Accelerated Life Testing Design Using Geometric Process For Marshall-Olkin Extended Exponential Distribution With Type I Censored Data

Sadia Anwar, Sana Shahab, Arif Ul Islam

Abstract: In this paper the geometric process is used for the analysis of accelerated life testing for Marshall-Olkin Extended Exponential (MOEE) distribution using Type I censored data. Assuming that the lifetimes under increasing stress levels form a geometric process, the parameters are estimated by using the maximum likelihood method and the original parameters instead of the developing inference for the parameters of the log linear link function are used. The asymptotic interval estimates of the parameters of the distribution using Fisher information matrix are also obtained. The simulation study is conducted to illustrate the statistical properties of the parameters and the confidence intervals.

Index Terms: Asymptotic Confidence Interval, Fisher Information Matrix, Maximum Likelihood Estimation, Survival Function, Simulation Study, Type I Censoring.

1 INTRODUCTION

Manufacturing designs are improving continuously due to advancement in technology; therefore, it is becoming more and more difficult to obtain information about lifetime of products or materials with high reliability at the time of testing under normal conditions. In such problems, accelerated life tests (ALTs) are often used to quickly obtain information on the life time distribution of products by testing them at accelerated conditions than normal use conditions to induce early failures. There are mainly three types of life test methods in accelerated life testing design. The first method is constant stress ALT, second is step-stress ALT and the third is Progressive stress ALT. In constant stress accelerated life testing, we may have several stress levels, which are applied for different groups of the tested items. This means that every item is subjected to only one stress level until the item fails or the test is stopped for other reasons. In step stress accelerated life testing, the test items are subjected to successively higher levels of stress at pre-assigned test times. The level of stress is increased step by step until all items have failed or the test stops for other reasons. Progressive-stress loading is quite like the step stress testing with the difference that the stress level increases continuously. For more Information about ALTs one can consult Nelson [1, 2], Mann, Singpurwalla [3], Meeker and Escobar [4], Bagdonavicius and Nikulin [5], Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some of failure data are missing).

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Complete data consist of the exact failure time of test units, which means that the failure time of each sample unit is observed or known. In many cases when life data are analyzed, all units in the sample may not fail. This type of data is called censored or incomplete data. Due to different types of censoring, censored data can be divided into time-censored (or type I censored) data and failure-censored (or type II censored) data. Time censored (or type I censored) data is usually obtained when censoring time is fixed, and then the number of failures in that fixed time is a random variable. Failure censored (or type II censored) data is obtained when the test is terminated after a specified number of failures, and then time to obtain that fixed number of failures is a random variable. Constant stress ALT with different types of data and test planning has been studied by many authors. For example, Yang [6] introduced an optimal design of 4-level constant-stress ALT plans considering different censoring times. Xiong [7] proposed the Inferences on a simple step-stress model with type-II censored exponential data. Ahmad et al. [8], Islam and Ahmad [9], Ahmad and Islam [10], Ahmad, et al. [11] and Ahmad [12] discuss the optimal constant stress accelerated life test designs under periodic inspection and Type-I censoring. Watkins and John [13] considers constant stress accelerated life tests based on Weibull distributions with constant shape and a log linear link between scale and the stress factor which is terminated by a Type-II censoring regime at one of the stress levels. Ding et al. [14] dealt with Weibull distribution to obtain accelerated life test sampling plans under type I progressive interval censoring with random removals. Pan et al. [15] proposed a bivariate constant stress accelerated degradation test model by assuming that the copula parameter is a function of the stress level that can be described by a logistic function. Chen et al. [16] discuss the optimal design of multiple stresses constant accelerated life test plan on non-rectangle test region. Fan and Yu [17] discuss the reliability analysis of the constant stress accelerated life tests when a parameter in the generalized gamma lifetime distribution is linear in the stress level. The geometric process (GP) model is first introduced by Lam [18, 19] when he studied the problem of repair replacement. Lam and Zhang [20] introduced the geometric process model in the analysis of a two-component series system with one repairman. Lam and Chan [21] derived the maximum likelihood estimate of parameters of the GP with lognormal distribution. Lam [22] proposed the geometric process model for a multistate system and determined an optimal replacement policy to minimize the long run average cost per unit time. Lam [23] introduced least square and modified moment estimation of parameters for GP, and studied the asymptotic normal properties of these estimators. Zhang [24] used the geometrical process to model a simple repairable system with delayed repair. Large amount of studies in maintenance problems and system reliability have shown that a geometric process model is a good and simple model for analysis of data with a single trend or multiple trends. Chen [25] provided the Bayesian approach to the estimation of parameters in a GP with several popular life distributions including the exponential and lognormal distributions. So far, only Huang [26] utilizes the geometric process in the analysis of accelerated life test with complete and censored exponential samples under the constant stress. Kamal et al. [27] extended the GP model for the analysis of ALT with complete Weibull failure data under constant stress and Kamal et al. [28] used the geometric process model for the analysis of accelerated life test for Pareto distribution under constant stress. Anwar et al. [29] proposed the mathematical model of ALT for geometric process for Marshal-Olkin extended exponential distribution for complete data. This article is to focus on the maximum likelihood method for estimating the acceleration factor and the parameters of Marshall-Olkin Extended distribution. This work was conducted for CSALT with type I censored scheme. The confidence intervals for parameters are also obtained by using the asymptotic properties of normal distribution. In the last, the statistical properties of estimates and confidence intervals are examined through a simulation study.

2 MODEL DESCRIPTIONS

2.1 The Geometric Process

A GP is a stochastic process $\{X_n, n=1,2,\ldots\}$ such that $\{\lambda^{n-1}X_n, n=1,2,\ldots\}$ forms a renewal process where $\lambda>0$ is real valued and called the ratio of the GP. It is easy to show that if $\{X_n, n=1,2,\ldots\}$ is a GP and the probability density function of X_1 is $f(x)$ with mean μ and variance σ^2 then the probability density function of X_n will be $\lambda^{n-1}f(\lambda^{n-1}x)$ with mean μ/λ^{n-1} and variance $\sigma^2/\lambda^{2(n-1)}$. It is clear to see that a GP is stochastically increasing if $0<\lambda<1$ and stochastically decreasing if $\lambda>1$. Therefore, GP is a natural approach to analyze the data from a series of events with trend.

2.2 The Marshall-Olkin Extended Exponential Distribution (MOEE)

Marshall and Olkin [29] proposed a new method for adding a parameter to a family of distributions. Suppose we have a given distribution with survival function (SF) $\overline{F}(x), -\infty<x<\infty$ then the Marshall-Olkin extended distribution is defined by the SF

$$\overline{G}(x)=\frac{\alpha\overline{F}(x)}{1-\overline{\alpha}\overline{F}(x)} \qquad -\infty<x<\infty, \alpha>0, \overline{\alpha}=1-\alpha \tag{1}$$

If the survival function of exponential distribution is $\overline{F}(x)=e^{-\theta x}, x,\theta>0$, and put it in equation (1), we obtain the SF

$$\overline{G}(x)=\frac{1-\overline{\alpha}}{e^{\theta x}-\overline{\alpha}} \qquad -\infty<x<\infty, \alpha>0, \overline{\alpha}=1-\alpha \tag{2}$$

The distribution with the survival function (2) is called the MOEE with parameters α and θ. The probability density function (pdf) and the cumulative distribution function (cdf) and the hazard rate of the Marshall-Olkin extended exponential with the survival function (2), respectively are given by

$$g(x;\alpha,\theta)=\frac{\alpha\theta e^{\theta x}}{[e^{\theta x}-\overline{\alpha}]^2}, \qquad -\infty<x<\infty, \alpha>0, \overline{\alpha}=1-\alpha \tag{3}$$

$$G(x;\alpha,\theta)=\frac{e^{\theta x}-1}{[e^{\theta x}-\overline{\alpha}]} \tag{4}$$

$$r(x)=\frac{\theta e^{\theta x}}{[e^{\theta x}-\overline{\alpha}]} \tag{5}$$

when $\alpha=1$, the pdf, cdf, SF and hazard rate reduce to those of the exponential distribution.

2.3 Assumptions

- Let there are s increasing stress levels and under each stress level n items are inspected. The index for stress level is denoted by $k\,;k=1,2,\ldots\ldots s$ and the index for test items under each stress level is denoted by $i\,;i=1,2,\ldots\ldots\ldots n$. In the Type I censoring scheme, the test at each stress level terminates at time t .An item's exact failure time is observed only if its lifetime $x_{ki}\le t$. Assume at the k^{th} stress level we observed r_k failure before the test is terminated, where $0\le r_k\le n$. Correspondingly, $(n-r_k)$ units survive the entire test without failing. The observed ordered failure times under the k^{th} stress level can be written as $x_{k(1)}\le x_{k(2)}\le\ldots\ldots\ldots\le x_{k(r_k)}$ .Note that t is fixed in advance and r_k is random.
- The product life follows a Marshall-Olkin Extended Exponential distribution given by (1) at any stress.
- Let the sequence of random variables $X_0, X_1, X_2,\ldots\ldots, X_n$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily. We assume $\{X_k, k=1,2,3,\ldots..s\}$ is a geometric process with ratio $\lambda>0$.
- At any constant stress level S_k, the mean life $1/\theta_k$ is a log linear function of stress, that is, $\log(1/\theta_k)=a+bS_k$, where a and b are unknown parameters that depend on the nature of the product

and the test method. When $k = 0$, the above equation depicts the relation of the mean life and the designed stress level.

2.4 GP Assumption in ALT

From the above assumption, it can be easily shown that

$$\log\left(\frac{\theta_k}{\theta_{k+1}}\right) = b(S_{k+1} - S_k) = b\Delta S$$

Above relation can be written as

$$\left(\frac{\theta_{k+1}}{\theta_k}\right) = e^{-b\Delta S}$$

It tells that when the increased stress levels form an arithmetic sequence with a constant difference ΔS, the mean life under each stress level forms a geometric sequence with the ratio $e^{-b\Delta S}$. Let $e^{-b\Delta S} = \lambda$, it is clear from the above relation

$$\theta_k = \lambda\theta_{k-1} = \lambda^2\theta_{k-2} = \ldots\ldots\ldots = \lambda^k\theta.$$

In case of MOEE distribution, the pdf of a test item at the k^{th} stress level is:

$$g_{X_k}(x|\alpha,\theta,\lambda) = \frac{\alpha\theta_k e^{\theta_k x}}{[e^{\theta_k x} - \bar{\alpha}]^2} = \frac{\alpha\lambda\theta_{k-1} e^{\lambda\theta_{k-1} x}}{[e^{\lambda\theta_{k-1} x} - \bar{\alpha}]^2} = \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x}}{(e^{\theta\lambda^k x} - \bar{\alpha})^2}$$

This implies that

$$g_{X_k}(x) = \lambda^k g_{x_0}(\lambda^k x)$$

Therefore the pdf of a test item at the k^{th} stress level is

$$g_{X_k}(x) = \lambda^k g(\lambda^k x) = \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x}}{(e^{\theta\lambda^k x} - \bar{\alpha})^2} \quad (6)$$

Consequently, the CDF of the test item at the k^{th} stress level is:

$$G_{X_k}(x \mid \alpha,\theta,\lambda) = \frac{e^{\theta\lambda^k x} - 1}{e^{\theta\lambda^k x} - \bar{\alpha}} \quad (7)$$

The probability that an item censored at time t is:

$$\bar{G}_{X_k}(t) = \frac{1-\bar{\alpha}}{e^{\theta\lambda^k t} - \bar{\alpha}} \quad (8)$$

For those r_k items that fail at time t, their order statistics can be denoted by $X_{k(i)}$ with PDF:

$$g_{X_{K(i)}}(x) = \frac{n!}{(i-1)!(n-i)!} g_{X_{k(i)}} [G_{X_k}(x)]^{i-1} [1 - G_{X_k}(x)]^{n-i}, i = 1,2,\ldots\ldots\ldots, r_k. \quad (9)$$

3 Maximum Likelihood Estimation

The likelihood function of observed data at the k^{th} stress level can be expressed as

$$L_k = \frac{n!}{(n-r_k)!} g_X(x_{k(1)}) \ldots\ldots\ldots\ldots g_{X_k}(x_{k(r_k)}) [\bar{G}_{X_k}(t)]^{n-r_k} \quad (10)$$

We derive the ML estimates of $\alpha, \theta,$ and λ from the likelihood function given by (6). The substitution of (6) and (8) in (10) gives

$$L_k = \frac{n!}{(n-r_k)!} \left(\prod_{i=1}^{r_k} \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x_{ki}}}{\left[e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right]^2} \right) \left(\frac{\alpha}{e^{\theta\lambda^k t} + \alpha - 1} \right)^{n-r_k}$$

$$0 \le x_{k(1)} \le \ldots\ldots\ldots \le x_{k(r_k)} \le t \quad (11)$$

It follows that the likelihood function of observed data in a total s stress levels is:

$$L = \prod_{k=1}^{s} \left[\frac{n!}{(n-r_k)!} \left(\prod_{i=1}^{r_k} \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x_{ki}}}{\left[e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right]^2} \right) \left(\frac{\alpha}{e^{\theta\lambda^k t} + \alpha - 1} \right)^{n-r_k} \right]$$

$$0 \le x_{k(1)} \le \ldots\ldots\ldots\ldots\ldots\ldots\ldots \le x_{k(r_k)} \le t \;; 1 \le k \le s. \quad (12)$$

The log-likelihood function corresponding (8) can be rewritten as

$$l = \sum_{k=1}^{s} \left(\frac{n!}{(n-r)!} \right) + \sum_{k=1}^{s} \sum_{i=1}^{r_k} \left[k\,ln\lambda + ln\alpha + ln\theta + \theta\lambda^k x_{ki} - 2ln\left(e^{\theta\lambda^k x_{ki}}\right) \right]$$

$$+ \sum_{k=1}^{s} (n - r_k) \left[ln\alpha - ln\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right) \right]$$

MLE's of α, θ and λ are obtained by solving the equations

$$\frac{\partial l}{\partial \alpha} = 0, \frac{\partial l}{\partial \theta} = 0 \text{ and } \frac{\partial l}{\partial \lambda} = 0, \text{ where}$$

$$\frac{\partial l}{\partial \alpha} = \frac{ns}{\alpha} - 2\sum_{k=1}^{s}\sum_{i=1}^{r_k} \frac{1}{\left(e^{\theta\lambda^k x_{ki}} + \alpha - 1\right)} - \sum_{k=1}^{s} (n - r_k) \frac{1}{\left(e^{\theta\lambda^k t} + \alpha - 1\right)}$$

(13)

$$\frac{\partial l}{\partial\theta}=\frac{r_k s}{\theta}+\sum_{k=1}^{s}\sum_{i=1}^{r_k}\lambda^k x_{ki}-2\sum_{k=1}^{s}\sum_{i=1}^{r_k}\frac{\lambda^k x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)}-\sum_{k=1}^{s}\frac{(n-r_k)\lambda^k t e^{\theta\lambda^k t}}{\left(e^{\theta\lambda^k t}+\alpha-1\right)} \quad (14)$$

$$\frac{\partial l}{\partial\lambda}=\sum_{k=1}^{s}\sum_{i=1}^{r_k}\frac{k}{\lambda}+\theta\sum_{k=1}^{s}\sum_{i=1}^{r_k}k\lambda^{k-1}x_{ki}-2\theta\sum_{k=1}^{s}\sum_{i=1}^{r_k}\frac{k\lambda^{k-1}x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)}-\sum_{k=1}^{s}\frac{(n-r_k)\theta k\lambda^{k-1}t e^{\theta\lambda^k t}}{\left(e^{\theta\lambda^k t}+\alpha-1\right)} \quad (15)$$

The MLEs of $\alpha,\theta,$ and λ exist but do not have a closed form. The Newton Iterative Method is applied to obtain $\hat{\alpha},\hat{\theta},$ and λ.

4 Fisher Information Matrix

The Fisher information matrix F is obtained by taking the negative second partial derivatives of the log-likelihood function and can be written

$$F=\begin{bmatrix}-\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\theta} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda}\\ -\frac{\partial^2 l}{\partial\theta\partial\alpha} & -\frac{\partial^2 l}{\partial\theta^2} & -\frac{\partial^2 l}{\partial\theta\partial\lambda}\\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda\partial\theta} & -\frac{\partial l}{\partial\lambda^2}\end{bmatrix}$$

Elements of Fisher Information matrix are

$$\frac{\partial^2 l}{\partial\alpha^2}=-\frac{ns}{\alpha^2}+2\sum_{k=1}^{s}\sum_{i=1}^{r_k}\frac{1}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}+\sum_{k=1}^{s}\frac{(n-r_k)}{\left(e^{\theta\lambda^k t}+\alpha-1\right)^2}$$

$$\frac{\partial^2 l}{\partial\theta^2}=-\frac{r_k s}{\theta^2}-2\sum_{k=1}^{s}\sum_{i=1}^{r_k}\left[\frac{(\alpha-1)\lambda^{2k}x_{ki}^2 e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}\right]-\sum_{k=1}^{s}\left[\frac{(n-r_k)(\alpha-1)\lambda^{2k}t^2 e^{\theta\lambda^k t}}{\left(e^{\theta\lambda^k t}+\alpha-1\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\lambda^2}=-\sum_{k=1}^{s}\sum_{i=1}^{r_k}\frac{k}{\lambda^2}+\theta\sum_{k=1}^{s}\sum_{i-1}^{r_k}k(k-1)\lambda^{k-2}x_{ki}$$

$$-2\sum_{k=1}^{s}\sum_{i=1}^{r_k}\left[\frac{\theta k(k-1)\lambda^{k-2}x_{ki}e^{2\theta\lambda^k x_{ki}}+(\alpha-1)\left\{\theta k(k-1)\lambda^{k-2}x_{ki}e^{\theta\lambda^k x_{ki}}+\theta^2k^2\lambda^{2k-2}x_{ki}^2e^{\theta\lambda^k x_{ki}}\right\}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}\right]$$

$$-\sum_{k=1}^{s}(n-r_k)\left[\frac{\theta k(k-1)\lambda^{k-2}te^{2\theta\lambda^k t_i}+(\alpha-1)\left\{\theta k(k-1)\lambda^{k-2}te^{\theta\lambda^k t}+\theta^2k^2\lambda^{2k-2}t^2e^{\theta\lambda^k t}\right\}}{\left(e^{\theta\lambda^k t}-1+\alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\alpha\partial\theta}=\frac{\partial^2 l}{\partial\theta\partial\alpha}=2\sum_{k=1}^{s}\sum_{i=1}^{r_k}\left[\frac{\lambda^k x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}\right]+\sum_{k=1}^{s}\left[\frac{(n-r_k)\lambda^k te^{\theta\lambda^k t}}{\left(e^{\theta\lambda^k t}-1+\alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\alpha\partial\lambda}=\frac{\partial^2 l}{\partial\lambda\partial\alpha}=2\sum_{k=1}^{s}\sum_{i=1}^{r_k}\left[\frac{\theta k\lambda^{k-1}x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}\right]+\sum_{k=1}^{s}\left[\frac{\theta k\lambda^{k-1}te^{\theta\lambda^k t}}{\left(e^{\theta\lambda^k t}-1+\alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\theta\partial\lambda}=\frac{\partial^2 l}{\partial\lambda\partial\theta}=\sum_{k=1}^{s}\sum_{i=1}^{r_k}k\lambda^{k-1}x_{ki}$$

$$-2\sum_{k=1}^{s}\sum_{i=1}^{r_k}\left[\frac{k\lambda^{k-1}x_{ki}e^{2\theta\lambda^k x_{ki}}+(\alpha-1)\{k\lambda^{k-1}x_{ki}e^{\theta\lambda^k x_{ki}}+\theta k\lambda^{2k-1}x_{ki}^2e^{\theta\lambda^k x_{ki}}\}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}\right]$$

$$-\sum_{k=1}^{s}(n-r_k)\left[\frac{k\lambda^{k-1}te^{2\theta\lambda^k t}+(\alpha-1)\{k\lambda^{k-1}x_{ki}e^{\theta\lambda^k t}+\theta k\lambda^{2k-1}x_{ki}^2e^{\theta\lambda^k t}\}}{\left(e^{\theta\lambda^k t}-1+\alpha\right)^2}\right]$$

5 Asymptotic Confidence Interval Estimates

The variance covariance and covariance matrix of the parameter can be written as

$$\Sigma=\begin{bmatrix}-\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\theta} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda}\\ -\frac{\partial^2 l}{\partial\theta\partial\alpha} & -\frac{\partial^2 l}{\partial\theta^2} & -\frac{\partial^2 l}{\partial\theta\partial\lambda}\\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda\partial\theta} & -\frac{\partial l}{\partial\lambda^2}\end{bmatrix}^{-1}$$

$$=\begin{bmatrix}AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\theta}) & ACov(\hat{\alpha}\hat{\lambda})\\ ACov(\hat{\theta}\hat{\alpha}) & AVar(\hat{\theta}) & ACov(\hat{\theta}\hat{\lambda})\\ ACov(\hat{\lambda}\hat{\alpha}) & ACov(\hat{\lambda}\hat{\theta}) & AVar(\hat{\lambda})\end{bmatrix}$$

The $100(1-\beta)\%$ asymptotic confidence interval for α,θ and λ are then given respectively as

$$\left[\hat{\alpha}\pm Z_{1-\frac{\beta}{2}}\sqrt{AVar(\hat{\alpha})}\right],$$

$$\left[\hat{\theta}\pm Z_{1-\frac{\beta}{2}}\sqrt{AVar(\hat{\theta})}\right]$$

and

$$\left[\hat{\lambda} \pm Z_{1-\frac{\beta}{2}} \sqrt{AVar(\hat{\lambda})}\right]$$

6 Simulation Study

Simulation of data is the initial task for studying different properties of parameters. A simulation is an attempt to model a hypothetical situation to study how a function works. By changing variables in the simulation, predictions may be made about the behaviour of the function. Main motive of simulation is to obtain MLEs of estimator and judge their performance for this model. For this task following steps are involved which are as follows:

1. Initial Values of parameter is suppose to be θ=0.2, α =0.9 and λ =4 for stress value s=2 & 4. The termination time t is chosen to be 2 and 4. The combinations are chosen as s= (2, 4) and t= (2, 4).
2. Random sample of size n=40, 60, 80,100 & 200 are generated.
3. MLEs $\hat{\theta}, \hat{\lambda}$ and $\hat{\alpha}$ are estimated from equation (13), (14) and (15) by Newton iterative method.
4. Measure of accuracy like Mean Square Error (MSE) is calculated for checking the performance of estimator. Lesser the values of MSE better will be the estimated results.
5. Finally 95% confidence interval coverage is also calculated.

The results obtained in the above simulation study are summarized in Table 1, 2, 3 & 4. It shows the MLE, MSE, 95% confidence interval and probability coverage of the estimators for different combinations of stress and termination time t for same value of θ, λ and α.

TABLE 1
SIMULATIONS RESULTS BASED ON CENSORED DATA FROM GP MOEE WITH θ=0.2, α =0.9 and λ =4 FOR s=2 & t=2

n	PARAMETER	MLE	MSE	95% CONFIDENCE INTERVAL LCL	UCL	95% CONFIDENCE INTERVAL COVERAGE
	θ	1.2635	0.7541	1.1821	1.4714	0.93823
40	α	0.0624	0.0632	0.0514	0.0765	0.92847
	λ	2.7351	1.3612	2.1338	2.9644	0.93947
	θ	1.2725	0.6431	1.1721	1.4762	0.93161
60	α	0.0725	0.0643	0.0654	0.0876	0.92235
	λ	2.4744	1.3182	2.2356	2.9762	0.93989
	θ	1.2475	0.3212	1.1781	1.4672	0.94612
80	α	0.0725	0.0545	0.0622	0.0965	0.92812
	λ	2.4173	1.2635	2.2951	2.4189	0.92962
	θ	1.2262	0.3651	1.1862	1.4183	0.93912
100	α	0.0619	0.0251	0.0544	0.0678	0.93891
	λ	2.2173	1.1173	2.1836	2.3826	0.95751
	θ	1.2624	0.2532	1.1853	1.4825	0.94715
200	α	0.0601	0.0133	0.0573	0.0633	0.95150
	λ	2.2018	1.1026	2.1831	2.2030	0.95514

TABLE 2

SIMULATIONS RESULTS BASED ON CENSORED DATA FROM GP MOEE WITH θ=0.2, α =0.9 and λ =4 FOR s=2 & t=4

n	PARAMETER	MLE	MSE	95% CONFIDENCE INTERVAL LCL	UCL	95% CONFIDENCE INTERVAL COVERAGE
	θ	1.2811	0.6448	1.1179	1.4344	0.93297
40	α	0.0872	0.0776	0.0534	0.0912	0.92179
	λ	2.6902	1.3289	2.2841	2.9857	0.92901
	θ	1.2689	0.5989	1.2421	1.3765	0.93673
60	α	0.0781	0.0574	0.0543	0.0978	0.94863
	λ	2.5428	1.3174	2.3632	2.6574	0.93876
	θ	1.2451	0.3494	1.2086	1.3864	0.95873
80	α	0.0680	0.0598	0.0579	0.0733	0.93960
	λ	2.4834	1.3565	2.3865	2.4986	0.93863
	θ	1.2385	0.2969	1.2018	1.4876	0.95957
100	α	0.0545	0.0380	0.0496	0.0686	0.94644
	λ	2.2542	1.1796	2.0576	2.3674	0.95968
	θ	1.2468	0.1978	1.1563	1.3667	0.95875
200	α	0.0583	0.0183	0.0579	0.0590	0.96743
	λ	2.2046	1.1064	2.2035	2.2058	0.96987

TABLE 3

SIMULATIONS RESULTS BASED ON CENSORED DATA FROM GP MOEE WITH θ=0.2, α =0.9 and λ =4 FOR s=4 & t=2

n	PARAMETER	MLE	MSE	95% CONFIDENCE INTERVAL LCL	UCL	95% CONFIDENCE INTERVAL COVERAGE
	θ	1.2071	0.3763	1.1478	1.2689	0.93751
40	α	0.0581	0.0611	0.0515	0.0679	0.93437
	λ	2.5816	0.8754	2.2678	2.6460	0.94789
	θ	1.1387	0.3987	1.1298	1.4688	0.93468
60	α	0.0573	0.0374	0.0475	0.0689	0.94682
	λ	2.3715	0.7621	2.2353	2.4877	0.94896
	θ	1.1216	0.2451	1.1997	1.3789	0.95879
80	α	0.0478	0.0375	0.0254	0.0571	0.94758
	λ	2.2711	0.5832	2.1765	2.3686	0.94780
	θ	1.0614	0.2274	1.0145	1.0897	0.95699
100	α	0.0371	0.0231	0.0146	0.0476	0.95789
	λ	2.1735	0.5727	2.1797	2.1758	0.96895
	θ	1.0163	0.1836	1.0573	1.0174	0.95835
200	α	0.0172	0.0184	0.0154	0.0468	0.95975
	λ	2.0812	0.4725	2.0143	2.0976	0.96573

TABLE 4

SIMULATIONS RESULTS BASED ON CENSORED DATA FROM GP MOEE WITH θ=0.2, α =0.9 and λ =4 FOR s=4 & t=4

n	PARAMETER	MLE	MSE	95% CONFIDENCE INTERVAL LCL	UCL	95% CONFIDENCE INTERVAL COVERAGE
	θ	1.1689	0.3358	1.1577	1.1957	0.94588
40	α	0.0535	0.0586	0.0504	0.0646	0.93536
	λ	2.4779	0.7542	2.3579	2.5694	0.94743
	θ	1.1487	0.2648	1.1258	1.3648	0.95864
60	α	0.0442	0.0356	0.0364	0.0598	0.95242
	λ	2.3563	0.6335	2.2578	2.4352	0.94785
	θ	1.1185	0.2578	1.1056	1.2568	0.95544
80	α	0.0367	0.0334	0.0277	0.0425	0.95584
	λ	2.2543	0.4866	2.2948	2.2754	0.94442
	θ	1.0733	0.2146	1.0264	1.0786	0.95464
100	α	0.0346	0.0244	0.0279	0.0578	0.96463
	λ	2.1654	0.4686	2.1896	2.1780	0.96774
	θ	1.0164	0.1245	1.0265	1.0178	0.95953
200	α	0.0143	0.0154	0.0125	0.0170	0.95683
	λ	2.0476	0.3669	2.0356	2.0865	0.96785

7 Conclusions

Use of Geometric process not only makes the calculations easier but helps in getting good estimator also. As we can see from tables that estimator have very less MSE and probability coverage is also satisfactory. It is quite noticeable from table 3 & 4 that the values of estimators are more stable for combinations (s=2, t=4) & (s=4, t=4). The proposed estimator $\hat{\theta}$, is consistently larger than the true value of θ, which means we tend to underestimate the mean lifetime of the product at the normal stress level. For the same sample size, the mean squared errors of the estimators slightly decrease as the value of t increases. This is quite expected because large values of t would lead to more failures before the test ends, and thus increase the efficiency of the estimators. In comparison with complete case of Marshall-Olkin extended exponential distribution [29] the values of estimators are relatively low and stable. Thus this model works well for censored case.

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References

[1] W. B. Nelson, "Accelerated life testing: step-stress models and data analyses", *IEEE Transactions on Reliability*, vol 29, pp. 103-108, 1980.

[2] W. B. Nelson, "Accelerated testing: Statistical models, test plans and data analyses", Wiley, New York, 1980.

[3] N.R. Mann and N.D. Singpurwalla, "Life Testing: Encyclopaedia of statistical sciences", Wiley, USA, Vol. 4, pp. 632-639, 1983.

[4] W.Q. Meeker and L.A. Escobar, "Statistical methods for reliability data", Wiley, New York, 1980.

[5] V. Bagdonavicius and M. Nikulin, "Accelerated life models: Modeling and statistical analysis", Chapman & Hall, Boca Raton, 2002.

[6] G.B. Yang, "Optimum constant-stress accelerated life-test plans, *IEEE Transactions on Reliability",* vol. 43, no. 4,pp. 575-581,1994.

[7] C. Xiong, "Inferences on a simple step-stress model with type-II censored exponential data", *IEEE Transactions on Reliability*, vol. 47, pp. 142-146, 1998

[8] Ahmad et.al, "Optimal Design of Accelerated Life Test Plans under Periodic Inspection and Type I Censoring: The Case of Rayleigh Failure Law", *South African Statistical Journal,* vol. 28, pp. 27-35, 1994.

[9] A. Islam and N. Ahmad, "Optimal design of accelerated life tests for the Weibull distribution under periodic inspection and type I censoring", *Microelectronics Reliability,* vol. 34, no. 9, pp. 1459-1468, 1994.

[10] N. Ahmad and A. Islam, "Optimal accelerated life test designs for Burr type XII distributions under periodic inspection and type I censoring", *Naval Research Logistics*, vol. 43, pp.1049-1077,1996 .

[11] N. Ahmad, A. Islam and A. Salam, "Analysis of optimal accelerated life test plans for periodic inspection: The case of Exponentiated Weibull failure model", *International Journal of Quality & Reliability Management,* vol. 23, no. 8, pp. 1019-1046,2006.

[12] N. Ahmad, "Designing Accelerated Life Tests for Generalized Exponential Distribution with Log-linear Model", *International Journal of Reliability and Safety,* vol. 4, no. 2/3, pp. 238-264, 2010.

[13] A.J. Watkins, and A.M. John, "On constant stress accelerated life tests terminated by Type II censoring at one of the stress levels", *Journal of Statistical Planning and Inference,* vol. 138, no. 3, pp 768-786,2008.

[14] C. Ding, C. Yang and S.K. Tse, "Accelerated life test sampling plans for the the Weibull distribution under type I progressive interval censoring with random removals", *Journal of Statistical Computation and Simulation,* vol. 80, no. 8, pp. 903-914, 2010.

[15] Z. Pan, N. Balakrishnan, and Quan Sun, "Bivariate constant-stress accelerated degradation model and inference", *Communications in Statistics-Simulation and Computation,* vol. 40, no.2, 247–257, 2011.

[16] Chen et. al, "Optimal design of multiple stress constant accelerated life test plan on non-rectangle test region", *Chinese Journal of Mechanical Engineering*, vol. 25, no. 6, pp. 1231-1237,2012.

[17] T.H. Fan, and C.H. Yu, "Statistical Inference on Constant Stress Accelerated Life Tests under Generalized Gamma Lifetime Distributions", *Quality and Reliability Engineering International,* 2012.

[18] Y. Lam, "Geometric process and replacement problem", *Acta Mathematicae Applicatae Sinica*, vol. 4, pp. 366-377, 1988 a.

[19] Y. Lam, "A note on the optimal replacement problem", *Advances in Applied Probability*, vol. 20, pp. 479-482, 1988b.

[20] Y. Lam and Y.L. Zhang, "Analysis of a two-component series system with a geometric process model", *Naval Research Logistics*, vol. 32, pp. 491-502, 1996.

[21] Y. Lam and S.K. Chan, "Statistical inference for geometric processes with lognormal distribution", *Computational Statistics and Data Analysis,* vol. 27, pp. 99-112, 1998.

[22] Y. Lam, "A monotone process maintenance model for multistate system", *Journal of Applied Probability*, vol. 32, pp. 212-223, 2005.

[23] Y Lam, "The geometric process and its application", *World Scientific Publishing Co. Pte. Ltd., Singapore*, 2007.

[24] Y.L. Zhang, "A geometrical process repair model for a repairable system with delayed repair", *Computers and Mathematics with Applications*, vol. 55, pp. 1629-1643, 2008.

[25] J. Chen, "Bayesian computation for geometric process in maintenance problems", *Mathematics and Computers in Simulation,* vol. 81, pp. 771-781, 2010.

[26] S.Huang, "Statistical inference in accelerated life testing with geometric process model", Master's thesis, San Diego State University, 2011.

[27] M. Kamal, S. Zarrin, S. Saxena and A. Islam, "Weibull Geometric Process Model for the Analysis of Accelerated Life Testing with Complete Data", *International Journal of Statistics and Applications*, vol. 2, no. 5, pp. 60-66,2012.

[28] M. Kamal, S. Zarrin, S. Saxena, and A.Islam, "Accelerated life test design using geometric process for Pareto distribution", vol.1, no.2, pp. 25-31, 2013.

[29] S. Anwar, M. Kamal and A. Islam, "Mathematical model of accelerated life testing using geometric process for Marshall-Olkin extended exponential distribution" , *International Journal of Innovative Research in Science , Engineering and Technology,* vol.2, no.12, pp. 7382-7390, 2013

[30] A.N. Marshall and I. Olkin, "A new method for adding a parameter to a family of distributions with applications to the exponential and Weibull families", *Biometrica,* Vol. 84, pp. 641-652,1997.