On Comparative Modeling Of Gls And Ols Estimating Techniques

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ABSTRACT: In this study GLS and OLS estimating techniques were compared. To achieve the goal, GLS and OLS estimating techniques were applied on a simultaneous equation models (that is Per Capital Gross Domestic Product equation model and Foreign Direct Investment equation model). Annual data was collected for Per Capital Gross Domestic Product, Foreign Direct Investment, Lending Rate of Interest and Exchange Rate Index for the period of 1983 to 2008 from the National Bureau of Statistics (NBS) and Central Bank of Nigeria Statistical Bulleting (2009). Results from the analysis showed that GLS and OLS estimating techniques produced the same values of coefficients and standard errors in the two equations. The study however concluded that the two estimators are both efficient alike, which shows that the GLS estimator is an OLS estimator of a transformed isomorphic model. The R-package of statistical software was adopted. The two estimators provide BLUE (Best Linear Unbiased Estimator) under heteroscedasticity/serial correlation.

KEY WORDS: Generalized Least Squares (GLS), Ordinary Least Square (OLS)

1. INTRODUCTION

In statistics, Generalized Least Squares (GLS) is a technique for estimating the unknown parameters in a linear regression model. The GLS is applied when the variances of the observations are unequal (heteroscedaticity), or when there is a certain degree of correlation between the observations. In these cases, Ordinary Least Squares (OLS) can be statistically inefficient, or even give misleading inferences. On the other hand, OLS which emerged in the early years of the nineteenth century was centered on dominant and powerful estimating principle. However, OLS which is the cornerstone of most econometric theory was developed and published by Carl Friedrich Gauss which have been discussed in (Gujarat, 2003; Upton et al, 2002), is an estimation technique used in regression analysis, which is a method that studies the relationship between two or more variables. Supposed that Gauss – Markov assumptions hold for all equations, thus the transformed variables in the equation satisfy the conditions under which OLS is BLUE (Best Linear Unbiased Estimator). The coefficient vector from the OLS regression of Y on X is the Generalized Least Squares (GLS) estimator. However, by using the GLS method to estimate the equations jointly, efficiency is obtained. (http://en.wikipedia.org/wiki/generalizedleast-squares-regression).

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Efficiency is one of the properties of a good estimator. The concept refers to the one with smallest variance for any given sample size. (Udom, Akaninyene Udo, 2005). In this study, the GLS and OLS estimating technique will be adopted on a simultaneous equation models (that is Per Capital Gross Domestic Product equation model and Foreign Direct Investment equation model and then confirm which estimator is most efficient).

2. MODEL SPECIFICATION

I. Generalized Least Square

The Generalized Least Square (GLS) model consists of $\{y_i, x_{ij}\}$; i=1… n, j = 1… p on n statistical units. The response values are placed in a vector $Y = (y_1, ..., y_n)'$, and the predictor values are placed in the design matrix $X = \|X_{ij}\|$, where X_{ij} is the values of the j_{th} predictor variable for the i_{th} unit. The model assumes that the conditional mean of Y given X is a linear function of X, whereas the conditional variance of Y given X is a known matrix Ω. This is usually written as $Y_i = X_i\beta_i + \varepsilon_i, \quad i = 1,2,3..., m$ Where y_i (n x 1), X_i (n x k_i), β_i (k_i x 1) $E[\varepsilon / x] = 0, \quad \mathrm{var}[\varepsilon / x] = \Omega$. Here β is a vector of unknown "regression coefficient" that must be estimated from the data. If ε_{it} is the t^{th} element of ε_i, we assumed that $(\varepsilon_i, \varepsilon_{2t}, ..., \varepsilon_{mt})$ is iid with $E(\varepsilon_{it}) = 0$.

$$E(\varepsilon_{it} \quad \varepsilon_{js}) = \begin{cases} \sigma_{ij} & if \quad t = s \\ 0 & if \quad t \neq s \end{cases}$$

(Non zero contemporaneous correlation) If we stack the m equations we have

$$\begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_m \end{pmatrix} = \begin{pmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ . & . & \dots & \\ . & . & & \\ . & . & \dots & \\ 0 & 0 & \dots & x_m \end{pmatrix}_{\left(nm\sum_{i=1}^{m} k_i\right)} \begin{pmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_m \end{pmatrix}_{\left(\sum_{i=1}^{m} k_i, 1\right)} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ . \\ \varepsilon_m \end{pmatrix}_{(n,m,1)}$$

that is $Y = X\beta + \varepsilon$, $E(\varepsilon) = 0$

$$E(\varepsilon\varepsilon)^{/} = v = \sum \otimes I_n = \begin{bmatrix} \sigma_{11}I_n & \sigma_{12}I_n & \dots \sigma_{im}I_n \\ \sigma_{21}I_n & \sigma_{22}I_n & \dots \sigma_{2m}I_n \\ . & . & \\ . & . & \\ . & . & . \\ \sigma m1 & \sigma m2 & \dots \sigma_{mn}I_n \end{bmatrix}$$

The GLS estimator of β is:

$$\hat{\beta}_{GLS} = \left(X^{/}v^{-1}x\right)^{-1} X^{/}v^{-1}y = \left[X^{/}\left(\sum{}^{-1} \otimes I_n\right)X\right]^{-1} X^{/}\left(\sum{}^{-1} \otimes I_n\right)Y$$

Since from Kronecker product $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$

$$V\left(\hat{\beta}_{GLS}\right) = \left(X^{/}v^{-1}X\right)^{-1} = \left[X^{/}\left(\sum{}^{-1} \otimes I_n\right)X\right)^{-1}$$

Which have been discussed in Greene, 1998; Zellers, 1962; Griffiths et al 1993; Ferdinand et al, 2008; GLS (http://200.32.4.58/econometrical/ssem.pdf); GLS (http://psweb.sbs.ohiostate.edu).

II. Ordinary Least Squares

The method of least squares is a standard approach to the approximate solution of over determined systems i.e. sets of equations in which there are more equations than unknown. The least squares method is usually credited to Carl Friedrich Gauss (1794), but it was first published by Adrien – Marie Legendre. "Least Squares" means that the overall solution minimizes the sum of the squares of the errors made in the result of every single equation. (http://en.wikipedia.org/wiki/leastsquares). Multiple regression model is a type of model in which a response variable Y is determine by two or more predictor/explanatory variables X_1, X_2, …, X_k (A. I. Arua et al, 1999). The model is of the form Y=Xβ + ε and the desire formula is $\hat{\beta} = \left(X^{/}X\right)^{-1} X^{/}Y$. The variance is given by V (β) $=\sigma^2(X^{/}X)^{-1}$, we determine the scalar σ^2, being discussed in (Pindyck and Rubinfield, 1981). The coefficient of determination (denoted by R^2) measures the proportionate reduction of total variation in Y associates with the use of the set of X variable X_1, … X_{p-1}. (Peter et al, 2003). It is given by $R^2 = \dfrac{SS_{total} - SS_e}{SS_{total}}$ have values of R^2 close to 0 indicate a poor fit.

III. Simultaneous Equation Models

An economic model may contain multiple equations, which are explanatory of each other on the surface. They are not estimating the same response variable; they have different explanatory variable e.t.c. (http://en.wikipedia.org/wiki/seemingly-unrelated-regression).

IV. The Model

The models of interest are

$PGDP = \alpha_0 + \alpha_1 ERI + \alpha_2 LDR + \alpha_3 FDI + \varepsilon_1$

$FDI = \beta_0 + \beta_1 ERI + \beta_2 LDR + \varepsilon_2$

Here we have two equations with two endogenous variables (PGDP and FDI) and two exogenous variables (ERI and LDR).

V. Definition of Terms

PGDP – Per Capital Gross Domestic Product

FDI – Foreign Direct Investment

LDR – Lending Rate of Interest

ERI – Exchange Rate Index

3. MATERIAL

The data used for this study was obtained from the National Bureau of Statistics; Annual Abstract of Statistics 2009 Federal Republic of Nigeria, December, 2009.

CBN Statistical Bulletin, December, 2009

PERIOD: - 1983 to 2008

NOTE:- GDP per capital was generated by dividing National Income by population i.e. $\dfrac{National.Income}{Population}$

4. DISCUSSION OF RESULTS

Two different estimation techniques used to obtain the estimated parameters are GLS and OLS. The R – package of statistical software was adopted to obtain the results and necessary discussions were made.

(i) Per Capital Gross Domestic Product model for GLS and OLS estimators are displayed below:-

Estimator: Generalized Least Squares

Equation 1: GLS estimates using 26 observations 1983 – 2008
Dependent Variable: Per Capital Gross Domestic Product.

From the GLS output in table 1(b) below, the model becomes

PGDP = -2071.6403 + 155.8892ERI + 399.1797LDR + 0.1218FDI

Table 1a: OLS OUTPUT

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-2071.640276	4090.281852	-0.50648	0.61755908
ERI	155.889165	59.980389	2.59900	0.01638170 *
LDI	399.179655	216.685190	1.84221	0.07896469
FDI	0.121842	0.032123	3.79299	0.00099792

Table 1b: GLS OUTPUT

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	8874.821	26485.955	0.33508	0.74060
ERI	1700.887	160.631	10.58877	2.5697e-10 ***
LDI	-166.971	1406.099	-0.11875	0.90651

Mean of dependent variable = 20428.18

Standard deviation of dependent variable = 20244.61

Sum of squared residuals = 727703018.385395

Standard error of residuals = 5751.296369

R – Squared = 0.928978

Estimator: Ordinary Least Squares
Equation 1: OLS estimates using 26 observations 1983 – 2008

Dependent variable: Per Capital Gross Domestic Product

From the OLS output in the table 1(a) above, the model becomes:

PGDP = -2071.6403 + 155.8892 ERI + 399.17971LDR + 0.1218FDI

Mean of dependent variable = 20428.18

Standard deviation of dependent variable = 20244.61
Sum of squared residuals = 727703018.385395

Standard error of residuals = 5751.296369

R – Squared = 0.928978

Interpretation: The two estimators produced the same value of coefficients and standard errors. This result revealed that both of them are efficient alike, which shows that the GLS estimator is an OLS estimator of a transformed isomorphic model.

(ii) Foreign Direct Investment model for GLS and OLS estimation are displayed below:-

Estimator: Generalized Least Squares
Equation 2: GLS estimates using 26 observations 1983 – 2008

Dependent variable: Foreign Direct Investment

From the GLS output in 2(b) below, the model becomes

FDI = 8874.8206 + 1700.8870 ERI – 166.9712LDR

Table 2a: OLS OUTPUT

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-2071.640276	4090.281852	-0.50648	0.61755908
ERI	155.889165	59.980389	2.59900	0.01638170 *
LDI	399.179655	216.685190	1.84221	0.07896469
FDI	0.121842	0.032123	3.79299	0.00099792 *

Table 2b: GLS OUTPUT

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	8874.821	26485.955	0.33508	0.74060
ERI	1700.887	160.631	10.58877	2.5697e-10 ***
LDI	-166.971	1406.099	-0.11875	0.90651

Mean of dependent variable = 71840.95

Standard deviation of dependent variable = 91823.35

Sum of squared residuals = 32055238567.287

Standard error of residuals = 37332.372342

R – Squared = 0.847927

Estimator: Ordinary Least Squares
Equation 2: OLS estimates using 26 observations 1983 – 2008

Dependent variable: Foreign Direct Investment

From the OLS output in the table 2(a) above, the model becomes:

FDI = 8874.8206 + 1700.8870 ERI – 166.9712LDR

Mean of dependent variable = 71840.95

Standard deviation of dependent variable = 91823.35

Sum of squared residuals = 32055238567.287

Standard error of residuals = 37332.372342

R – Squared = 0.847927

Interpretation: The two estimators produced the same value of coefficients and standard errors. This result revealed that both of them are efficient – alike, which shows that the GLS estimator is an OLS estimator of a transformed isomorphic model.

5. CONCLUSION

In this study, the two estimators – GLS and OLS were compared. Result from the analysis showed that GLS and OLS estimators produced the same values of coefficients and standard errors in the simultaneous equations. This study therefore concluded that the two estimators are efficient alike, which shows that this GLS estimator is an OLS estimator of a transformed isomorphic model.

6. REFERENCES

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