New Analysis Algorithm For Solving The Nearest Facility Pre-Location Problem On A Plane

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Abstract: There are two types of Facility Location Problem (FLP) that have been presented. The first type of FLP is minisum rectilinear distance single FLP which have been solved by using median method and also linear programming formulation which solved by using linear programming software, TORA. The second type of FLP is minisum Euclidean distance single FLP which solved by using Weiszfeld's method and also extension of Weiszfeld's method. Given weighted demand points, the new facility location point was find to minimize the total demand weighted cost between the demand and the new facility location (minisum objective function). All of the methods have been applied successfully. The results obtained have been analyze and compared by the total distances.

Index Terms: Facility Location Problem, Median Method, Minisum Facility Location, Weiszfeld's Method.

1. INTRODUCTION

ACCORDING to (Farahani et al., 2009) there are four components that describe location problems: customers, who are assumed to be already located at points or on routes, facilities that will be located, a space in which customers and facilities are located, and a metric (standard) that indicates distances or time between customers and facilities. In short, the four main components are Objective Function, Distance Measure, Feasible Space and Number of Facilities. This paper consider in solving the minisum single facility location problem with two different types of distance measure, which are rectilinear and Euclidean. For the rectilinear distance measure problem, the median method was applied to so solve the problem. Then, this paper apply an early but interesting optimal method first proposed by Endre Weiszfeld to solve the Euclidean distance single facility location problem. In this paper, the extended of Weiszfeld's method and the result was compared to the result from the original Weiszfeld's method.

2 MINISUM SINGLE FACILITY LOCATION PROBLEM

In this paper, the problem focused on minisum single facility location problem with two different types of distance measure which are rectilinear and Euclidean distance. The formulation of this two problem was as follow. Equation 2.1 is the formulation for rectilinear distance measure while equation 2.2 is the formulation of Euclidean distance problem.

$$\min \ f(x,y)=\sum_{i=1}^{m} w_i |x_i-a_i|+\sum_{i=1}^{m} w_i |y_i-b_i| \quad (1)$$

$$\text{Min} \ f(x,y)=\sum_{i=1}^{m} W_i \left[(x-a_i)^2+(y-b_i)^2\right] \quad (2)$$

3 MEDIAN METHOD

According to (Moradi & Bidkhori, 2009) the median method finds the median location (defined later) and assigns the new facility to it. This method is used for single-facility location problems with rectilinear distance. The procedure of this method was as follow:

Step 1. List the existing facilities in increasing order of the x coordinates.
Step 2. Find the j th x coordinate in the list (created in step 1) at which the cumulative weight equals or exceeds half the total weight for the first time;

$$\sum_{i=1}^{j} W_i \geq \sum_{i=1}^{m} \frac{W_i}{2} \quad (3)$$

Step 3. List the existing facilities in no decreasing order of the y coordinate.
Step 4. Find the kth y coordinate in the list (created in step 3) at which the cumulative weight equals or exceeds half the total weight for the first time:

$$\sum_{i=1}^{k} W_i \geq \sum_{i=1}^{m} \frac{W_i}{2} \quad (4)$$

The optimal location of the new facility is given by the j^{th} x coordinate and the k^{th} y coordinate identified in steps 2 and 4, respectively.

4 WEISZFELD'S METHOD

Step 1: Consider a trial location = (0, 0)
Step 2: Modify the x and y values by using equation.

$$x_j=\frac{\sum W_i a_i D_{ij}}{\sum W_i / D_{ij}} \quad (5)$$

$$y_j=\frac{\sum W_i b_i D_{ij}}{\sum W_i / D_{ij}} \quad (6)$$

Step 3: If one or both of (x, y) changes, repeat the process with the modified (x_i, y_i). Go to step 2. If there is no improvement occurs in the estimate of the optimum location for the new facility which means the solution has reached the terminating condition.

$$\left\| \left(x^j, y^{kj}\right) - \left(x^{j-1}, y^{j-1}\right) \right\| \quad (7)$$

5 EXTENDED WEISZFELD'S METHOD

For the extended Weiszfeld's method,the calculation steps is same with Weiszfeld's method except for the extension of Weiszfeld's method, the demand points, are modeled as 2-dimensional shape. In the extended of Weiszfeld's method each 2-dimensional shape is divided into numerous points and the points are used as an input for Weiszfeld's algorithm.

6 RESULTS AND DISCUSSIONS

TABLE 1
MINISUM RECTILINEAR DISTANCE FLP RESULTS COMPARISON

	x-coordinate	y-coordinate	Total distance
Median method	-82.9874	39.9889	31.0112
TORA	-82.9874	39.9889	31.0112
Existing journal (Lee & Wong, 2000)	-82.8746	40.3294	33.0510

Table 1 show the results of solving the minisum rectilinear distance FLP that obtained from the median method, TORA software and also the existing journal that used the mean centre method to solve the FLP. The distance measure was calculated by using the coordinate of each method and being compared to each other. It seems that the median method and the TORA software give the same answer with the same total distance which is 31.0112 units while the answer from the existing journal gives the total distance 33.0510 units which is larger than the total distance from the median method and TORA software. It shows that the median method gives better answer than the mean centre method.

TABLE 2
MINISUM EUCLIDEAN DISTANCE FLP RESULTS COMPARISON

	x-coordinate	y-coordinate	Total distance
Weiszfeld's method	-82.9874	39.9892	6.7795
Extended Weiszfeld's method	-82.9950	39.9901	6.7577

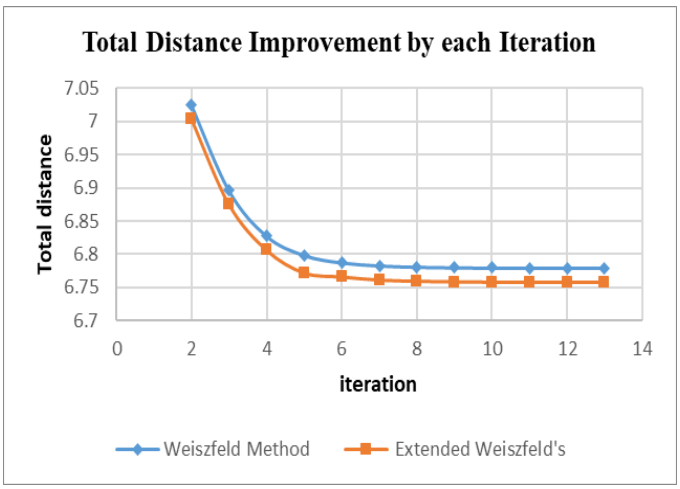

Fig. 1*. Total distance improvement by each iteration.*

Table 2 present the results of solving the minisum Euclidean distance FLP that obtained from solving the problem by using the original Weiszfeld's method and also the extended Weiszfeld's method. The total distance obtained from the answer of each method were compared. The total results from Weiszfeld's method gives the total distance 6.7795 units while the extended Weiszfeld's method gives the total distance 6.7577 units. From the obtained results, it shows that the extended Weiszfeld's gives better answer compare to the original Weiszfeld's method. Fig. 1 shows the total distance improvement for each method by iteration. From the graph, it shows that the total distance was improved in each iteration for both of the method. However, the extended Weiszfeld's method gives the better final coordinates compared to the original Weiszfeld's method.

7 CONCLUSION

The result of solving the minisum rectilinear distance single FLP by using median method and also linear programming software TORA gives lower total distance compare to the results from the mentioned existing journal which solved the problem by using the mean centre formula. This indicates that median method is a reliable method that can be used to solved the minisum rectilinear distance single FLP. As for the minisum Euclidean distance single FLP the results obtained from solving the problem by using modified Weiszfeld's method is found to be the better result which gives the new facility location with the lowest total distance compare to the result obtained from the original Weiszfeld's method.

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