}^{013}, g_{11}^{013}] = [5,5]$, $[e_{12}^{013}, g_{12}^{013}] = [2,4]$,<br>$[e_{21}^{013}, g_{21}^{013}] = [0,0]$ $[e_{24}^{013}, g_{24}^{013}] = [12,14]$,<br>$[e_{33}^{013}, g_{33}^{013}] = [10,10]$, $[e_{34}^{013}, g_{34}^{013}] = [0,0]$<br>with total transportation cost is [80,164]. | $[f_{11}^{013}, h_{11}^{013}] = [0,0]$, $[f_{12}^{013}, h_{12}^{013}] = [0,0]$,<br>$[f_{21}^{013}, h_{21}^{013}] = [5,7]$, $[f_{24}^{013}, h_{24}^{013}] = [0,0]$,<br>$[f_{33}^{013}, h_{33}^{013}] = [3,5]$, $[f_{34}^{013}, h_{34}^{013}] = [3,3]$<br>with total transportation cost is [29,60]. |
| 14 | $[e_{11}^{014}, g_{11}^{014}] = [4,4]$, $[e_{12}^{014}, g_{12}^{014}] = [2,4]$,<br>$[e_{21}^{014}, g_{21}^{014}] = [1,1]$ $[e_{24}^{014}, g_{24}^{014}] = [12,14]$,<br>$[e_{33}^{014}, g_{33}^{014}] = [10,10]$, $[e_{34}^{014}, g_{34}^{014}] = [0,0]$<br>with total transportation cost is [79,162]. | $[f_{11}^{014}, h_{11}^{014}] = [1,1]$, $[f_{12}^{014}, h_{12}^{014}] = [0,0]$,<br>$[f_{21}^{014}, h_{21}^{014}] = [4,6]$, $[f_{24}^{014}, h_{24}^{014}] = [0,0]$,<br>$[f_{33}^{014}, h_{33}^{014}] = [3,5]$, $[f_{34}^{014}, h_{34}^{014}] = [3,3]$<br>with total transportation cost is [30,62]. |
| 15 | $[e_{11}^{015}, g_{11}^{015}] = [3,3]$, $[e_{12}^{015}, g_{12}^{015}] = [2,4]$,<br>$[e_{21}^{015}, g_{21}^{015}] = [2,2]$ $[e_{24}^{015}, g_{24}^{015}] = [12,14]$,<br>$[e_{33}^{015}, g_{33}^{015}] = [10,10]$, $[e_{34}^{015}, g_{34}^{015}] = [0,0]$<br>with total transportation cost is [78,160]. | $[f_{11}^{015}, h_{11}^{015}] = [2,2]$, $[f_{12}^{014}, h_{12}^{014}] = [0,0]$,<br>$[f_{21}^{014}, h_{21}^{014}] = [3,5]$, $[f_{24}^{014}, h_{24}^{014}] = [0,0]$,<br>$[f_{33}^{014}, h_{33}^{014}] = [3,5]$, $[f_{34}^{014}, h_{34}^{014}] = [3,3]$<br>with total transportation cost is [31,64]. |
| 16 | $[e_{11}^{016}, g_{11}^{016}] = [5,5]$, $[e_{12}^{016}, g_{12}^{016}] = [2,4]$,<br>$[e_{21}^{016}, g_{21}^{016}] = [0,0]$ $[e_{24}^{016}, g_{24}^{016}] = [11,13]$,<br>$[e_{33}^{016}, g_{33}^{016}] = [10,10]$, $[e_{34}^{016}, g_{34}^{016}] = [1,1]$<br>with total transportation cost is [82,166]. | $[f_{11}^{016}, h_{11}^{016}] = [0,0]$, $[f_{12}^{016}, h_{12}^{016}] = [0,0]$,<br>$[f_{21}^{016}, h_{21}^{016}] = [5,7]$, $[f_{24}^{016}, h_{24}^{016}] = [1,1]$,<br>$[f_{33}^{016}, h_{33}^{016}] = [3,5]$, $[f_{34}^{016}, h_{34}^{016}] = [2,2]$<br>with total transportation cost is [27,58]. |
| 17 | $[e_{11}^{017}, g_{11}^{017}] = [4,4]$, $[e_{12}^{017}, g_{12}^{017}] = [2,4]$,<br>$[e_{21}^{017}, g_{21}^{017}] = [1,1]$ $[e_{24}^{017}, g_{24}^{017}] = [11,13]$,<br>$[e_{33}^{017}, g_{33}^{017}] = [10,10]$, $[e_{34}^{017}, g_{34}^{017}] = [1,1]$<br>with total transportation cost is [81,164]. | $[f_{11}^{017}, h_{11}^{017}] = [1,1]$, $[f_{12}^{017}, h_{12}^{017}] = [0,0]$,<br>$[f_{21}^{017}, h_{21}^{017}] = [4,6]$, $[f_{24}^{017}, h_{24}^{017}] = [1,1]$,<br>$[f_{33}^{017}, h_{33}^{017}] = [3,5]$, $[f_{34}^{017}, h_{34}^{017}] = [2,2]$<br>with total transportation cost is [28,60]. |
| 18 | $[e_{11}^{018}, g_{11}^{018}] = [3,3]$, $[e_{12}^{018}, g_{12}^{018}] = [2,4]$,<br>$[e_{21}^{018}, g_{21}^{018}] = [2,2]$ $[e_{24}^{018}, g_{24}^{018}] = [11,13]$,<br>$[e_{33}^{018}, g_{33}^{018}] = [10,10]$, $[e_{34}^{018}, g_{34}^{018}] = [1,1]$<br>with total transportation cost is [80,162]. | $[f_{11}^{018}, h_{11}^{018}] = [2,2]$, $[f_{12}^{018}, h_{12}^{018}] = [0,0]$,<br>$[f_{21}^{018}, h_{21}^{018}] = [3,5]$, $[f_{24}^{018}, h_{24}^{018}] = [1,1]$,<br>$[f_{33}^{018}, h_{33}^{018}] = [3,5]$, $[f_{34}^{018}, h_{34}^{018}] = [2,2]$<br>with total transportation cost is [29,62]. |
| 19 | $[e_{11}^{019}, g_{11}^{019}] = [5,5]$, $[e_{12}^{019}, g_{12}^{019}] = [2,4]$,<br>$[e_{21}^{019}, g_{21}^{019}] = [0,0]$ $[e_{24}^{019}, g_{24}^{019}] = [10,12]$,<br>$[e_{33}^{019}, g_{33}^{019}] = [10,10]$, $[e_{34}^{019}, g_{34}^{019}] = [2,2]$<br>with total transportation cost is [84,168]. | $[f_{11}^{019}, h_{11}^{019}] = [0,0]$, $[f_{12}^{019}, h_{12}^{019}] = [0,0]$,<br>$[f_{21}^{019}, h_{21}^{019}] = [5,7]$, $[f_{24}^{019}, h_{24}^{019}] = [2,2]$,<br>$[f_{33}^{019}, h_{33}^{019}] = [3,5]$, $[f_{34}^{019}, h_{34}^{019}] = [1,1]$<br>with total transportation cost is [25,56]. |
| 20 | $[e_{11}^{020}, g_{11}^{020}] = [4,4]$, $[e_{12}^{020}, g_{12}^{020}] = [2,4]$,<br>$[e_{21}^{020}, g_{21}^{020}] = [1,1]$ $[e_{24}^{020}, g_{24}^{020}] = [10,12]$,<br>$[e_{33}^{020}, g_{33}^{020}] = [10,10]$, $[e_{34}^{020}, g_{34}^{020}] = [2,2]$<br>with total transportation cost is [83,166]. | $[f_{11}^{020}, h_{11}^{020}] = [1,1]$, $[f_{12}^{020}, h_{12}^{020}] = [0,0]$,<br>$[f_{21}^{020}, h_{21}^{020}] = [4,6]$, $[f_{24}^{020}, h_{24}^{020}] = [2,2]$,<br>$[f_{33}^{020}, h_{33}^{020}] = [3,5]$, $[f_{34}^{020}, h_{34}^{020}] = [1,1]$<br>with total transportation cost is [26,58]. |
| 21 | $[e_{11}^{021}, g_{11}^{021}] = [3,3]$, $[e_{12}^{021}, g_{12}^{021}] = [2,4]$,<br>$[e_{21}^{021}, g_{21}^{021}] = [2,2]$ $[e_{24}^{021}, g_{24}^{021}] = [10,12]$,<br>$[e_{33}^{021}, g_{33}^{021}] = [10,10]$, $[e_{34}^{021}, g_{34}^{021}] = [2,2]$<br>with total transportation cost is [82,164]. | $[f_{11}^{021}, h_{11}^{021}] = [2,2]$, $[f_{12}^{021}, h_{12}^{021}] = [0,0]$,<br>$[f_{21}^{021}, h_{21}^{021}] = [3,5]$, $[f_{24}^{021}, h_{24}^{021}] = [2,2]$,<br>$[f_{33}^{021}, h_{33}^{021}] = [3,5]$, $[f_{34}^{021}, h_{34}^{021}] = [1,1]$<br>with total transportation cost is [27,60]. |
| 22 | $[e_{11}^{022}, g_{11}^{022}] = [5,5]$, $[e_{12}^{022}, g_{12}^{022}] = [2,4]$,<br>$[e_{21}^{022}, g_{21}^{022}] = [0,0]$ $[e_{24}^{022}, g_{24}^{022}] = [9,11]$,<br>$[e_{33}^{022}, g_{33}^{022}] = [10,10]$, $[e_{34}^{022}, g_{34}^{022}] = [3,3]$<br>with total transportation cost is [86,170]. | $[f_{11}^{022}, h_{11}^{022}] = [0,0]$, $[f_{12}^{022}, h_{12}^{022}] = [0,0]$,<br>$[f_{21}^{022}, h_{21}^{022}] = [5,7]$, $[f_{24}^{022}, h_{24}^{022}] = [3,3]$,<br>$[f_{33}^{022}, h_{33}^{022}] = [3,5]$, $[f_{34}^{022}, h_{34}^{022}] = [0,0]$<br>with total transportation cost is [23,54]. |
| 23 | $[e_{11}^{023}, g_{11}^{023}] = [4,4]$, $[e_{12}^{023}, g_{12}^{023}] = [2,4]$,<br>$[e_{21}^{023}, g_{21}^{023}] = [1,1]$, $[e_{24}^{023}, g_{24}^{023}] = [9,11]$, | $[f_{11}^{023}, h_{11}^{023}] = [1,1]$, $[f_{12}^{023}, h_{12}^{023}] = [0,0]$,<br>$[f_{21}^{023}, h_{21}^{023}] = [4,6]$, $[f_{24}^{023}, h_{24}^{023}] = [3,3]$, |

| | | |
|---|---|---|
| | $[e_{33}^{023}, g_{33}^{023}] = [10,10]$, $[e_{34}^{023}, g_{34}^{023}] = [3,3]$ with total transportation cost is [85,168]. | $[f_{33}^{023}, h_{33}^{023}] = [3,5]$, $[f_{34}^{023}, h_{34}^{023}] = [0,0]$ with total transportation cost is [24,56]. |
| 24 | $[e_{11}^{024}, g_{11}^{024}] = [3,3]$, $[e_{12}^{024}, g_{12}^{024}] = [2,4]$, $[e_{21}^{024}, g_{21}^{024}] = [2,2]$ $[e_{24}^{024}, g_{24}^{024}] = [9,11]$, $[e_{33}^{024}, g_{33}^{024}] = [6,10]$, $[e_{34}^{024}, g_{34}^{024}] = [3,3]$ with total transportation cost is [84,166]. | $[f_{11}^{024}, h_{11}^{024}] = [2,2]$, $[f_{12}^{024}, h_{12}^{024}] = [0,0]$, $[f_{21}^{024}, h_{21}^{024}] = [3,5]$, $[f_{24}^{024}, h_{24}^{024}] = [3,3]$, $[f_{33}^{024}, h_{33}^{024}] = [3,5]$, $[f_{34}^{024}, h_{34}^{024}] = [0,0]$ with total transportation cost is [25,58]. |

## 5 CONCLUSION

In this paper, we presented a split and bind method for TSFIITP. The proposed method provides all the possible solutions and thus it can be served as an essential tool for the decision-making persons to select a suitable solution based on their present circumstances in medication problems of logistics.

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