Graph Coloring In Optimization Total Waste Transport Vehicles In Bandung

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Abstract: Waste is a material that is produced from human activities that are no longer used and disposed of. Waste management really needs support and commitment from the government to produce sustainable solutions. However, the government also has limited funding in the waste handling system. PD. Kebersihan Kota Bandung is a regional company engaged and responsible in the field of cleaning the City of Bandung. Based on the daily report of waste collection, the costs incurred for the operation is quite large. These operational costs are dominated by transportation service costs (driver). Meanwhile, not all vehicles operate every day. Graph theory provides an alternative solution to solve this problem. Welch-Powell algorithm, indicate that to facilitate the transportation of garbage from the temporary landfill to the landfill, the Bandung Wetan area is sufficient to provide a minimum of 4 units of transport trucks.

Index Terms: Graph theory, Graph coloring, Optimization, Problem solving, Waste handling system, Waste transportation, Welch-Powell algorithm

1 INTRODUCTION

Waste is unwanted residual material after the end of a process. In human life, a large amount of waste comes from industrial activities, for example mining, factories, and consumption. In Indonesia, one of the biggest producers of waste is household [1], with an average of almost 1 kg every day each person producing waste per day. If not managed properly, garbage that accumulates will worsen the land and sea conditions. Waste problem is closely related to people’s lifestyle. Therefore, its handling requires broad community participation. The amount of waste every year continues to increase with increasing population and quality of life of the people accompanied by advances in science and technology resulting in a shift in lifestyle with a tendency to be consumptive [2]. The problem of garbage in the city of Bandung is very important, after the designation of the city of Bandung as the City of Flowers turned into a City of Trash [3]. Until now, the Bandung local government continues to innovate to find solutions to overcome waste problems [4].

Efforts made by the government in overcoming the waste problem are still getting pros and cons from the public [4] [5]. One of them is the environmental tax imposed on each industrial product which will eventually become junk. Industrial products with packaging on each product, of course, will provide packaging waste after consumption. Therefore the industry is required to pay waste handling fees for each product produced. This environmental tax is known as the Polluter Pay Principle. Waste management really needs support and commitment from the government to produce sustainable solutions. However, the government also has limited funding in the waste handling system. PD Kebersihan Kota Bandung is a regional company that is engaged and responsible in the field of cleaning up the City of Bandung.

Based on Kota Bandung Regulation No. 09 of 2011 concerning waste management, one of the duties of PD Kebersihan is to transport waste from the TPS (Temporary Disposal Site) to the Processing Site and/or TPA (Final Disposal Site). Based on the daily report of research and development (R & D) of PD Kebersihan Kota Bandung, the costs incurred for the operation is quite large [7]. These operational costs are dominated by transportation service costs (driver). Meanwhile not all available vehicles operate every day. Therefore it is necessary to make efforts to optimize transportation schedules to reduce operational costs without reducing the operational effectiveness of waste management.

Graph Theory provides alternative solutions to solve this problem [8] [9].

2 GRAPH THEORY

Graph theory is an important part of mathematics and logic. In graph theory, a mathematical model for each set of discrete objects, where several pairs of elements from the set are bound according to certain rules. In everyday life, there are many problems that can be represented as problems related to sets and binary relations, where the logic of the problem is often represented by graphs [10]. Informally, a graph is a node or set of objects connected by edges. This theory can be modeled into various types of relationships and processes in optimization[11].

Definition 1 [5] The graph G is defined as a set of pairs (V, E), written with the notation G = (V, E), in which case V is a non-empty set of vertices, and E is a side set (edges or arcs), which connects a pair of vertices.

Graphs are used to describe problem models, represent discrete objects and the relationships between them. The most important thing from applying a graph is how to read the problem, then define what will be a discrete object to describe the problem. Visual representation of a graph is to declare an object as a vertex and the relationship between objects expressed by edge.

Definition 2 G = (V, E, f ), v1, v2 ∈ V is non directed simpel graph. Graph G are said to be adjacent if { v1, v2 } ∈ E.

If f (e) = { v1, v2 } or e = f (v1, v2 )then e is said incidenceto v1 and
The vertices $v_1$ and $v_2$ are then said to be the end vertices of edge $e \in E$.

**Definition 3** Let $G = (V, E, f)$ be an undirected graph. The degree of a vertex $v \in V$ on $G$ is the number of edges that incidence with vertex $v$, provided that the number of edges in the form of a loop is counted twice. The degree of $v$ is denoted by $\deg(v)$. The problem represented in the graph, can be solved by several methods of finding solutions that are easier to do than other analytic methods or theories [13]. One method in graph theory that is often used is graph coloring [14][15]. The term graph coloring refers to the vertex coloring of a graph.

**Definition 4** $G = (V, E)$ as a simple graph. Graph $G$ coloring is labeling colors in the vertices of $G$ so that the two vertices that are adjacent to $G$ have different colors.

The minimum number of colors that can be used to color a vertex is called the G graph chromatic number, symbolized by $\chi(G)$. The Welch-Powell algorithm can be used to find chromatic numbers as a scheduling solution [16]. Following is Welch-Powell algorithm:

1. Sort all vertices of $G$ in descending degree
2. Use one color to color the first node (which has the highest degree) and other vertices (in sequence) that are not connected with first node.
3. Color the next high degree vertex in the sorted list that has not been colored, and repeat the process of coloring the vertex with the second color
4. Repeat the addition of colors until all vertices have been colored

**3 METHOD**
As explained in the introduction, that graph theory can find an alternative solution for scheduling garbage transportation trucks from temporary landfill (TPS) to landfill (TPA). Many areas are managed by PD. Kebersihan Kota Bandung, so that this paper will only provide an example of scheduling for the sub-district of Bandung Wetan and trial methods for Bojongloa Kidul area, with the hope that this theory can be implemented for scheduling other regions.

**3.1 GRAPH REPRESENTATION**
Based on the data taken from TPS to TPA House Waste Transportation schedule of PD. Kebersihan Kota Bandung, here is the Bandung Wetan District transportation data:

**Table 1.**
House Waste Transportation Schedule of District Bandung Wetan

<table>
<thead>
<tr>
<th>Node</th>
<th>TPS Name</th>
<th>Transport Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>TPS Pasar Bunga</td>
<td>Friday</td>
</tr>
<tr>
<td>B</td>
<td>SC Gelaek</td>
<td>Tuesday</td>
</tr>
<tr>
<td>C</td>
<td>TPS Ambon</td>
<td>Monday-Saturday</td>
</tr>
<tr>
<td>D</td>
<td>TPS Taman Cibeunyi</td>
<td>Monday-Saturday</td>
</tr>
<tr>
<td>E</td>
<td>SC Cimandiri</td>
<td>Wednesday</td>
</tr>
<tr>
<td>F</td>
<td>SC Balubur</td>
<td>Wed, Fri, and Sun</td>
</tr>
<tr>
<td>G</td>
<td>SC Gasibu</td>
<td>Sunday</td>
</tr>
</tbody>
</table>

*Source: http://PPID.Bandung.go.id*

The next step is to formulate the data contained in table 1 into graphs. In order to be formulated, vertex and edges must be determined in the scheduling problem. In the case of this scheduling, the TPS as a vertex, and if between TPS sending vehicles on the same day to the TPA then the TPS are adjacent. So that the edge represents the relationship between polling stations that have the same day delivery schedule. The following table is a grouping schedule for waste transportation on the same day:

**Table 2.**
TPS vehicle sending schedule

<table>
<thead>
<tr>
<th>Day</th>
<th>Node</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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</thead>
<tbody>
<tr>
<td>Mon</td>
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<td>Sun</td>
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<td>*</td>
</tr>
</tbody>
</table>

The (*) sign illustrates that the TPS is sending the vehicle on the day that has been marked, while the (-) sign means the TPS is not sending the vehicle. The next step is to represent the data in the form of a graph as follows:

![Garbage Truck Transportation Schedule Graph](image)

3.2 GRAPH COLORING
The problem of waste transporting that will be discussed in this paper is to determine how many minimum transport trucks PD. Kebersihan Kota Bandung must provide in order to be able to transport waste from the TPS to the TPA according to the schedule given in table 1. The Welch-Powell algorithm will be used to find a solution [17][18] by first determining the degree of each vertex. The vertices in the graph are then sorted descending according to their degree, as follows:

<table>
<thead>
<tr>
<th>Degree</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>3</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
<td>C</td>
<td>D</td>
<td>F</td>
<td>A</td>
<td>E</td>
<td>B</td>
<td>G</td>
</tr>
</tbody>
</table>

Then give colors starting from the highest degree:

<table>
<thead>
<tr>
<th>Deg</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>3</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
<td>C</td>
<td>D</td>
<td>F</td>
<td>A</td>
<td>E</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>Col</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
Give the color vertex C which is first in color 1, then to vertex D which is second. Check whether vertex D is adjacent to C, if yes then it must be colored differently to it then give color 2. The next step is to give color to the third order vertex, which is vertex F. F is adjacent with C and D, so it must be given a different color from both, color 3. Vertex the fourth order is vertex E, after checking, D is adjacent to C, F, and A, so it must be given a different color, namely color 4. Vertex in the next sequence is vertex E, this vertex is adjacent to C, D, and F, but not adjacent to A, so that it is colored 4. And so on, until all vertices in the sequence are checked. Vertex B is adjacent to C and D, but not adjacent to F, so it can be colored 3. Vertex G is only next to vertex F so that it can be colored 1. After all the algorithms are complete, it is known that 4 different colors are obtained which means chromatic number \( \chi(G) = 4 \). The chromatic number shows that the PD. Kebersihan Kota Bandung must provide at least 4 transport trucks in order to provide waste transportation services from the TPA to the TPS in the Bandung Wetan sub-district. So that it can be described in the following graph:

![Fig 2 Schedule Graph Colored](image)

### 4. EXPERIMENT AND RESULT

The main thing in finding solutions to problems using graph theory is to make representations of problems in the form of graphs, so vertex and edge must be determined with certainty so that the problem can be modeled in graphs. It has been shown that this waste transport problem after being modeled in graph form and optimization using the Welch-Powell algorithm can reduce the number of garbage trucks that must be provided by PD. Kebersihan Kota Bandung, Bandung Wetan area. Next will be shown whether this algorithm applies to other regions. Based on the data taken from TPS to TPA House Waste Transportation schedule of PD. Kebersihan Kota Bandung, here is the Bandung Wetan District transportation data:

<table>
<thead>
<tr>
<th>Node</th>
<th>TPS Name</th>
<th>Transport Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>RS Imanuel</td>
<td>Tuesday, Thursday</td>
</tr>
<tr>
<td>B</td>
<td>Pasar Leuw Panjang</td>
<td>Sunday-Saturday</td>
</tr>
<tr>
<td>C</td>
<td>Terminal Leuw Panjang</td>
<td>Monday, Wednesday.</td>
</tr>
</tbody>
</table>

Bojongloa Kidul area has 6 TPS, less than the Bandung Wetan area with a higher transportation volume. This happens because in Bojongloa Kidul area there are several public access places such as hospitals and markets which produce more garbage and must be immediately transported to landfills because it will pollute the environment. The next step is a grouping schedule for waste transportation on the same day:

**Table 4.**

TPS vehicle sending schedule

<table>
<thead>
<tr>
<th>Day</th>
<th>Node</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
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<td>Mon</td>
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<td>Sun</td>
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</tr>
</tbody>
</table>

After the transportation schedule is grouped by the same day, the next step is to represent the data in the form of graphs as follows:

![Fig 3 Garbage Truck Transportation Schedule Graph](image)

This graph representation shows the adjacency of each vertex, so that it can be known the degree of each vertex that will be sorted descending as input Welch-Powell algorithm, as follows:

<table>
<thead>
<tr>
<th>Degree</th>
<th>5</th>
<th>5</th>
<th>4</th>
<th>4</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td>B</td>
<td>F</td>
<td>A</td>
<td>D</td>
<td>C</td>
<td>E</td>
</tr>
</tbody>
</table>

Then give colors starting from the highest degree:

<table>
<thead>
<tr>
<th>Degree</th>
<th>5</th>
<th>5</th>
<th>4</th>
<th>4</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td>B</td>
<td>F</td>
<td>A</td>
<td>D</td>
<td>C</td>
<td>E</td>
</tr>
</tbody>
</table>

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After checking vertex F adjacent to vertex B so it must be colored 2, then examine vertex A adjacent to F and B so that it is colored 3. The next vertex in sequence is vertex D, which after being examined is adjacent to vertex B, F, and A so given color 4. Next checks vertex C which turns out to be adjacency with vertex B, F, and A but not with A, so that it can be color 4. Finally is checking vertex E. Vertex E is adjacent to B, F and A, so it can be given color 4 because it's not adjacent with D and C.

4 CONCLUSION
The results of data processing of the Bandung Wetan garbage truck transport schedule using Welch-Powell algorithm, obtained an optimal number of 4. These results indicate that to facilitate the transportation of garbage from the TPS to the landfill, the Bandung Wetan area is sufficient to provide a minimum of 4 units of transport trucks. To anticipate interference with one of the units so that it cannot operate, it is recommended that PD Kebersihan Kota Bandung prepare 1 additional unit. The reduced number of garbage trucks operating, the operational costs can be reduced by not reducing the volume of transportation, so that the effectiveness of transporting waste from temporary landfill to landfill is maintained. This method can be developed for the same case in other districts.

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6 REFERENCES

