Numerical Calculation Of Lyapunov Exponents In Various Nonlinear Chaotic Systems

Joan Jani, Partizan Malkaj

Abstract: In this paper we study the meaning and importance of Lyapunov exponents through methods of analysis of experimental data applied in physics, and especially in chaotic circuits. The Lyapunov exponents play a very important role in detecting chaos, which occurs in many areas of science and technology. So, the question belongs to the theory of chaotic dynamical systems and generally all dynamical systems, which should analyzed correctly and accurately to obtain the correct conclusions regards Lyapunov exponents. The purpose of the study is to find the Lyapunov exponents for various dynamical systems and the explanation of the results for the dynamic behavior of each system. We also present applications science where Lyapunov exponents play an important role in explaining the main algorithms to calculate these exponents under different implementation and different computer packages.

Index Terms: chaos, nonlinear sience, lyapunov exponents

1 INTRODUCTION

The issue of an unstable situation is known in the science. As an example, in practice is not possible to balance a ball on top of a mountain, even if the form of perfectly balanced ball on top is stable situation. The problem is that the orbit of any initial position of the ball close, but not exactly in the steady state will evolve away from it. A kind of conduct for initial condition that begins near an unstable steady state is removed and pulled from a stable steady state, or perhaps a stable periodic statement.[1] We consider an initial state that is close to a source (source, type fixed point characterized by unstable and repels neighboring trajectories in phase space) an imaging. At the beginning of this trajectory appears unstable behavior. Exponential separation means that the distance between the point of the track and the source increases in an exponential rate. Each iteration multiplies spaced apart. . We suppose that the exponential rate of separation is by repetition.This means, at least in principle that small separations grow. After some wandering, the trajectory can be attracted to a sink (sink, type fixed point characterized by stable balance, and attracts the neighboring trajectories in phase space) q. As we near the sink, the trajectory display convergent behavior-the distance between the orbital point and the sink will change by a factor. As the orbit approaches the attractor short distances increasingly diminish.[2] It is common to see such a phenomenon in which the unstable behavior is transitional and eventually leads to stable behavior.

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But there is no reason why an initial starting position near a source is forced to come attracted to a periodic sink or sink.[3] A chaotic orbit is an orbit such that everything continues to have an unstable behavior near one source, but is not constant or periodical. Near each point of such wheels are points arbitrarily to be removed from the point during further iteration. This abnormality is quantified by numbers Lyapunov exponents and Lyapunov. We define the number Lyapunov as the average rate of divergence per step of nearby elements in along the track, and the Lyapunov exponent as the natural algorithm of the number Lyapunov. Chaos Chaos is defined by a Lyapunov exponent greater than zero.[4]

2 Lyapunov Exponent

The Lyapunov exponents are an algorithmic and computational model, a quantitative measure of the degree of chaotic motion of an orbit. Speaking roughly, the Lyapunov exponents of a trajectory characterize the average exponential divergence of neighboring trajectories of this. The characterization of the chaotic motion trajectories in phase space for deviation of adjacent tracks presented by the first and HénonHeiles (1964) and further investigated by Zaslavski and Chirikov (1972), FroeschleScheideker (1973), and Ford (1975) and other.[5]

2.1 Wolf's Algortithm

The well-known technique of reconstruction phase space with coordinatesdelay makes it possible to obtain a time series of an attractorLyapunov spectrum which is identical to the original attractor.[6] The Lyapunov exponents can be defined by the evolution of the phase spacesphere statements. Attempts to apply this definition figure equations of motion to fail since the computational limitations not allow the initial sphere constructed quite small. The approachODE one avoids this problem by working in tangent spacea conventional track so always take infinitesimally main vectorsaxis. The remaining spreads easily fadeorthogonalization Gram-Schmidt. The ODE approach is not directly applicable to experimental datawhen the linear system is not available. We consider systems that holdat least one positive exponent To estimate the λ_1 , witnessing the evolution of a long periodsingle pair of nearby orbits. The reconstructed attractor ouralthough only one set of track, can provide points that can beconsidered to be in different orbits. We choose points whosetemporal separation in the original time series is at least an average orbitalsentence because a pair of points with a much shorter separation characterized by a zero exponent Lyapunov. Two data points canbe regarded as defining the early state of the first principal axis as thespatial distance is small. When the distance is large, we would likeperform the GSR to vector defining

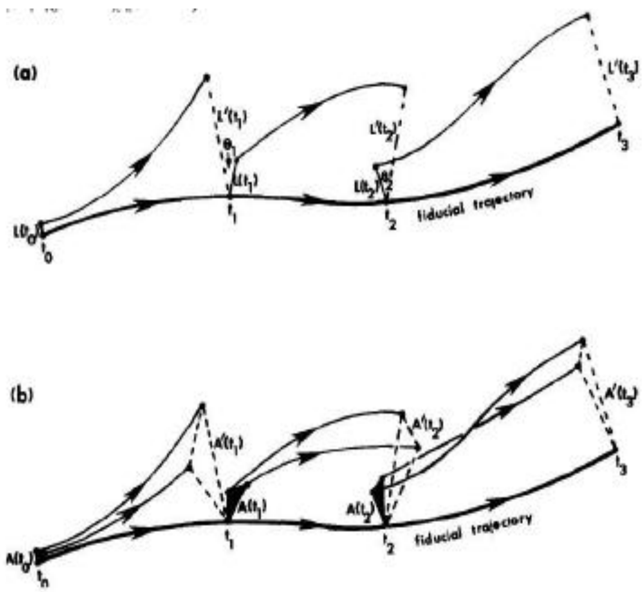

Figure 1

3.1 Roesistein

This algorithm follows exactly the definition of the maximum Lyapunov exponent and is accurate because it exploits all the necessary data. Algorithm is fast, easy to implement, and robust to changes in the following amounts: immersion dimension, size of data, delay reconstruction and noise level. Moreover, one can use the algorithm to calculate the correlation dimension simultaneously. Also, a sequence of calculations will lead to an assessment and the level of chaos and complexity of the system.[7] As we mentioned for the time series generated by dynamicsystems, the presence of a positive Lyapunov characteristic exponent indicateschaos. Moreover, in many applications it is sufficient to calculate only the maximumexponent Lyapunov (L1) . However, existing methods for estimating the L1suffer from at least one of the following disadvantages:

4 Experimental results

In this part, presented various systems of continuous and discrete time, which will calculate exponent Lyapynov. At the end we will compare the results obtained and will arrive at ways and algorithm works better. Systems that will build and simulate is particularly the following.

4.1 Lorenz Atractor

The Lorenz attractor is an attractor defined by a simplified system of equations describing the two-dimensional fluid flow and are known as the equations Lorenz.

$$\dot{X} = \sigma(\mathrm{Y} - \mathrm{X})$$
$$\dot{\mathrm{Y}} = -\mathrm{XZ} + rX - Y$$
$$\dot{Z} = XY - bz$$

Also, the Lorenz realized that the solution to these equations holds sensitive dependence on initial conditions. This is the phenomenon called butterfly proposed inability term weather forecasting.

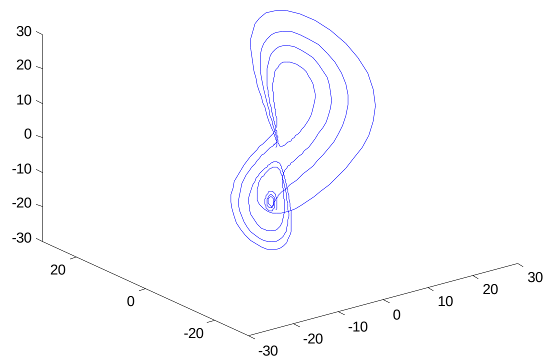

Figure 2: *Chaoticattractorof the Lorenzsystem.*

We observe that the track does not intersect itself. For p = 10, b = 8/3, the Lorenz found numerically that the system behaves chaotic whenever r exceeds a critical value r ≈ 24.74. This isbecause all solutions appear to be sensitive to initial conditions, and nearly all of the is evidently periodic solutions either converge to periodic solutions or equilibrium. We observe that the Lyapunov exponents are λ1 = 1.49734, and λ2 = -0.00404149,λ3= - 22.4933 and the Lyapunov dimension is 2.0622. We will try to use the parameters of Lorenz as the previous algorithms. So initially we observe that for p = 10, b = 8/3 and r = 23.5, the system goes very quickly from chaotikotita or not. Eventually the the last value is not chaotic system is for r = 22.9.

4.2 Hénon Map

The depiction of Hénon is a discrete-time dynamical system. It is one of the most studied examples of dynamical systems with chaotic behavior. The depiction of Hénon gets a point (x_o, y_o) in space and illustrates a new item.

$$x_{n+1} = y_n + 1 - ax_n^2$$
$$y_{n+1} = bx_n$$

The display depends on two parameters, a and b, which for normal representation the Hénon map has values a = 1.4 and b = 0.3. For normal display of the prices Hénon chaotic. For other values of a and b, the display can be chaotic, or converges to a periodic orbit. For normal display, a starting point in space or close a set of points known as a strange attractor of Hénon, or diverges to infinity.

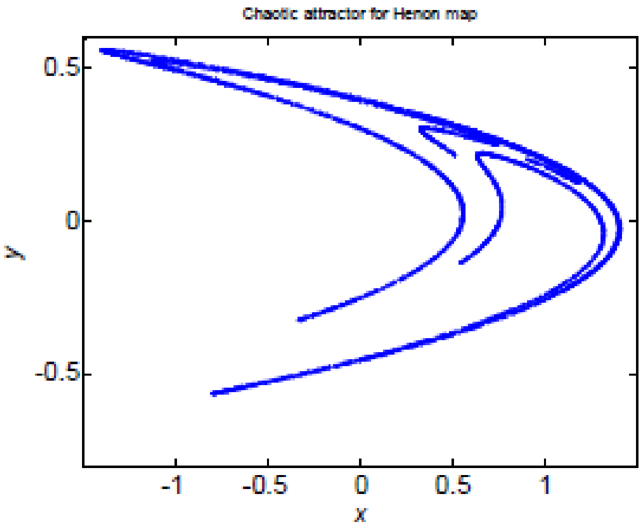

Figure 3Chaoticattractorof the Lorenzsystem

Points close to this fixed point and along the slope 1.924 will approach the fixed point and points along the slope -0.156 will move away from the fixed point. These slopes arise from the linearization of the stable manifold and unstable manifold of the fixed point. The unstable manifold of the fixed point in the attractor is contained in the strange attractor of the Hénon map.

4.2 LogisticMap

The logistic map was first used in 1845 by the Belgian mathematician Pierre Francois Verhulst, as a model describing the time evolution of a population of a colony of living beings. The Verhulst considered that if p_n and p_{n+1} is the population of the colony time's n and n +1, respectively, then the growth rate of population growth.[8]

$$K = \frac{p_{n-1} - p_n}{p_n}$$

is proportional to the possibility of the system pn population suffer further development within the existing ecosystem. This means that if we denote the unit with the total capacity of system, and p_n the current value of the population, then the remainingability is equal to 1 - p_n, and therefore the time evolution of the population will begiven by the equation

$$\frac{p_{n-1} - p_n}{p_n} = r(1 - p_n) \quad (5)$$

The parameter r corresponds to a constant value. If you resolve the (5) in term of p_{n-1}, this will take the form

$$p_{n-1} = p_n + rp_n(1 - p_n) \quad (6)$$

The above equation can be simplified even further if use a new variable, and to define a new $x_n = (r/(r+1))\,p_n$ constant λ, as λ = r +1. In this case, the equation is transformed into the form

$$x_{n+1} = \lambda x_n (1 - x_n)$$

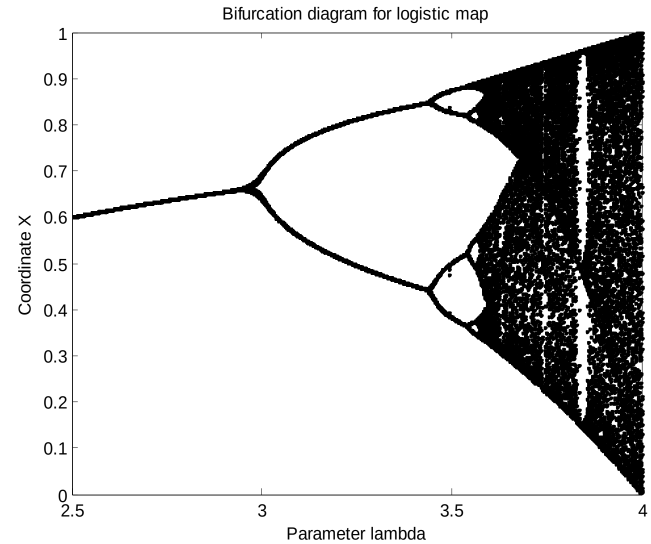

Figure 4: Difurcation diagram for the logisticmap

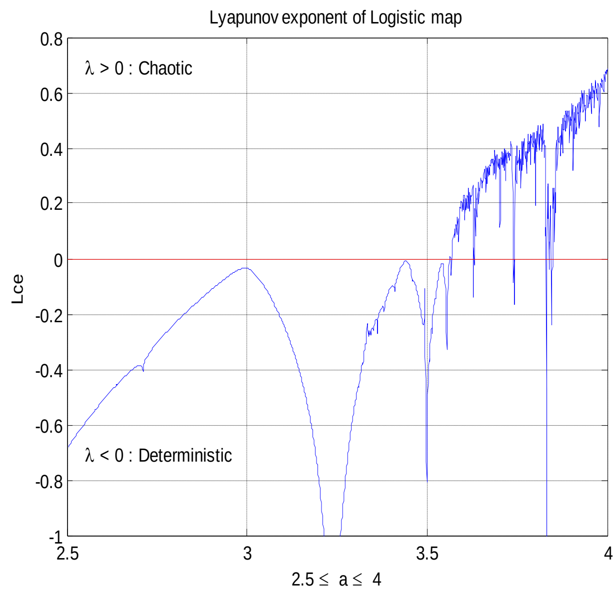

Figure 5: Lyapunovexponentsfor the logisticmap as x varies

Conclusion

After the above theoretical analysis and the experimental procedure,the main conclusion is that it is very difficult to calculate the exponentsLyapunov using algorithms. The algorithms used notconverge to the same result and thus introduce several errorsarithmetic, but even the acts of the tables on thereversal and cleavage QR. As we mentioned above, there are two main sources of error: thediscretization of differential equations and replacing the limits ofcontinuous variable of time from a sequence of time points. Alsoerrors introduced by linear algebra computations thatconsidered within the discretization errors. The bad news however foralgorithms is that these errors accumulate. In addition, we observed that there is difference in the results betweenthe continuous QR algorithms and discrete. Continued consideredsuperior to discrete algorithms, because the latter estimate with pooraccuracy negative Lyapunov exponents and require more time toCPU. In contrast, continuous algorithms do not have a problem incalculation of Lyapunov exponents and further requiring fewer steps and isless expensive. Also, the results deteriorated due to noise enters themeasurements but this can be avoided by filters.The number of data also plays an important role in the implementation of algorithms. If the data is a few can lead to completely different results than what you should arrive. Therefore, forproper performance of the algorithms required for enough data as well as thetime is enough.

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