Prime Labeling For Some Crown Related Graphs

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Abstract: - A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integer 1,2,3… $|V|$ such that for edge xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper we investigate prime labeling for some Crown related graphs. We also discuss prime labeling in the context of some graph operations namely fusion and duplication switching, path union in Crown graph C_n^*.

Keywords: - Prime Labeling, Fusion, Duplication, switching, path union.

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1 Introduction

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notations and terminology we refer to Bondy and Murthy [1]. The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout. A (1982 P 365-368). [2] Many researchers have studied prime graph. For example Fu.H.(1994 P 181-186) [5] have proved that path P_n on n vertices is a Prime graph. Deretsky.T (1991 p359 – 369) [4] have proved that the C_n on n vertices is a prime graph. Lee.S (1998 P.59-67) [2] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Etringer conjectured that all tress have prime labeling which is not settled till today. The prime labeling for planner grid is investigated by Sundaram.M (2006 P205-209) [6] In [8] S.K.Vaidhya and K.K.Kanmani have proved the prime labeling for some cycle related graphs

Definition 1.1

Let $G = (V(G), E(G))$ be a graph with p vertices. A bijection $f: V(G) \to \{1,2, \dots p\}$ is called a prime labeling if for each edge $e = uv$, $gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.2

Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by identifying (fusing) two vertices u and v by a single vertex x in such that every edge which was incident with either u or v in G now incident with x in G.

Definition: 1.3

Duplication: Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_k^I with $N(v_k^I) = N(v_k)$. In other words a vertex v_k^I is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k^I

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Definition: 1.4

Switching: A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.5 (Union of Two Graph)

Let $G_1, G_2, \dots G_n, n \geq 2$ be n copies of a fixed graph G. The graph obtained by adding an edge between G_i and G_{i+1} for $i = 1,2, \dots\dots n-1$ is called the path union of G.

Definition 1.6

The Crown graph C_n^* is obtained from a cycle C_n by attaching a pendant edge at each vertex of the n-cycle. In this paper we have proved that the graph obtained by fusing any two vertices v_1 and v_k of degree greater than 3 in C_n^*, the graph obtained by duplication of any vertex v_k in C_n^*, the graph obtained by switching of any vertex v_k in C_n^*, the path union of k copies of C_n^* and the graph obtained by the path union of two copies of C_n^* by a path of length k are all prime graphs.

2 Theorems

2.1 Theorem: 1

Let G_k be the graph obtained by fusing any two vertices v_1 and v_k of degree 3 in C_n^* then G_k admits prime labeling.

Proof:

Let $V(C_n^*) = \{c, v_1, v_2, v_3 \dots v_n, v_1', v_2' \dots v_n'\}$

$$E(C_n^*) = \{v_i v_i' / 1 \leq i \leq n\} U \{v_i v_{i+1} / 1 \leq i \leq n-1\} U \{v_1 v_n\}$$

Let G_k be the graph obtained by fusing any two vertices of degree 3 say v_1 and v_k in c_n^*

Here $|V(G_k)| = 2n - 1$

Define a labeling $f: V(G_k) \to \{1,2,3 \dots 2n-1\}$ as follows

Let
$$f(v_1) = 1$$
$$f(v_i) = 2i - 1 \quad for\ 2 \leq i \leq k-1$$
$$f(v_i') = 2i - 2 \quad for\ 2 \leq i \leq k-1$$
$$f(v_i) = 2i - 3 \quad for\ k+1 \leq i \leq n$$
$$f(v_i') = 2i - 4 \quad for\ k+1 \leq i \leq n$$
$$f(v_1') = 2n - 1$$
$$f(v_k') = 2n - 2$$

Then for any edge $e = v_i v_j \in G_k$ $\gcd(v_i, v_j) = 1$ and for edge $v_i v_i' \in G_k$ $\gcd(v_i, v_i') = 1$

The function defined above gives prime labeling for G_k. Thus G_k is a prime graph.

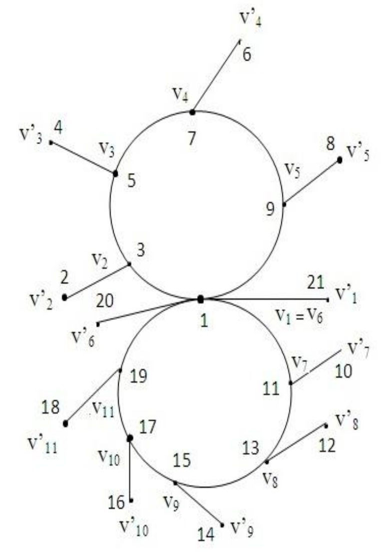

Fig.1. Example 1: Fusing of v_1 and $v_6$in C_{11}

2.2 Theorem: 2

Let G_k be the graph obtained by duplicating any vertices v_k of degree 3 in C_n^* then G_k admits prime labeling.

Proof:

Let $V(C_n^*) = \{c, v_1, v_2, v_3 \dots v_n, v_1^{'}, v_2^{'} \dots v_n^{'}\}$

$E(c_n^*) = \{v_i v_i^{'}, 1 \le i \le n\} U \{v_i v_{i+1}, 1 \le i \le n-1\} U \{v_1 v_n\}$

Let G_k be the graph obtained by duplicating any vertex v_k of degree 3 in C_n^*

Let v_k^* be the duplication of v_k in G_k.

Here $|V(G_k)| = 2n + 1$

Define a labeling $f: V(G_k) \to \{1,2,3 \dots 2n+1\}$ as follows

Let

$$
\begin{aligned}
&f(v_k) = 1 \\
&f(v_k^{'}) = 3 \\
&f(v_k^*) = 2 \\
&f(v_{k+1}) = 5 \\
&f(v_{i+1}) = f(v_i) + 2 && for\ k+1 \le i \le n-1 \\
&f(v_1) = f(v_n) + 2 \\
&f(v_i) = f(v_{i-1}) + 2 && for\ 2 \le i \le k-1 \\
&f(v_i^{'}) = f(v_i) - 1 && for\ 1 \le i \le n, i \ne k
\end{aligned}
$$

Then for any edge $e = v_i v_j \in G_k, \gcd(f(v_i), f(v_j)) = 1$ and for any edge $v_i v_i^{'} \in G_k, \gcd(f(v_i), f(v_i^{'})) = 1$ and also $\gcd(f(v_k^*), f(v_{k-1})) = 1$ and $\gcd(f(v_k^*), f(v_{k+1})) = 1$. The labeling defined above is a prime labeling for G_k. Thus G_k is a prime graph.

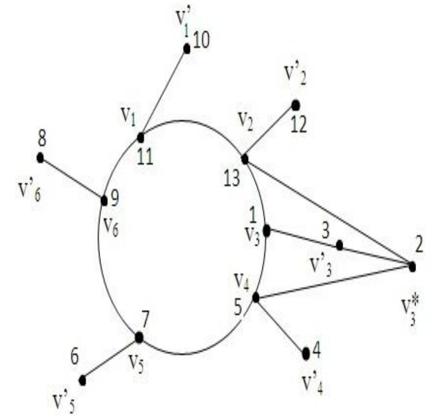

Fig.2. Duplication of v_6 in C_6^*

2.3 Theorem: 3

Let G be the graph obtained by switching any vertices v_k of degree 3 in C_n^* then G is a prime graph.

Proof:

Let $V(C_n^*) = \{c, v_1, v_2, v_3 \dots v_n, v_1^{'}, v_2^{'} \dots v_n^{'}\}$

$E(c_n^*) = \{v_i v_i^{'} / 1 \le i \le n\} U \{v_i v_{i+1} / 1 \le i \le n-1\} U \{v_1 v_n\}$

Let G be the graph obtained by switching any vertices v_k of degree 3 in G

Here $|V(G)| = 2n$

Define a labeling $f: V(G) \to \{1,2,3 \dots 2n\}$ as follows

$$
\begin{aligned}
&f(v_k) = 1 \\
&f(v_k^{'}) = 2 \\
&f(v_{k+i}) = 2i + 1 && for\ 1 \le i \le n-k \\
&f(v'_{k+i}) = 2i + 2 && for\ 1 \le i \le n-k \\
&f(v_i) = 2(n-k) + 2i + 1 && for\ 1 \le i \le k-1 \\
&f(v_i^{'}) = 2(n-k) + 2i + 2 && for\ 1 \le i \le k-1
\end{aligned}
$$

Then for any edge $e = v_i v_j \in G, \gcd(f(v_i), f(v_j)) = 1$

For edge $e = v_k v_i \in G, \gcd(f(v_k), f(v_i)) = 1$

For edge $e = v_i v_i^{'} \in G, \gcd(f(v_k), f(v_i^{'})) = 1$

Then f admits prime labeling.

Thus G is a prime graph.

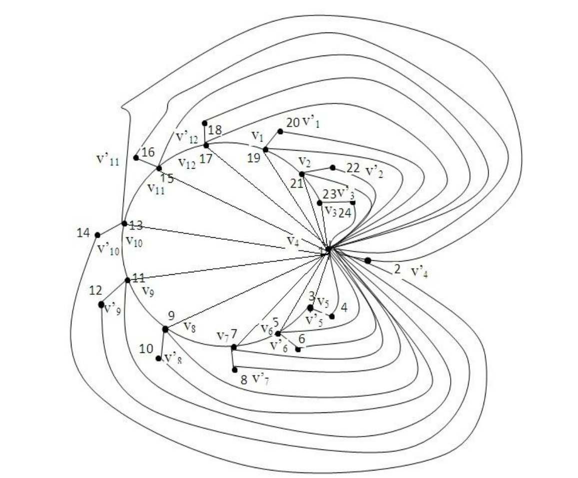

Fig.3. switching the vertex v_4 in c_{12}

2.4 Theorem: 4
The graph obtained by the path union of finite number of copies of the crown graph C_n^* is a prime graph for all $n \geq 3$

Proof
Let G be the path union of the crown graph C_n^* and $G_1, G_2, G_3 \dots . G_k$ be k copies of the crown graph C_n^*

Here $|V(G)| = 2nk$

Let us denote the vertices of the cycle on G_i as $u_{i1}, u_{i2} \dots . u_{in}$ where $1 \leq i \leq k$ and the pendant vertices attached to the above vertices in G_i as $u_{i1}', u_{i2}' \dots u_{in}'$ respectively. Let $e_i = u_{i1}u_{i+1\,1}$ be the edge joining G_i and G_{i+1} for $i = 1,2, \dots k-1$

Define a labeling $f: V(G) \rightarrow \{1,2,3 \dots 2n_k\}$ by considering the following cases.

Case (i):
If $n \not\equiv 1\ (mod\ 3)$ or $n \not\equiv 1\ (mod\ 5)$

Let $f(u_{ij}) = 2j - 1 \quad for\ 1 \leq j \leq n$
$f(u_{ij}) = 22(i-1) + 2j - 1\ for\ 1 \leq j \leq n$ and $2 \leq i \leq k$
$f(u_{ij}') = f(u_{ij}) - 1 \quad for\ 1 \leq j \leq n$ and $1 \leq i \leq k$
except $f(u_{11}') = 2nk$ Then for any edge
$e = u_{ij}u_{ij+1} \in G, \gcd(f(u_{ij}), f(u_{ij+1})) = 1$
$\gcd(f(u_{i1}), f(u_{i+11})) = 1$ for $1 \leq i \leq k$. Because
$\gcd(f(u_{i1}), f(u_{i+1\,1})) = \gcd(22(i-1)+1, 22i+1) = 1$

Case (ii):
If $n \equiv 1\ (mod\ 3)$
Then in the above labeling defined in case (i) interchange the labels of v_{m1} and v_{m1}'

if $m = 3l - 1$ where $l = 1,2,3 \dots$. Thus resulting labeling is prime.

Case (iii) If $n \equiv 1\ (mod\ 5)$
Then in the above labeling f defined in case (i) interchange the labels of v_{mn} and v_{mn}' if $m = 5l - 2$ where $l = 1,2,3 \dots$. then the resulting labeling is prime. Thus in all the cases G admits prime labeling. Hence G is a prime graph.

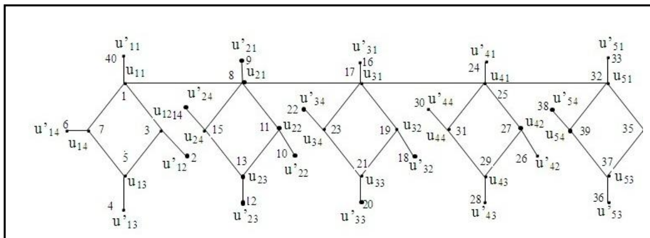

Fig.4. Path union of 5 copies of C_4^* $n \equiv 1\ (mod\ 3)(\ case\ (ii)\ m = 2,5)\ with\ k = 5$

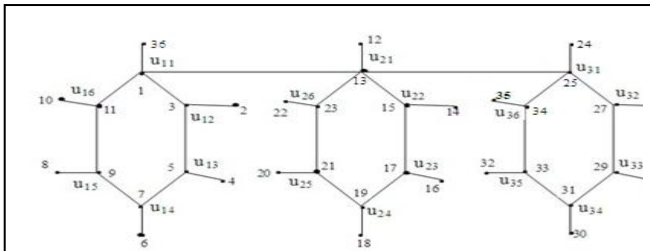

Fig.5. Path union of 3 copies of c_6 $(n \equiv 1\ (mod\ 3))$

Theorem 5:
The graph obtained by joining two copies of the Crown graph C_n^* by a path P_k is a prime graph for all $n \geq 3$ and for all k.

Proof:
Let G be the graph obtained by joining two copies of the Crown graph C_n^* by a path P_k. We note that $|V(G)| = 4n + k - 2$. Let $u_1, u_2 \dots u_n$, $u_1', u_2', \dots u_n'$ be the vertices of first copy of the Crown C_n^* and let $v_1, v_2 \dots v_n$, $v_1', v_2', \dots v_n'$ be the vertices of second copy of the Crown C_n^*. Let $w_1, w_2 \dots w_k$ be the vertices of the path P_k with $u_1 = w_1$ and $v_1 = w_k$

Define a labeling $f: V(G) \rightarrow \{1,2,3 \dots 4n + k - 2\}$ by considering the following cases.

Case (i):
If $n \not\equiv 1\ (mod\ 3)$

Let
$f(u_i) = 2i - 1 \quad for\ 1 \leq i \leq n$
$f(u_i') = 2i \quad for\ 1 \leq i \leq n$
$f(v_i) = 2n + (2i - 3) \quad for\ 2 \leq i \leq n$
$f(v_i') = 2(n) + (2i - 2) \quad for\ 2 \leq i \leq n$
$f(v_1) = 4n - 1$
$f(v_1') = 4n$
$f(w_{k-i}) = 4n + i \quad for\ 1 \leq i \leq k - 2$

Case (ii):
If $n \equiv 1\ (mod\ 3)$

Then in the above labeling f defined in case (i) interchange the labels of v_2 and v_2'. Then we get a prime labeling. Then f admits prime labeling. Thus G is a prime graph.

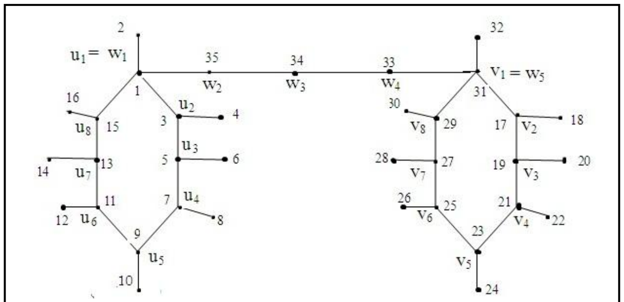

Fig.6. Path union of 2 copies of c_8 *by* the path P_5

3 CONCLUSION

Here we investigate 5 results corresponding to prime labeling. Analogues work can be carried out for other families also.

ACKNOWLEDGMENT

The author wish to thank university grants commission. This work was supported by UGC minor research project.

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