Investigation On Transmission Characteristics Of SMF-PCF-SMF Fibre Optic Structure

Anupam Kumar, Manoj Kumar

Abstract: The transmission characteristic of single mode fibre – photonic crystal fibre- single mode fibre (SMF-PCF-SMF) structure is investigated. The propagation constants of the cladding modes are calculated and their respective field patterns are simulated. The dispersion characteristics of the cladding modes are investigated. The modes are practically restricted within the outer silica structure. This analysis also reveals the penetration of evanescent fields in outer medium which should be helpful in the design of the devices for sensing applications.

Index terms: Cladding mode, effective index modal, evanescent fields, modal field, Photonic crystal fibre

1 INTRODUCTION

In the precedent years, Photonic crystal fibre (PCF) has come into limelight as a substitute to conventional optical fibre in communication as well as other application such as sensing. PCFs are the micro structured optical fibre having only one index of refraction consisting of missing central hole acting as core enclosed by hexagonal array of cylindrical air holes running along the axis of fibre [1]. The electromagnetic wave propagates in these fibres either by total internal reflection [2] or by the photonic bandgap effect [3]. In contrast to conventional optical fibres, PCFs have tailor made properties like endlessly single mode, high birefringence, variable mode area and anomalous dispersion. There are previously several algorithms used to investigate the electromagnetic wave propagation in photonic crystal fibres, such as the localized basis function method [4], Block-iterative frequency-domain methods [5], finite element method (FEM) [6, 7, 8], finite difference method [9] and the plane-wave expansion method (PWM) [10]. Even though these techniques equip us with precise solution but on the other hand they require a large computation time and huge memory. Several models have also been put forward to study the approximate behaviour of electromagnetic waves in PCF such as scalar effective index method (SEIM) [11] and fully vectorial effective index method (FVEIM) [12] which permit us substitute PCF with equivalent convention step index fibre. The cladding index of such corresponding step index fibre is identical to effective index of fundamental space filling mode. Most of these studies are proposed to study the core mode of the fibre. However, PCFs are widely used for sensing application exploits core as well as cladding modes of the fibre. Thus, it turns out to be very essential to study the behaviour of cladding modes which propagates in PCFs. In this paper, we have studied the dispersion in cladding mode using three layer equivalent model. One of the most important parameters in studying the above models is fundamental space filling mode [1,2] which is the modes supported in the array of air holes with highest propagation constant. The modal index of such modes is termed as n_{fsm}. Further transmission response of a SMF-PCF-SMF structure is also investigated.

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2 THEORETICAL MODEL AND SIMULATION METHOD

We consider a SMF-PCF-SMF structure consisting of two identical single mode fibres spliced (assume it to be axially aligned) at both the ends of a photonic crystal fibre of length L. The photonic crystal fibre is an index guiding solid core fibre (LMA-8). The fractional transmitted power through this structure can be calculated using the overlap integral between the two modal fields of SMF to PCF at L= 0 and PCF to SMF at L=L. The modal field pattern (normalized with respect to the entire cross-section of the optical fibre) of the single mode fibre is given as:

$$\Psi_s(r) = \frac{\sqrt{2}}{\omega_s \pi} e^{-r^2/\omega_s^2} \quad (1)$$

where ω_s is the Gaussian spot size of the mode supported by the optical fibre.
In the case of step index single mode fibre this can be approximately expressed as [13]

$$\frac{w_s}{a_s} = 0.65 + \frac{1.619}{V_S^{3/2}} + \frac{2.879}{V_S^6} \qquad 0.8 \le V_S \le 2.5 \quad (2)$$

where V_s is the V parameter of the single mode fibre and a_s is the core radius of the SM fibre. The above formula is correct within the error of less than 1%. However, when the fundamental mode of SMF fibre enters in the collapsed region of PCF it starts to diverge due to diffraction. This diffraction of the Gaussian mode is governed by equation

$$w^2 = w_0^2 \left[1 + \left(\frac{\lambda l}{\pi n w_0^2}\right)^2\right] \quad (3)$$

where w_o is spot size at the tip of collapsed region, λ is wavelength, n is refractive index, l is length of collapsed region. We consider PCFs with claddings of triangular lattice of circular air holes, as illustrated in "Fig. "1. Λ is the lattice pitch; R is the radius of the air-holes. The core and cladding modes of PCF is complex and cannot be determined analytically because of large core cladding index difference. The modes of PCF have been studied using computational techniques which require large simulation time. However, several models have also been proposed to study modes of PCF such as effective index model.

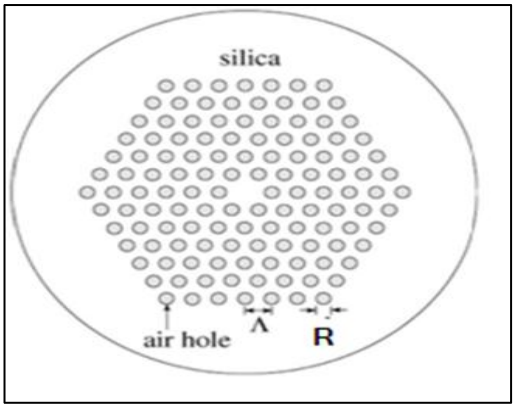

Figure 1 *Typical cross-section of PCF*

The characteristic transcendental equation for the determination of effective index [14] is given by equation 4.

$$W\frac{I_1(aW)}{I_0(aW)}\left(J_0(aU)-Y_0(aU)\frac{J_1(UR)}{J_1(UR)}\right) = -U\left(J_1(aU)-Y_1(aU)\frac{J_1(UR)}{J_1(UR)}\right) \quad (4)$$

Where J_l, Y_l and I_l are the Bessel's function of first kind, Bessel's function of second kind and modified Bessel's function of first kind respectively and a is radii of hole while R is radii of equivalent circular lattice. Several values of R have been proposed however $R = \Lambda/2$ yields the results which agree with numerical results [12] reported so far, where Λ is distance between two holes usually referred as pitch of the fibre. Also, U and W are given as

$$U = \sqrt{(k^2 n_{silica}^2 - \beta^2)} \quad (5)$$

and

$$W = \sqrt{(\beta^2 - k^2 n_{air}^2)} \quad (6)$$

We have solved for β(ω) and the effective refractive index of the fundamental space filling mode. The effective cladding index, is defined as

$$n_{eff}(\omega) = \frac{\beta(\omega)}{k} = \frac{c\beta(\omega)}{\omega} \quad (7)$$

Once we obtained n_{eff} (ω), we have solved the characteristic equation for the propagation constant
β_c (ω), of the LP_{01} mode of the approximate step-index fibre

$$u(\omega)\frac{J_1[u(\omega)r_c]}{J_0[u(\omega)r_c]} = w(\omega)\frac{K_1[w(\omega)r_c]}{K_0[w(\omega)r_c]} \quad (8)$$

where K_0 and K_1 are also modified Bessel functions, and

$$u^2(\omega) = n_c^2(\omega)\frac{\omega^2}{c^2} - \beta_c^2(\omega) \quad (9)$$

and

$$w^2(\omega) = \beta_c^2(\omega) - n_{eff}^2(\omega)\frac{\omega^2}{c^2} \quad (10)$$

with $n_c(\omega)$ being the refractive index of the core material and r_c being the core radius.
The radial part of field of the cladding mode dependency is expressed in terms of Bessel's function

$$R(r) = \begin{cases} A_l I_l(\frac{ur}{R}) & r < a \\ B_l J_l\left(\frac{vr}{R_o}\right) + C_l Y_l(\frac{vr}{R_o}) & a < r < b \\ D_l K_l(\frac{wr}{R_o}) & r > b \end{cases} \quad (11)$$

where

$$u = R\sqrt{\beta^2 - k_0^2 n_{eff}^2} \quad (12)$$

$$v = R_o\sqrt{k_0^2 n_{silica}^2 - \beta^2} \quad (13)$$

and

$$w = R_o\sqrt{\beta^2 - k_0^2 n_a^2} \quad (14)$$

When propagation constant β is in between n_{fsm} and n_{silica} the modes are guided along the fibre. Here, radial field $R(r)$ represents electric as well as magnetic field and β, the modal propagation constant of the cladding modes. Under the weakly guiding approximation, the longitudinal field components (E_z and H_z) are insignificant as compared to the transverse components (E_r, E_ϕ, H_r and H_ϕ). The linearly polarized (LP) modes are the sum of similar modes which are nearly degenerate. Thus $R(r)$ in Eq. 11 then represents the total field polarized normal to the fibre axes rather than one of the separate components of electric or magnetic field. The fields and their derivatives must be continues at the interface of the two regions which yields the transcendental equation. [15]

$$\left(u\frac{I_l'(u)}{I_l(u)} - v\frac{J_l'\left(\frac{vR}{R_o}\right)}{J_l\left(\frac{vR}{R_o}\right)}\right)\left(w\frac{K_l'(w)}{K_l(w)} - v\frac{Y_l'(v)}{Y_l(v)}\right)J_l\left(\frac{vR}{R_o}\right)Y_l(v) = \left(u\frac{I_l'(u)}{I_l(u)} - v\frac{Y_l'\left(\frac{vR}{R_o}\right)}{Y_l\left(\frac{vR}{R_o}\right)}\right)\left(w\frac{K_l'(w)}{K_l(w)} - v\frac{J_l'(v)}{J_l(v)}\right)Y_l\left(\frac{vR}{R_o}\right)J_l(v) \quad (15)$$

Equation 15 determines the eigenvalue of propagation constant β. The modes supported have been evaluated numerically. The modal field at the interface joining collapsed region and holy region can be expressed as sum of core and cladding mode fields.

$$\Psi_s = A_{core}\Psi_{core} + \sum_{m=1}^{\infty}(a_m\Psi_m) \quad (16)$$

Where A_{core} is the amplitude of core mode of PCF and a_m is the amplitude of m^{th} circular cladding mode of PCF. The overlap integral of input incident and output excited field will give the value of A_{core} and a_m respectively. The power coupled from lead in SMF fibre to the core mode of PCF is calculated using the overlap integral of the field of the two fibres. For the Gaussian beam of spot size w_1 and w_2 the power coupled to core mode is calculated as [16]

$$A_{core} = \frac{2w_1w_2}{{w_1}^2 + {w_2}^2} \quad (17)$$

and a_m is evaluated numerically. The mismatch in core field results in the excitation of cladding fields.

3 RESULT AND DISCUSSION

The core mode and excited cladding mode are dispersed in the PCF which are computed numerically using MATLAB. The propagation constants of different cladding mode are computed and plotted in the "Fig. "2. It can be seen that all the modes have almost similar variation in propagation constant against wavelength.

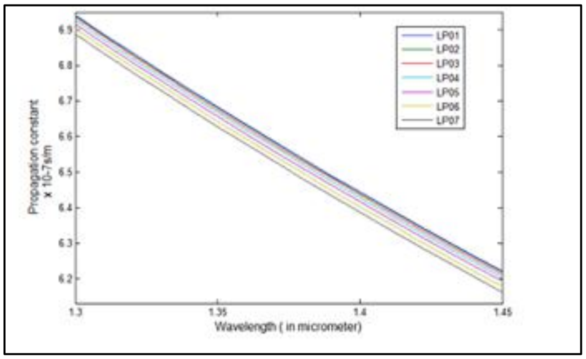

Fig. 2 *Dispersion of circular symmetric cladding modes*

This results in phase difference at the second splice among the various excited cladding modes which are also computed. At the second splice these dispersed modes again coupled from PCF to single mode fibre. The power couple to lead out SMF is also computed using overlap integral with relative phase. Thus the output power of SMF-PCF-SMF structure is dependent on the phase difference and relative excitation of different modes of PCF. We have computed the transmission characteristics of the SMF-PCF-SMF structure. The SMF fibre used for the simulation is SMF 28 with core radii of 4.15 micrometer and relative index difference of 0.0036 having pure silica core. A PCF LMA- 8 of length 3 cm with a collapsed region of a typical 200 micron at each splice with SMF is used for simulating the results. The LMA 8 fibre consists of a pure silica core with air hole of radii 3.15 μm and their interspacing 5.6 μm. The transmission characteristic of the above structure is simulated in "Fig. "3 the using the method discussed which shows multiple peaks and dips. This suggests the excitation of several cladding modes of the PCF along with the core mode.

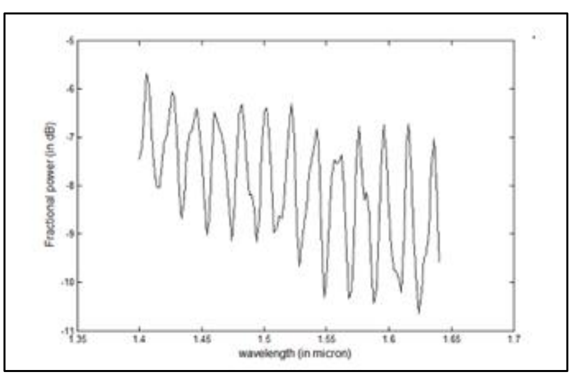

Fig. 3 *Variation of fractional output power as a function of wavelength*

4 CONCLUSION

In conclusion, the cladding ring modes of PCF is analysed and their dispersion characteristics have been studied. Also, light propagation through SMF-PCF-SMF structures has been investigated using the MATLAB under weakly guiding conditions and scalar effective model. The modal interference on the fibre structure performance has been studied and simulated numerically. Results indicate that the SMF-PCF-SMF structure transmission characteristic is very sensitive on the length of PCF, refractive index of PCF and its surrounding medium which in turn depends on the physical parameters like temperature, strain, bends etc. The structure considered in this paper, covers the theoretical aspects which resembles with the experimental works carried so far. Moreover, by further optimizing a number of physical parameters for the proposed fibre device length of PCF, alignment of SMF-PCF, air hole radii and its pitch may further increase its sensitivity.

5 REFERENCES:

[1] J.C. Knight, T.A. Birks, P.St.J. Russell, D.M. Atkin, Opt"All-silica single-mode optical fibre with photonic crystal cladding." Optics Letters Vol. 21, Issue 19 21.19 (1996): 1547-1549.
[2] T.A. Birks, J.C. Knight, and P.St.J. Russell, "Endlessy single-mode photonic crystal fibre," Opt. Lett. 22,961-963 (1997).
[3] J.C. Knight, J. Broeng, T.A. Birks, P.St.J. Russell, Science 282 (1998) 1476.
[4] Erik Knudsen' , Anders Bjarklev, "Modelling photonic crystal fibres with Hermite–Gaussian functions", Optics Communications, Volume 222, Issues 1–6, 1 July 2003, Pages 155–160
[5] S.G. Johnson and J.D. Joannopoulos, "Block-iterative frequency-domain methods for Maxwell's equations in a planewave basis," Opt. Express 8, 173-190 (2001),
[6] K. Saitoh and M. Koshiba, "Full-vectorial imaginary-distance beam propagation method based on a finite element scheme: application to photonic crystal fibres," IEEE J. Quantum Electron. 38, 927-933 (2002).
[7] Cucinotta, S. Selleri, L. Vincetti, and M. Zoboli, "Holey fibre analysis through the finite element method," IEEE Photon. Technol. Lett. 14, 1530-1532 (2002).
[8] F. Brechet, J. Marcou, D. Pagnoux and P. Roy, "Complete analysis of the characteristics of propagation into photonic crystal fibres, by the finite element method," Opt. Fibre Technol. 6, 181–191 (2000).
[9] Z. Zhu and T. Brown, "Full-vectorial finite-difference analysis of microstructured optical fibres," Opt. Express 10, 853-864 (2002).
[10] J. Arriaga, J.C. Knight, and P.St. Russell, "Modeling Photonic Crystal Fibres", Physica E 17, Elsevier Science, pp. 440-442, 2003M. Koshiba and K. Saitoh, "Applicability of classical optical fibre theories to holey fibres," Opt. Lett. 29, 1739-1741 (2004).
[11] Yan-feng Li *, Ching-yue Wang, Ming-lie Hu, "A fully vectorial effective index method for photonic crystal fibres: application to dispersion calculation" Optics Communications 238 (2004) 29–33

[12] y. Li, c. Wang, y. chen, m. hu, b. Liu, l. chai "Solution of the fundamental space-filling mode of photonic crystal fibres: numerical method versus analytical approaches" Appl. Phys. B 85, 597–601 (2006).
[13] Marcuse D., Gaussian approximation of the fundamental modes of graded-index fibres, J. Opt. Soc. Am. , 68, 103–109.(1978)
[14] Y. Li, C. Wang, M. Hu, "A fully vectorial effective index method for photonic crystal fibres: application to dispersion calculation"Opt. Commun. 238, 29 (2004)
[15] Kumar A., Kumar M. Cladding Mode Analysis of Photonic Crystal Fibre Using Scalar Effective Index Model. Lecture Notes in Electrical Engineering, vol 511. Springer, Singapore. (2019)
[16] W. Snyder and J. D. Love, Optical Waveguide Theory (Chapman & Hall, New York, (1983).