The Arithmetical Edifices of Strength of Connectedness in Intuitionistic Fuzzy Soft Graphs

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Abstract : Fuzzy graphs and soft graphs are two different mathematical frameworks for modeling ambiguous and imprecise data in real-time applications. The concept of a fuzzy soft graph is more generalized as compared with fuzzy graphs and soft graphs. Hence, the fuzzification of soft graph theory has attracted the interest of several researchers, and investigations in this domain have exaggerated over the last few years. The analysis of links (i.e., edges) plays a significant role in Intuitionistic fuzzy soft graphs (IFSG) to understand the nature of the entire real-world problems. In this paper, we present various arithmetical edifices related to types of links such as sturdy, feeble and δ* a weak edge to analyze the strength of connectedness of IFSG. Also, we implement these concepts to solve a real-time decision-making problem.

Keywords: decision making; edges; intuitionistic fuzzy soft graphs; strength of connectedness;

1 INTRODUCTION

Imprecision, uncertainty, and fuzziness are ubiquitous in current real-time applications, and these issues cannot be resolved successfully by conventional mathematical frameworks. Some of the innovative concepts exploited to handle these restrictions include fuzzy set theory [1], soft set theory [2] and intuitionistic fuzzy set theory [3]. Over the past fifty years, the concept of fuzzy set proposed by Zadeh, have paved the way to handle vagueness, ambiguous, and fuzziness of information in real-life problems. Based on the fundamental description of the fuzzy set theory introduced by Zadeh, Kauffman developed the preliminary concept of fuzzy graphs [4]. Fuzzy graphs are now used as an innovative technique for handling perceptions related to an imprecise and uncertain situation. They are hastily entering into the mainstream of science and technology fields including computer science, electrical and electronics engineering, data mining approaches, communication system, information coding, and image processing where the rate of required data of the system changes with multiple levels of precision. Consequently, Bhattacharya [5] proposed some essential insights into fuzzy graphs. Mordeson and Nair performed some basic operations on fuzzy graphs [6].

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Recently, fuzzy graphs having gained more attention from several researchers. Akram et al. extended the notion of fuzzy graphs to the interval-valued fuzzy graph [7], intuitionistic fuzzy graphs [8], intuitionistic fuzzy hyper graphs structures [9], and bipolar fuzzy graphs [10]. Samanta et al. suggested some important concepts on irregular bipolar fuzzy graphs, fuzzy planar graphs, regularity and completeness of the generalized fuzzy graph, and vague graphs [11-13]. However, the main issue in implementing a fuzzy graph is that a membership degree in [0, 1] is fixed for each element in the set for a specific application. Then, it was ratified that a single membership function might not replicate the uncertainty of a real-time environment completely. To overcome this limitation, Molodtsov proposed soft sets as an effective mathematical tool for handling uncertainty problems. The concept of soft set delivers a parameterized view for handling and evaluating uncertainties [14]. The utilization of the soft set is rising rapidly and a number of researchers are aiming at real-world problems with imperfect and vague data. Molodtsov's approach revealed the applicability of soft sets in various domains and realized some noteworthy results continuously improved by other researchers like Maji et al. [15] and Aktas and Cagman [16], and others. Ali et al. discussed some interesting features of soft sets [17]. Sezgin and Atagun studied the theoretical characteristics of the soft sets by performing some elementary operations on soft sets [18]. At present, soft sets are finding several applications in several fields to handle data with uncertainties. In order to evade the uncertainty issues that are integral in each of these notions, some investigators have preferred to integrate the above-mentioned notions in order to implement novel fuzzy-based hybrid frameworks. The most popular among these include fuzzy soft sets [19], [20] interval-valued fuzzy soft sets [21], intuitionistic fuzzy set [22], vague soft sets [23]. But all of these notions have their intrinsic difficulties to assign the membership function in each case. This led to the evolution of the Intuitionistic fuzzy soft graphs model [20], and, successively, the progress and extension of this concept [24]. At present, the theory of IFSG is used to handle vague data in the domain of engineering, environmental science, social science, medical science, and economics. This article explores various kinds of links in IFSG and categorizes the links as sturdy, feeble and δ_a^* –weak. Furthermore, we also exploit the concept of IFSG to find appropriate solutions for a decision-making problem.

2 PRELIMINARIES AND NOTATIONS

We start this section by reviewing some fundamental concepts related to IFSG.

Definition 2.1: A fuzzy set of a non-empty base set $X = \{x_1, x_2, \ldots\ldots x_n\}$ is defined by its degree of membership function U; where $U: X \longrightarrow [0,1]$ assigning to all $x_1 \in X$, the degree to which $X \in U$.

Definition 2.2: A fuzzy graph $G = (X, E)$ is defined as a pair of function $U: X \longrightarrow [0,1]$ and $S: X \times X \longrightarrow [0,1]$, where $E(x_i, x_j) \leq U(x_i) \wedge U(x_j), \forall (x_i, x_j) \in X \times X, \forall x_i, x_j \in X$.
Here X and E are known as node and link of $G = (X, E)$ correspondingly.

Definition 2.3: Consider $G_1 = (x_1, E_1)$ and $G_2 = (x_2, E_2)$ are fuzzy graphs over the given set X. The union operation of G_1 and G_2 is provides a fuzzy graph $G_3 = (x_3, E_3)$ over the set X.
Here $x_3 = x_1 \vee x_2 = max\{U_1(x_i), U_2(x_i)\}$, $\forall x_i \in X$, and $i = 1,2,3 \ldots n$. Similarly $E_3(x_i, x_j) = max\{E_1(x_i, x_j), E_2(x_i, x_j)\}$, $\forall (x_i, x_j) \in X \times X$, where $i, j = 1,2,3 \ldots n$.

Definition 2.4: Consider a non-empty set X. An intuitionistic fuzzy set A in X is denoted as $A = \{X, U_a(x), S_a(x) | x \in X\}$ which is pigeonholed by a membership degree (m-deg) given by $U_a: X \longrightarrow [0,1]$ and the non-membership degree (nm-deg) $S_a: X \longrightarrow [0,1]$ and satisfying the following constraints

(i) $0 \leq U_a(x) + S_a(x) \leq 1, \forall x \in X$
(ii) $0 \leq U_a(x), S_a(x), F_a(x) \leq 1, \forall x \in X$
(iii) $F_a(x) = 1 - U_a(x) - S_a(x)$

Here F_a is known as the intuitionistic fuzzy index of the element $x \in X$; the F_a defines the degree of non-determinacy. If $F_a(x) = 0$ for all $x \in X$, then the intuitionistic fuzzy set X is just Zadeh's fuzzy set.

Definition 2.5: An intuitionistic fuzzy graph is denoted as $G = (X, E, U, S)$ where

1. The non-empty set X is defined as $X = \{x_1, x_2, \ldots\ldots x_n\}$ such that $U_1: X \longrightarrow [0,1]$ and $S_1: X \longrightarrow [0,1]$ define the m-deg and nm-deg of the element $x_i \in X$ correspondingly and $0 \leq U_1(x_i) + S_1(x_i) \leq 1, \forall x_i \in X$ and $i = 1,2,3 \ldots n$.

2. $E \subseteq X \times X$ where $U_2: X \times X \longrightarrow [0,1]$ and $S_2 = X \times X \longrightarrow [0,1]$ are such that
   a) $U_2((x_i, x_j) \leq min\{U_1(x_i), U_1(x_j)\}$
   b) $S_2((x_i, x_j) \leq max\{S_1(x_i), S_1(x_j)\}$ and
   c) $0 \leq U_2(x_i, x_j) + S_2(x_i, x_j) \leq 1$,
   $0 \leq U_2(x_i, x_j), S_2(x_i, x_j), F(x_i, x_j) \leq 1$

where $F(x_i, x_j) = 1 - U_2(x_i, x_j) - S_2(x_i, x_j)$, $\forall (x_i, x_j) \in E$ and $i, j = 1,2,3 \ldots n$.

Definition 2.6: Consider S be a universe of discourse, P be a set of attributes, $P(S)$ be the power set of S and $X \subseteq P$. A pair (F, X) is known as a soft set over S if and only if F is a mapping of X into the set of all subsets of the set S.
Definition 2.7: A pair $(\hat{F}, X)$ is known as a fuzzy soft set over S, where $\hat{F}$ is a mapping defined by $\hat{F}: X \longrightarrow M^S$; the M^S represents the group of all fuzzy subsets of S; and $A \subseteq P$.

Definition 2.8: A pair $(\widehat{F}, X)$ is known as an intuitionistic fuzzy soft set over S, where $\widehat{F}$ is a mapping defined by $\widehat{F}: X \longrightarrow \Gamma^S$; Γ^S represents the group of all intuitionistic fuzzy subsets of S; and $A \subseteq P$.

3 STRENGTH OF CONNECTEDNESS IN INTUITIONISTIC FUZZY SOFT GRAPHS

Definition 3.1: Let $G = (X, E)$ be a simple graph, X is a non-empty set and it is defined as $X = \{x_1, x_2, \ldots\ldots x_n\}$, $E \subseteq X \times X$, P (set of attributes) and $A \subseteq P$. Also consider

1) U_1 is a m-deg given by $U_1: A \longrightarrow \Gamma^S$
$a \mapsto U_1(a) = U_{1a}(say), a \in A$ and
$$U_{1a}: X \longrightarrow [0,1]$$
$$x_i \mapsto U_{1a}(x_i)$$
(A, U_1) denotes an intuitionistic fuzzy soft node (I-node) of m-deg

$$S_1: A \longrightarrow \Gamma^S$$
$a \mapsto S_1(a) = S_{1a}(say), a \in A$ and
$$S_{1a}: X \longrightarrow [0,1]$$
$$x_i \mapsto S_{1a}(x_i)$$
(A, S_1) denotes an I-node of the nm-deg such that $0 \leq U_{1a}(x_i) + S_{1a}(x_i) \leq 1, \forall x_i \in X$ and $a \in A$

2) U_2 is a m-deg given on E and given by
$U_2: A \longrightarrow \Gamma^S(X \times X)$
$a \mapsto U_2(a) = U_{2a}(say), a \in A$ and
$$U_{2a}: X \times X \longrightarrow [0,1]$$
$$(x_i, x_j) \mapsto U_{2a}(x_i, x_j)$$

S_2 is a nm-deg and defined on E by
$S_2: A \longrightarrow \Gamma^S(X \times X)$
$a \mapsto S_2(a) = S_{2a}(say), a \in A$ and
$$S_{2a}: X \times X \longrightarrow [0,1]$$
$$(x_i, x_j) \mapsto S_{2a}(x_i, x_j)$$
where (A, U_2) and $(A, S_2)$are IFSG-links (I-links) of m-deg and nm-deg satisfying

a) $U_{2a}((x_i, x_j) \leq min\{U_{1a}(x_i), U_{1a}(x_j)\}$
b) $S_{2a}((x_i, x_j) \leq max\{S_{1a}(x_i), S_{1a}(x_j)\}$ and
c) $0 \leq U_{2a}(x_i, x_j) + S_{2a}(x_i, x_j) \leq 1$,
$0 \leq U_{2a}(x_i, x_j), S_{2a}(x_i, x_j), F(x_i, x_j) \leq 1, \forall (x_i, x_j) \in X$

The graph $G_{A,X,E} = X, E, (A, U_1), (A, S_1), (A, U_2)(A, S_2)$ is known as the IFSG.

Example 3.1: Let $G = (X, E)$ be a simple graph, where $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{x_1x_2, x_2x_3, x_1x_3, x_1x_4\}$. Let $A = \{a_1, a_2, a_3\}$ be the set of attributes. Then the IFSG $G_{A,X,E} = X, E, (A, U_1), (A, S_1), (A, U_2)(A, S_2)$ is defined as below

$$U_1(a_1) = \{x_1|0.7, x_2|0.8\}$$
$$U_1(a_2) = \{x_1|0.8, x_2|0.5, x_3|0.4\}$$
$$U_1(a_3) = \{x_1|0.3, x_2|0.6, x_3|0.8, x_4|0.8\}$$

$$S_1(a_1) = \{x_1|0.2, x_2|0.1\}$$
$$S_1(a_2) = \{x_1|0.1, x_2|0.3, x_3|0.5\}$$
$$S_1(a_3) = \{x_1|0.5, x_2|0.3, x_3|0.1, x_4|0.1\}$$

$$U_2(a_1) = \{x_1x_2|0.6,\}$$
$$U_2(a_2) = \{x_1x_2|0.3, x_1x_3|0.1, x_2x_3|0.2\}$$
$$U_2(a_3) = \{x_1x_2|0.1, x_1x_3|0.2, x_2x_3|0.5, x_1x_4|0.1\}$$

$$S_2(a_1) = \{x_1x_2|0.1\}$$
$$S_2(a_2) = \{x_1x_2|0.1, x_1x_3|0.1, x_2x_3|0.2\}$$
$$S_2(a_3) = \{x_1x_2|0.3, x_1x_3|0.3, x_2x_3|0.3, x_1x_4|0.3\}$$

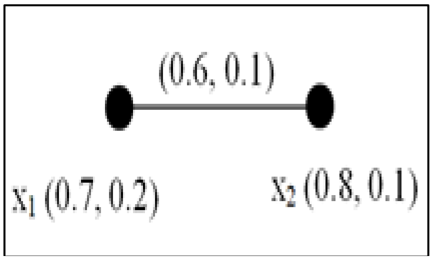

Figure 1(a):IFSG corresponding to a $_1$

Figure 1(a): IFSG corresponding to

Figure 1(b): IFSG corresponding to

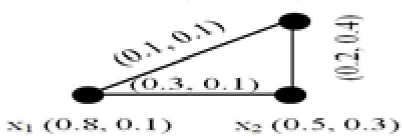

Figure 1(b):IFSG corresponding to a $_2$

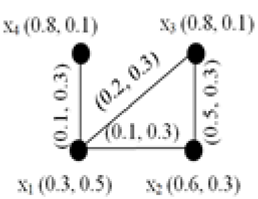

Figure 1(c): IFSG corresponding to

Figure 1(c):IFSG corresponding to a $_3$

Definition 3.2: An IFSG $G_{A,X,E} = X, E, (A, U_1), (A, S_1), (A, U_2)$ (A, S_2) is known as strong-IFSG if $U_{2a}((x_i, x_j)) = min\{U_{1a}(x_i), U_{1a}(x_j)\}$ and $S_{2a}((x_i, x_j)) = max\{S_{1a}(x_i), S_{1a}(x_j)\}, \forall (x_i, x_j) \in X$ and $a \in A$

Definition 3.3: The I-path $\wp$ of length l in an IFSG is defined as a sequence of distinct nodes $x_1, x_2, \ldots \ldots x_{l,}$ $\forall x_i \in X$, such that any one of the following constraint met for each attribute $a \in A$

a) $U_{2a}((x_i, x_j)) > 0$ and $S_{2a}((x_i, x_j)) = 0$ for some i, j
b) $U_{2a}((x_i, x_j)) = 0$ and $S_{2a}((x_i, x_j)) > 0$ for some i, j
c) $U_{2a}((x_i, x_j)) > 0$ and $S_{2a}((x_i, x_j)) > 0$ for some i, j

Definition 3.4: An I-path $\wp$ of length l with vertices $x_1, x_2, \ldots \ldots x_{l,}$ for the attribute $a \in A$ is called I-cycle of length l if $x_1 = x_l$.

Definition 3.5: The strength of m-deg of the I-path $\wp = x_1, x_2, \ldots \ldots x_{l,}$ $\forall x_i \in X$ for the attribute $a \in A$ is denoted as the minimum m-deg of all links in the I-path and it is symbolized by U_a^∞.

Definition 3.6: In an IFSG, the strength of nm-deg of the I-path $\wp = x_1, x_2, \ldots \ldots x_{l,}$ $\forall x_i \in X$ for the attribute $a \in A$ is denoted as the highest nm-deg of all links in the I-path and it is symbolized by S_a^∞.

The strength of an I-path $\wp$ for the attribute $a \in A$ is (U_a^∞, S_a^∞) and it is symbolized by δ_a^∞.

Definition 3.7: In an IFSG, the strength of connectedness (U_a) among vertices x_i and x_j for the attribute $a \in A$ is measured as the maximum strength of m-deg (U_a^∞) of all I-path connecting x_i and $x_j$excluding the link connecting x_i and x_j which is symbolized by $C_{U_a}(x_i, x_j)$.

Definition 3.8: In an IFSG, the strength of connectedness (S_a) among vertices x_i and x_j for the attribute $a \in A$ is measured as the minimum strength of nm-deg (S_a^∞) of all I-path connecting x_i and $x_j$excluding the link connecting x_i and x_j which is symbolized by $C_{S_a}(x_i, x_j)$.

Definition 3.9: In an IFSG, the strength of connectedness among vertices x_i and $x_j$for the attribute $a \in A$ is defined as the least strength of node m-deg (U_a^∞) of all I-path connecting x_i and $x_j$including the link connecting x_i and x_j is called total strength of connectedness and it is symbolized by $TC_{U_a}(x_i, x_j)$.

Definition 3.10: In an IFSG, the strength of connectedness among vertices x_i and $x_j$for the attribute $a \in A$ is measured as the least strength of node nm-deg (S_a^∞) of all I-path connecting x_i and x_j including the link connecting x_i and x_j is called Total strength of connectedness and symbolized by $TC_{S_a}(x_i, x_j)$.

Definition 3.11: In an IFSG $G_{A,X,E}$, a node $x_i \in X$ is called partially U_a –weakest vertex for the attribute $a \in A$ if it has minimum m-deg as compared to other vertexes for $a \in A$ in $G_{A,X,E}$.

Definition 3.12: In an IFSG $G_{A,X,E}$, a node $x_i \in X$ is called partially S_a –weakest vertex for the attribute $a \in A$ if it has maximum node nm-deg as compared to other vertexes for $a \in A$ in $G_{A,X,E}$.

Definition 3.13: In an IFSG $G_{A,X,E}$, a node $x_i \in X$ for the attribute $a \in A$ is called weakest vertex, if it is partially U_a-weakest and partially S_a –weakest vertex for $a \in A$. That is the node $x_i \in X$ for $a \in A$ has the minimum node m-deg and the highest node nm-deg as compared to other vertexes for the attribute $a \in A$ in $G_{A,X,E}$.

Definition 3.14: In an IFSG $G_{A,X,E}$, a node $x_i \in X$ for the attribute $a \in A$ is called partially weakest vertex for $a \in A$, if it is partially U_a –weakest or partially S_a –weakest vertex for $a \in A$ in $G_{A,X,E}$.

Definition 3.15: Let $(x_i, x_j) \in E$ is an I-link of $G_{A,X,E}$ and it is called partially U_a –weakest link for the attribute $a \in A$ if it has minimum link m-deg as compared to other vertexes for $a \in A$ in $G_{A,X,E}$.

Definition 3.16: Let $(x_i, x_j) \in E$ is an I-link of $G_{A,X,E}$ and it is called partially S_a –weakest link for the attribute $a \in A$ if it has least link nm-deg as compared to other vertexes for $a \in A$ in $G_{A,X,E}$.

Definition 3.17: Let $(x_i, x_j) \in E$ is an I-link of $G_{A,X,E}$ and it is called the weakest link for the attribute $a \in A$ if it is partially U_a- weakest and partially S_a –weakest link for $a \in A$ in $G_{A,X,E}$. That is if $(x_i, x_j) \in E$ has minimum link m-deg and maximum link nm-deg as compared to other vertexes $a \in A$ in $G_{A,X,E}$.

Definition 3.18: Let $(x_i, x_j) \in E$ is an I-link of $G_{A,X,E}$ and it is called partially weakest link for the attribute $a \in A$ if it has minimum link m-deg or maximum link nm-deg as compared to other vertexes for $a \in A$ in $G_{A,X,E}$.

Example 3.18: Consider the same problem given in Example 1. In that example, x_1 is the weakest vertex for the attribute a_1, x_3 is the weakest vertex for the attribute a_2 and x_1 is the weakest one for a_3. Similarly the link (x_1, x_3) is the weakest link for a_2, (x_1, x_2) is partially U_{a3} –weakest link for a_3 and (x_1, x_3) is partially S_{a3} –weakest link for the attribute a_3.

Definition 3.19: Let $(x_i, x_j) \in E$ is an I-link of IFSG $G_{A,X,E} = \{X, E, (A, U_1), (A, S_1), (A, U_2)(A, S_2)\}$ is said to be

a) $\alpha - U_a$strong if $U_{2a}(x_i, x_j) > C_{U_a}(x_i, x_j)$ for some $a \in A$
b) $\alpha - S_a$ strong if $S_{2a}(x_i, x_j) < C_{S_a}(x_i, x_j)$ for some $a \in A$
c) $\beta - U_a$ strong if $U_{2a}(x_i, x_j) = C_{U_a}(x_i, x_j)$ for some $a \in A$
d) $\beta - S_a$ strong if $S_{2a}(x_i, x_j) = C_{S_a}(x_i, x_j)$ for some $a \in A$
e) $\gamma - U_a$ weak if $U_{2a}(x_i, x_j) < C_{U_a}(x_i, x_j)$ for some $a \in A$
f) $\gamma - S_a$ weak if $S_{2a}(x_i, x_j) > C_{S_a}(x_i, x_j)$ for some $a \in A$

Example 3.19: Consider an IFSG, $G_{A,X,E} = \{X, E, (A, U_1), (A, S_1), (A, U_2)(A, S_2)\}$ described as below:

$$U_1(a_1) = \{x_1|0.7, x_2|0.8, x_3|0.0, x_4|0.0\}$$
$$U_1(a_2) = \{x_1|0.8, x_2|0.5, x_3|0.4, x_4|0.0\}$$
$$U_1(a_3) = \{x_1|0.3, x_2|0.6, x_3|0.75, x_4|0.75\}$$

$$S_1(a_1) = \{x_1|0.2, x_2|0.1, x_3|0.0, x_4|0.0\}$$
$$S_1(a_2) = \{x_1|0.05, x_2|0.3, x_3|0.5, x_4|0.0\}$$
$$S_1(a_3) = \{x_1|0.5, x_2|0.3, x_3|0.1, x_4|0.1\}$$

$$U_2(a_1) = \{x_1x_2|0.6, x_1x_3|0.0, x_2x_3|0.0, x_1x_4|0.0\}$$
$$U_2(a_2) = \{x_1x_2|0.3, x_1x_3|0.1, x_2x_3|0.2, x_1x_4|0.0\}$$
$$U_2(a_3) = \{x_1x_2|0.01, x_1x_3|0.2, x_2x_3|0.5, x_1x_4|0.1\}$$

$$S_2(a_1) = \{x_1x_2|0.1, x_1x_3|0.0, x_2x_3|0.0, x_1x_4|0.0\}$$
$$S_2(a_2) = \{x_1x_2|0.2, x_1x_3|0.15, x_2x_3|0.2, x_1x_4|0.0\}$$
$$S_2(a_3) = \{x_1x_2|0.1, x_1x_3|0.3, x_2x_3|0.2, x_1x_4|0.3\}$$

Using regular calculation, the value of the strength of m-deg and nm-deg of I-path, U_a –strength and S_a –strength of connectedness among each pair of nodes and the total U_a –strength and S_a –strength of connectedness among each pair of nodes for the attributes a_1, a_2 and a_3 are estimated and tabulated in Table 1.

Table 1: *Strength of connectedness*

Attributes	*End Nodes*	*I-paths*	U_a^∞	S_a^∞	C_{U_a}	TC_{U_a}	C_{S_a}	T
a_1	x_1x_2	$x_1 \leftrightarrow x_2$	0.60	0.10	0.00	0.60	0.00	0.10
a_2	x_1x_2	$x_1 \leftrightarrow x_2$	0.30	0.20	0.10	0.30	0.20	0.20
		$x_1 \leftrightarrow x_3 \leftrightarrow$	0.10	0.20				
	x_1x_3	$x_1 \leftrightarrow x_3$	0.10	0.15	0.20	0.20	0.20	0.15
		$x_1 \leftrightarrow x_2 \leftrightarrow$	0.20	0.20				
	x_2x_3	$x_2 \leftrightarrow x_3$	0.20	0.20	0.10	0.20	0.20	0.20
		$x_2 \leftrightarrow x_1 \leftrightarrow$	0.10	0.20				
a_3	x_1x_2	$x_1 \leftrightarrow x_2$	0.01	0.10	0.20	0.20	0.30	0.10
		$x_1 \leftrightarrow x_3 \leftrightarrow$	0.20	0.30				
	x_1x_3	$x_1 \leftrightarrow x_3$	0.20	0.30	0.01	0.20	0.20	0.20
		$x_1 \leftrightarrow x_2 \leftrightarrow$	0.01	0.20				
	x_2x_3	$x_2 \leftrightarrow x_3$	0.50	0.20	0.01	0.50	0.30	0.20
		$x_2 \leftrightarrow x_1 \leftrightarrow$	0.01	0.30				

From the above table, it is noticed that the link $x_1 \leftrightarrow x_2$ is $\alpha - U_{a2}$ –strong as well as $\beta - S_{a2}$ –strong for the attribute a_2 and $\gamma - U_{a3}$ –weak and $\alpha - S_{a3}$ –strong for a_3. Also the link $x_1 \leftrightarrow x_3$ is $\gamma - U_{a2}$ –weak and $\alpha - S_{a2}$ –strong for the attribute a_2 and $\alpha - U_{a3}$ –strong and $\gamma - S_{a3}$ –weak for the attribute a_3 and the link $x_2 \leftrightarrow x_3$ is $\alpha - U_{a2}$ –strong and $\beta - S_{a2}$ –strong for a_2 and $\alpha - U_{a3}$ –strong and $\alpha - S_{a3}$ –strong for a_3. Therefore it can be observed that a link which is $\alpha - U_a$ –strong or $\beta - U_a$ –strong for an attribute $a \in A$ and does not mean that it is $\alpha - S_a$ –strong and $\beta - S_a$ –strong for the attribute $a \in A$. Furthermore an I-link which is $\alpha - U_a$ –strong or $\beta - U_a$ –strong for some attribute $a \in A$ need not means that it is $\alpha - U_a$ –strong or $\beta - U_a$ –strong for some other attribute $a \in A$.

Definition 3.20: An IFSG is considered as connected if there is an I-path connecting each pair of nodes for all attributes $a \in A$.

Definition 3.21: Let (x_i, x_j) is an I-link and it is named as a U_a −strong link if it is $\alpha - U_a$ −strong or $\beta - U_a$ −strong for some $a \in A$.

Definition 3.22: Let (x_i, x_j) is an I-link and it is named as a S_a −strong link if it is $\alpha - S_a$ −strong or $\beta - S_a$ −strong for some $a \in A$.

Definition 3.23: Let (x_i, x_j) is an I-link and it is named as a sturdy link if it is both U_a −strong and S_a −strong for some $a \in A$.

Definition 3.24: Let (x_i, x_j) is an I-link and it is named as a feeble link if it is either $\gamma - U_a$ −weak or $\gamma - S_a$ −weak link for some $a \in A$.

Definition 3.25: Let (x_i, x_j) is an I-link and it is named as a δ_a^* weak link for the attribute $a \in A$ if it is $\gamma - U_a$ −weak and $\gamma - S_a$ −weak link.

Definition 3.26: In an IFSG, an I-path $\wp$ for the attribute $a \in A$ is called firm I-path if it comprises of only the sturdy links.

Definition 3.27: In an IFSG, an I-path $\wp$ is infirm I-path if it consists of only δ_a^* −weak links.
Consider the problem given in Example 3. In this example, the links $x_1 \leftrightarrow x_2$ and $x_2 \leftrightarrow x_3$ are sturdy for the attribute a_2 and $x_2 \leftrightarrow x_3$ is a sturdy link for a_3. The link $x_1 \leftrightarrow x_3$ is feeble for the attributes a_2 and a_3.

Definition 3.28: In an IFSG, an I-path $\wp$ from x_i to x_j is said to be the strongest I-path for the attribute $a \in A$ if its strength of m-deg and nm-deg equals $TC_{U_a}(x_i, x_j)$ and $TC_{S_a}(x_i, x_j)$. That is $U_a^\infty = TC_{U_a}(x_i, x_j)$ and $S_a^\infty = TC_{S_a}(x_i, x_j)$, for $a \in A$, $x_i, x_j \in X$.

In Example 3.19, the I-paths $x_1 \leftrightarrow x_2$, $x_2 \leftrightarrow x_3$ are the strongest I-paths for the attribute a_2.

4 APPLICATION OF IFSG IN DECISION-MAKING PROBLEM

Most of the problems in Engineering, Medical science, Economics, Environments, etc., have several uncertain information to handle. These uncertainties arise from different sources and cannot be captured within a single mathematical model. The concept of the fuzzy graph is used in many applications for modeling real-time systems. As IFSG provides more accuracy, compatibility, and flexibility to the system than other the fuzzy-based frameworks, this paper demonstrates an application of link analysis in a real-time decision-making problem. An educational institution desires to fill the position of Professor for a single post from the three shortlisted applicants after a discussion and interview over. The selection panel gave positive grade (m-deg) and negative grade (nm-deg) to each applicant according to their performance by considering "Educational qualification, Experience, Research and development activities, Knowledge level, and Communication skill". Let us model this problem with an IFSG. Consider the set of three applicants as the attributes and denoted as X, Y, Z. The kinds of stuff EQ, EX, RD, KL, and CS are considered as nodes of IFSG where 'EQ' denotes educational qualification, 'EX' denotes experience, 'RD' denotes Research and development activities, 'KL' denotes Knowledge level, and 'CS' denotes Communication skill. The selection panel also assign m-deg and nm-deg to combination of stuff like {(EQ, EX), (EQ, KL), (EQ, CS), (EX, RD), (EX, KL), (EX, CS), (RD, CS), (RD, KL), (KL, CS)} by considering combination. For instance, the m-deg or nm-deg of the I-link EQ−EX for attribute X denotes how the applicant X good or not good both in 'Educational qualification and Experience'. The path relating the nodes EQ−KL−CS−RD−EX for the attribute X denotes the grade given to the applicant X by considering all those kinds of stuff. Table 2 and Figure 2 denote the IFSG analogous to the scenario. It is very difficult to relate these three applicants here to provide the right recruitment since all the three applicants have a close degree in various kinds of stuff and their combination. Once we denote the information in the form of an IFSG, we can select a suitable applicant for the post by examining the I-links. Using regular calculation and definitions given in section 2, we observed that the IFSG in Figure 2(b) has more sturdy links and strong paths as compared to IFSGs in Figure 2(a) and Figure 2(c) and moreover it comprises of only one δ_a^* −weak link. The applicant X has two sturdy links (EX−CS) and (EX−KL) and it has firm I-paths (EX−KL), (EX – CS) and (KL−EX−CS). Also, it contains a δ_a^* −weak link (EX–RD). So (EX–RD) is an infirm I-path for the attribute X.

Table 2: *The degree given to the applicants by the selection panel*

Object/Object Combination	*m-deg*	*nm-deg*		*Object/Object Combination*	*m-deg*	*nm-deg*
The degree given to applicant X by the selection panel						
EQ	*0.65*	*0.35*		*EQ - CS*	*0.53*	*0.35*
EX	*0.80*	*0.20*		*EX - RD*	*0.55*	*0.30*
RD	*0.60*	*0.40*		*EX - KL*	*0.80*	*0.15*
KL	*0.85*	*0.15*		*EX - CS*	*0.75*	*0.20*
CS	*0.75*	*0.25*		*RD - KL*	*0.60*	*0.20*
EQ - EX	*0.65*	*0.30*		*RD - CS*	*0.65*	*0.25*
EQ - KL	*0.64*	*0.35*		*KL - CS*	*0.71*	*0.11*
The degree given to applicant Y by the selection panel						
Object/Object Combination	*m-deg*	*nm-deg*		*Object/Object Combination*	*m-deg*	*nm-deg*
EQ	*0.85*	*0.15*		*EQ - CS*	*0.60*	*0.25*
EX	*0.75*	*0.25*		*EX - RD*	*0.60*	*0.30*
RD	*0.65*	*0.30*		*EX - KL*	*0.70*	*0.20*
KL	*0.80*	*0.15*		*EX - CS*	*0.60*	*0.30*
CS	*0.65*	*0.30*		*RD - KL*	*0.65*	*0.25*
EQ - EX	*0.70*	*0.2*		*RD - CS*	*0.60*	*0.20*
EQ - KL	*0.80*	*0.2*		*KL - CS*	*0.63*	*0.30*
The degree given to applicant Z by the selection panel						
Object/Object Combination	*m-deg*	*nm-deg*		*Object/Object Combination*	*m-deg*	*nm-deg*
EQ	*0.75*	*0.20*		*EQ - CS*	*0.65*	*0.15*
EX	*0.70*	*0.30*		*EX - RD*	*0.55*	*0.20*
RD	*0.60*	*0.20*		*EX - KL*	*0.70*	*0.12*
KL	*0.78*	*0.12*		*EX - CS*	*0.68*	*0.12*
CS	*0.70*	*0.11*		*RD - KL*	*0.60*	*0.20*
EQ - EX	*0.70*	*0.20*		*RD - CS*	*0.60*	*0.12*
EQ - KL	*0.70*	*0.20*		*KL - CS*	*0.70*	*0.11*

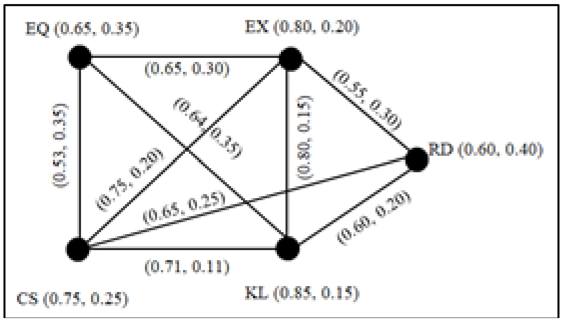

***Figure 2 (**a) : IFSG corresponding to X*

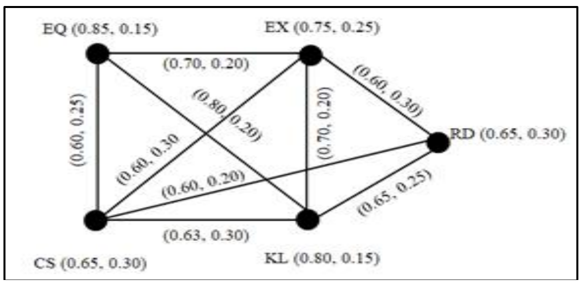

***Figure 2(**b): IFSG corresponding to Y*

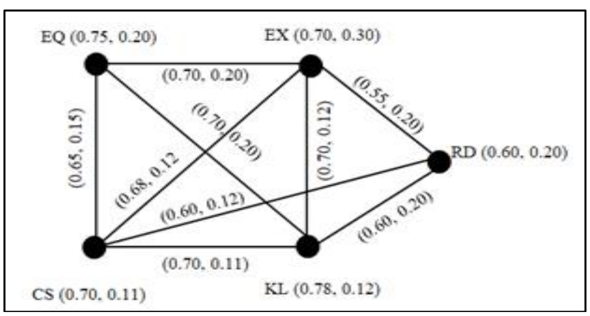

***Figure 2(**c): IFSG corresponding to Z*

When we consider the case of applicant Y, it has four sturdy links (EQ−EX), (EQ−CS), (EX–KL), (RD−KL) and has firm I-paths (EX−EQ–CS), (CS−EQ−EX−KL), (RD−KL−EX), (RD−KL−EX−EQ–CS), (RD−KL−EX–EQ) and has single infirm I-path (EQ−KL). When we consider the case of applicant Z, the sturdy links are (RD–KL) and (KL–CS) and the firm paths are (RD−KL−CS), (RD−KL), (KL−CS). But it has two δ_a^* −weak links (EX−RD) and (EX−KL). The candidate M has more sturdy links and firm I-paths. Therefore when we relate and examine the complete information, applicant Y is appropriate for recruitment. This simple example indicates that the concept of IFSG can be applied to solve decision-making problems effectively. Similarly, IFSG can be exploited in many more domains with uncertainties like machine learning applications, medical diagnosis, and so on.

5 CONCLUSION

Fuzzy graphs and soft graphs are two different mathematical frameworks for modeling ambiguous and imprecise data in real-time applications. The concept of the fuzzy soft graph is more generalized as compared with fuzzy graphs and soft graphs. Hence, the fuzzification of soft graph theory has attracted the interest of several researchers, and investigations in this domain have exaggerated over the last few years. The analysis of links (i.e., edges) plays a significant role in Intuitionistic fuzzy soft graphs (IFSG) to understand the nature of the entire real-world problems. In this paper, we present various arithmetical edifices related to types of links such as sturdy, feeble and δ∗ a weak edge to analyze the strength of connectedness of IFSG. Also, we implement these concepts to solve a real-time decision-making problem.

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