State-Of-The-Art Review On The Use Of Optimization Algorithms In Steel Truss

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Abstract: Structural design optimization is a mathematical approach that concerns in finding the maxima and minima function subject to some constraints. This involves various optimization technique to find the best possible design in terms of weight, reliability and thus the overall cost. Various researchers have worked on different optimization techniques in finding out the efficient and light weight structures that are essential for the actual design of tall structures. This paper summarizes the various techniques of optimization of steel truss or towers that have been used till now. For this purpose, different optimization techniques have been studied which involves the various geometric constraints like changing the base width, bracing pattern, area of cross section. By reviewing the literature of the works done, the common objective emphasizes the need for finding the minimum weight of the structure. From studies we see optimization using metaheuristic algorithm are effective in order to solve truss problems. Metaheuristic algorithms are nature -inspired and most widely used due to its applicability and feasibility to various types of structures with many numbers of design variables. In this paper a 25-bar space benchmark truss has been considered for demonstrating the performance of various optimization algorithm. A comparative study is done based on the performance in lowering the weight of the total truss. Results shows that optimal weight of the truss structure can be obtained effectively using Whale optimization algorithm and it proved to be robust and efficient than other algorithms.

Index Terms: Steel truss; Metaheuristic algorithm; Optimum weight; review

1 INTRODUCTION

In most engineering designs, civil engineers are engaged in designing buildings, dams, bridges, and different other structures in order to accomplish a minimum overall cost or maximum safety or both [1]. For this common design objective, structural optimization has emerged. The design process in optimization of structure as announced in 80's [2]. Optimization is a method of obtaining the best suitable result under a given circumstances. The main purpose is to obtain minimum or maximum value of the objective function [3,4]. Sometimes, optimization problem may have multiple objective functions. Structural optimization problems are of three different types. The first is the dimensional optimization which consider the sizing of the elements as design variables, which can continuously change or can be chosen from a list of available cross-sectional dimensions. The second is geometric optimization, taking into consideration the nodal coordinates. Finally, the topology optimization includes the number of elements. The minimization of cost or weight is the main objective to be achieved, and the constraints are associated with the design codes and requirements. The use of optimization algorithm has become popular recently as it deals exclusively with the mathematical form of the problem interfaced with computer model representing the physical structure. In this paper sizing optimization of space truss aimed at minimizing the weight of the structure is done under certain behavioral constraints on stress and displacements.

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In this paper, a 25-bar design example is considered to demonstrate the application of the optimization algorithms in determining the optimum weight of the total truss. A comparative study is done based on the performance of various algorithms in minimizing weight.

2 OPTIMIZATION TECHNIQUES

Optimization process involves four different stages: formulation of function as per requirement, conceptual design stage, optimization stage and detailing. The use of algorithm in the iterative stage was before the final solution is achieved. The optimization problem has the following characteristics: a) Objective function: It is a function, associated with the dimension of the analyzed problem, which will measure the performance of the system. For example: In a two-bar truss with a load P, as shown in Figure 1, the objective function is minimization of its mass, can be written as $w = \rho_i .l_i .a_i$, where ρ_i is the specific mass of the system, l_i is the length of the element, a_i is the cross-sectional area of each member i.

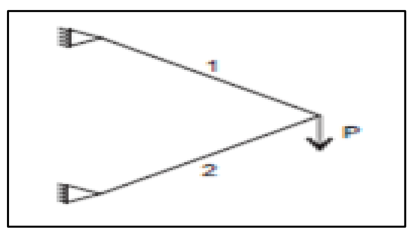

***Fig.1**. Truss example*

b) Design variables: they are parameters allowed to be modified to improve the objective function. They are written as vector $x = (x_1 \ldots x_n)$, where, n is the number of variables and E is the associated design space. For the above truss example, value of the design variables can be the value of the cross-sectional area of each member a_i. The vector x can be expressed as $x = (a_1, a_2)$. c) design space: these are constraints which are applied to limit the design space and subspace S of E. In the truss, to limit the values of cross-sectional areas between the minimum and maximum values, as $a_{min} \leq a_i \leq a_{max}$. the optimization problem is given by x, which minimizes weight given by w(x). where x belongs to subspace S. Various techniques have been studied to possibly optimize a truss structure [5]. Optimization of power transmission tower was done for making the truss more cost effective by varying the bracings, configuration and type of structure in order to reduce weight. Transmission line towers are tall structures which have much more height compared to their lateral dimensions. It is a steel space frames and supports the conductors and earth wires. They involve about 28 to 42% of the cost of the transmission line. After the optimization process the following results were obtained. Firstly, there was a saving in steel weight by about 21.2% which was achieved from three-legged tower compared to a four-legged type tower. Secondly, saving in steel weight resulted in optimization of cost by using different sections. Further the application of high-strength steel lead to a saving in cost by 4-16% [6]. Comparative studies were done between square and triangular sections for least weight. Using the triangular tower, the self-weight was noted to be 9.23% less as compared to square tower. Triangular tower behaves more rigidly than square tower as they have lesser amount of node deflection throughout the entire height. Analysis and design of two- self-supporting tower was studied [7]. A comparative study is conducted between three legged and four legged models using common parameters. Results show that there is a saving in steel weight by about 21.2% for three-legged tower than a four-legged type. Comparison were made between three-legged tube section and three-legged angle section. Weight was reduced by about 20.6% by using tube section. Optimization of high-strength steel was done having cost as the objective function [8]. Cross-sectional area was chosen as the design variable. The main aim was to find the minimum cost and minimum displacement. Analysis were performed and results shows that there was a saving by 4-16% in cost. A new method of optimization of tower design was done for extra high-voltage transmission lines taking tower weight and geometry as the desired function [9]. Nonlinear optimization method was incorporated taking into consideration the configuration and design of the tower as the prime interest. Fuzzy optimization technique was used for the double circuit transmission tower under multiple loading condition. Structural optimization of transmission tower was done for obtaining minimum weight using an optimization methodology that allows to obtain more efficient solution than conventional designs of towers [10]. Studies were done on design of a 33KV double circuit transmission line tower with rectangular base self-supporting lattice tower that optimizes the present geometry. The structural behavior of existing tower was studied and Excel programs are developed for calculation of load in STAAD-Pro. Analysis was performed in STAAD-Pro, and the axial stress, compressive stress of the tower member were obtained. While weight optimization, the height of the tower, base width and basic outline of the tower were design variables. Optimization of truss tower for least weight was studied for tower subjected to multiple combination of dead load, wind load and seismic load [11]. Optimization problem is divided into two design spaces. Hooke and Jeeves method were used for changing the coordinate variables. Design variables chosen are the area of the member. For designing the tower, three different sections are selected namely pipe section, angle section and tube section [12]. Results show that under anti-cascade loading condition, 1S2 tower with X-bracing and angle section, has a reduction in weight of about 14% compared to the weight before optimization, while in case of 2S2 tower, reduction was 24%. In case of pipe section, the weight reduced by 85% of that with angle section for 1S2 tower, and for 2S2 tower, the reduction was about 88%.

3 OPTIMIZATION ALGORITHMS

An optimization algorithm is the process which is iteratively executed by comparing the various solutions until an unless a satisfactory or optimum solution is obtained. These algorithms are based on iterative processes. They begin with an initial vector x0 and follows x_0, x_1…. x_n, until finally converge to a global minimum (or global maximum point). There are two distinct types of optimization algorithms used now a today. First there are algorithms which are deterministic, with specific rules for moving from one solution to the other. Secondly, there are algorithms which are stochastic transition rules. Metaheuristics algorithms are nature inspired and are now a days most popularly used as optimization algorithms. They carry many advantages when compared with the conventional algorithm.

3.1 Metaheuristics Algorithms

Metaheuristics algorithms emerged as the most effective approaches to tackle complex problems in the field of structural engineering. They are nature inspired and probabilistic-based search algorithms which are now a days most popularly used. They carry many advantages when compared with the conventional algorithm as because: (i) based on simple concepts and easy to implement; (ii) gradient information is not required; (iii) they can be used in a wide range of problems. These algorithms are of three different categories namely evolution-based, physics-based, and swarm-based methods. Evolution-based techniques includes genetic algorithms, Genetic Programming. Simulated Annealing, Central Force optimization, Gravitational Search Algorithm are few of the algorithms based on physics-based methods. Swarm-based techniques includes ant and bee algorithms, firefly algorithm, ant colony, differential evolution, particle swarm optimization harmony search, Cuckoo search and P-delta analysis. A methodology was studied for reducing the weight and cost of big steel structures during the early design phase [13]. Using genetic algorithms and SAP2000 software, a platform-tool was developed to support the automatic optimization of steel frames. The minimization of weight is the most used, and often there is an implicit assumption that the weight of a structure is the best measure for evaluating its cost. To optimize the weight of the transmission line tower Cuckoo search algorithm was developed [14]. In the first phase, the tower was analyzed and optimized using STAAD pro and the results are compared with the optimization results obtained from using algorithm. For case study, a 244-bar transmission tower model was considered. The objective function was weight of the tower and the constraints were the stress and deflection. The tower had 77 nodes and 244 members. Using Cuckoo search algorithm, results shows that for the

244-bar transmission line tower with 26 design variables, weight can be reduced by 8 percent whereas, for 472 bar transmission line tower with 49 design variables it can be reduced up to 10 percent. Ant colony optimization algorithm was used effectively for structural optimization problems with various new improvements [15]. The numerical simulation results demonstrated that this new methodology showed more obvious advantage for truss structure optimization problems in discrete system. Optimization of Transmission Tower was done using Genetic Algorithm, where weight of the transmission tower taken as the objective function while keeping the base width, height and basic outline of the tower constant [16]. The model was modeled and analyzed in ANSYS 16.2. Results obtained are workbench optimization is 69.46% and for optimization in MATLAB using algorithm showed a reduction of 78.66%.Optimal weight of transmission towers subjected to static loading was found by using Genetic Algorithm [17]. Optimization and Analysis procedure which integrated numerical methods and finite element analysis have been used for optimization. Also, a FORTRAN based genetic algorithm is implemented to search the optimum design. The results obtained using different design variables were compared. Optimization of Transmission Line Towers was carried out by using P-Delta Analysis [18]. A 400kV double circuit tower was modelled using Angle and Tubular sections using STAAD.Pro. Wind load analyzed by using linear static and P-delta analysis was done in order to study the importance of P-delta analysis for transmission tower. A comparative study has been made with respective to cost and displacement for both sections. Results showed that there was a saving in steel weight up to 20.9% using tubular section as compared to angular section. Whereas, the displacement values increased when tower is analyzed for P-delta as compared to static analysis.

3.1.1 Cuckoo Search Algorithm (CS)

One of the widely used metaheuristic algorithm is Cuckoo search algorithm which have been used for its good optimization ability [19]. This algorithm is inspired by the breeding action of cuckoo. This behavior gives a good balance between the local optimization and global optimization. Due to its promising performance and high efficiency, a good balance of local search and global exploration can be achieved for weight optimization and health monitoring. The specialty of the algorithm lies in its use of Levy flight behavior technique which improves the search efficiency of the technique. Cuckoo search algorithm was applied for weight minimization of truss, considering both continuous and discrete design variables. The breed behavior of cuckoo species is reviewed and performance of the algorithm was investigated [20].

3.1.2 Templeman Algorithm

This method deals in finding the optimum structural design with discrete member sizes. Optimization model for indeterminate trusses using discrete member sizes was proposed by Templeman and Yates [21] under the constraints of joint displacements and member stresses. Optimum design weight of truss has been achieved with discrete member sizes [22]. But this did not deal with constraints on the buckling of members which requires extra rounding processes after optimization by the simplex method. This has further increased the efficiency in calculating the optimum result.

3.1.3 Genetic Algorithm (GA)

These are search procedures based on the mechanics of natural genetics and natural selection. Genetic algorithms are computationally simple, but powerful in their search of the fittest with genetic operators abstracted from nature. They form a robust search mechanism and differ from traditional optimization algorithms [23]. This algorithm effectively exploits useful information contained in a population of solutions to generate new solutions. Discrete design variables have been used for optimizing structural systems by penalty-based transformation method [24].

3.1.4 Particle Swarm Optimization (PSO)

PSO algorithm are well-known metaheuristic algorithm which have shown success in a wide rage of problems. The development and applications of PSO are reviewed by many researchers [25] They are stochastic in nature and has a wide application in the field of structural design optimization. They are based on behavior of bird swarm concerned with grouping by social forces and it uses a velocity vector to update the current position of each particle in the swarm. The position of each particle in the swarm is updated based on the social behavior of the swarm, which adapts its environment by returning to the previously discovered promising regions of the space and searching for better positions over time. They provide better computational performance compared to the traditional algorithms used earlier. A new modified PSO algorithm (MPSO) was used for optimization of tall steel buildings [26]. In this sequentially PSO was utilized in multiple stage scheme where in each stage an initial swarm was generated based on the information obtained from the previous stage results.

3.1.5 Chaotic Swarming of Particles (CSP)

In case of particle swarm optimization, the chaotic dynamics is combined into the position updating rules of PSO in order to improve the diversity of solutions and avoid being trapped in the local optima. This incorporation of chaotic dynamic further enriches the search behavior. Chaos search has proved to be more efficient in solving global optimization problems than other stochastic algorithms [27,28]. Chaos methods has been introduced in system by randomly reinitializing the particle positions [29]. Chaotic particle swarm optimization has been introduced for route network problems and its effectiveness is tested for parking space guidance [30].

3.1.6 Whale Optimization Algorithm (WOA)

This metaheuristic optimization algorithm is based on the hunting behavior of humpback whales. A unique technique called bubble-net feeding behavior [31]. This is mathematically modelled in order to perform the optimization process. This new swarm-based optimization algorithm includes three operators to simulate the search process for the prey, encircling the prey and then the bubble-net foraging behavior of humpback whales. Due to the convergence behavior of this algorithm it has proved to be better and competitive enough than another metaheuristic method. Its search technique is much robust than other conventional techniques.

4 BENCHMARK TRUSS OPTIMIZATION

In this study, a 25-bar design example is considered to demonstrate the application of the optimization algorithms presented. The truss design along with the nodal numbers of a 25-bar spatial truss structure are shown in Figure 2. This 3D spatial truss is subjected to two loading conditions as shown in Table 1. The material density is taken as 0.1 lb/in^3 (2767.990 kg/m^3), and the modulus of elasticity is considered as 10000 ksi (68950 MPa). Maximum displacement limitations of $\pm$0.35 in ($\pm$8.89 mm) are imposed on every node in every direction, and the axial stress constraints vary for each group as shown in Table 2. All the 25 members are categorized into eight different groups as follows: (1) A_1, (2) A_{2-5}, (3) A_{6-9}, (4) A_{10-11}, (5) A_{12-13}, (6) A_{14-17}, (7) A_{18-21} and (8) A_{22-25}. The range of cross-sectional areas, a_i varies from 0.01 in^2 to 3.4 in^2 (or 0.6452 cm^2 to 21.94 cm^2). And the list of length was from the code American Institute of Steel Construction Manual as L_i.

L_i = {1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 16.9, 18.8, 19.9, 22.0, 22.9, 26.5, 30.0, 33.5} (in).

The set of available areas are A_i = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, and 3.4} (in²).

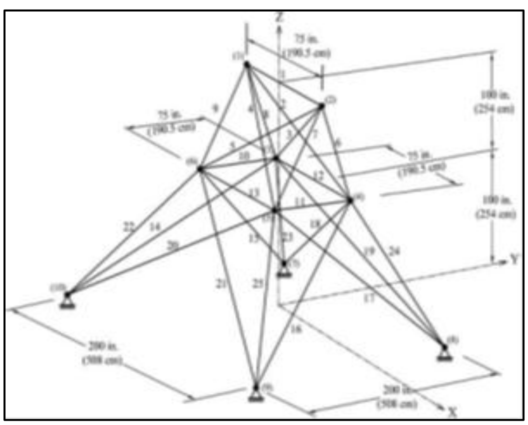

Fig.2. A 25-bar benchmark truss [19]

Table 1. *Loading conditions.*

	Case 1			Case 2		
Node	P_X kips(kN)	P_y kips(kN)	P_z kips(kN)	P_X kips(kN)	P_y kips(kN)	P_z kips(kN)
1	0	20.0 (89)	-5.0 (22.25)	1.0 (4.45)	10.0 (44.5)	-5.0 (22.25)
2	0	-20.0 (89)	-5.0 (22.25)	0	10.0 (44.5)	-5.0 (22.25)
3	0	0	0	0.5 (2.22)	0	0
4	0	0	0	0.5 (2.22)	0	0

Table 2*. Member stress limitation*

	Element group	Compressive stress limitations ksi (MPa)	Tensile stress limitations ksi (MPa)
1	A_1	35.092 (241.96)	40.0 (275.80)
2	A_{2-5}	11.590 (79.913)	40.0 (275.80)
3	A_{6-9}	17.305 (119.31)	40.0 (275.80)
4	A_{10-11}	35.092 (241.96)	40.0 (275.80)
5	A_{12-13}	35.092 (241.96)	40.0 (275.80)
6	A_{14-17}	6.759 (46.603)	40.0 (275.80)
7	A_{18-21}	6.959 (47.982)	40.0 (275.80)
8	A_{22-25}	11.082 (76.410)	40.0 (275.80)

Size optimization of the truss depends on obtaining the optimum values of the cross-sectional area A_i that would results in optimum weight W. The optimal design for minimizing the weight of the structure is given by equation 1 as shown below.

$$W(\{x\}) = \sum_{i=1}^{n} \rho i. Ai. Li \quad (1)$$

Where,

A_i is the cross-sectional area of the structural member, Li is the length of the member, ρ is the unit weight of the member. The steps involved in finding the optimal weight using algorithm are shown below in Figure 3.

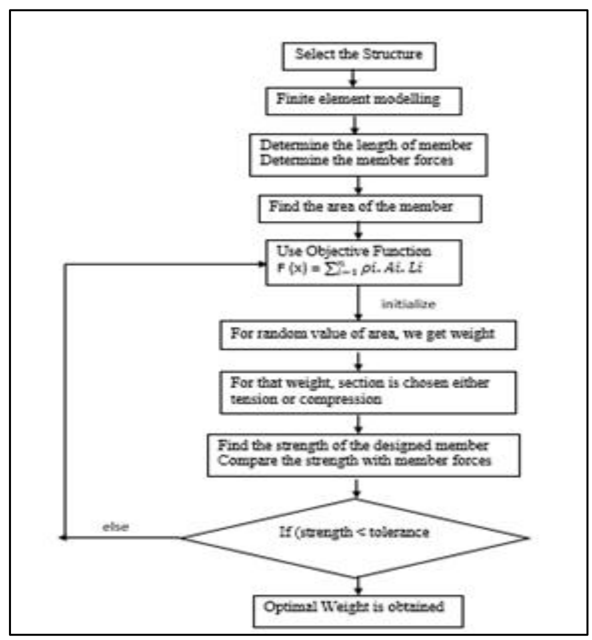

Fig.3. *Flowchart showing the implementation of algorithm*

5 RESULTS AND DISCUSSIONS

Optimization algorithm is to be chosen such that it gives successful result in less amount of time. Optimum results obtained in this study from various works in literature for this truss using the same load case, constraints and material properties has been shown in Table 3. The results produced by the Whale optimization algorithm have provided the optimum value when compared to the results obtained by using other algorithms. In the existing truss, WOA provide the lowest weight in total weight optimization of the truss.

Table 3. *Optimization results using different metaheuristic algorithms*

Design variable (area in^2)	Gandomi.A.H. et.al (2012) CS [19]	Zhu (1986) TEMPLEMAN [22]	Rajeev et.al (1992) GA [24]	Schutte et.al (2003) PSO [32]	Kaveh et.al (2014) CSP [33]	Mirjalili et.al (2016) WOA [31]
A1	0.01	0.10	0.10	0.01	0.01	0.01
A2	2.003	1.80	1.80	2.052	1.91	2.053
A3	3.007	2.30	2.30	3.001	2.798	3.004
A4	0.01	0.20	0.20	0.01	0.01	0.01
A5	0.01	0.10	0.10	0.01	0.01	0.01
A6	0.687	0.80	0.80	0.684	0.708	0.68
A7	1.655	1.80	1.80	1.616	1.836	1.61
A8	2.66	3.00	3.00	2.673	2.645	2.675
Weight (lb)	545.10	562.93	546.01	545.21	545.09	544.608

(CS stands for Cuckoo Search, GA is Genetic Algorithm, PSO is Particle Swarms Optimization, CSP is Chaotic Swarming of Particles, WOA is Whale Optimization Algorithm.) From the comparative study on the performance of various algorithm in weight optimization of truss, we know that Whale optimization algorithm is very efficient and proves to be superior among other optimization algorithms. The results obtained clearly shows that the proposed convergence search technique of whale optimization algorithm has shown better results as compared to other search techniques. Apart from 25 bar truss, WOA when tried on higher number of bar trusses, it has the potential to solve very effectively real problems with its unique search technique. Thus, WOA has proved to be robust and superior than other conventional algorithms.

6 CONCLUSION

The review work on the optimum design of the 25-bar truss structure using various metaheuristic algorithms is presented in this paper. For obtaining optimum weight using an optimization algorithm that allows to obtain more efficient solution than conventional designs methodology. The proposed metaheuristic optimization algorithms were studied and the results obtained from the previous work in literature were compared. In this paper, an approach is made to optimize weight using algorithms. The results produced with the Whale optimization algorithm have proved to be efficient in finding the optimum when compared to the results obtained by other algorithms. Thus, by involving different design variables and different degrees of complexity in constraints, Whale optimization algorithm has proved to be robust better solutions.

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