Wavelet Transforms And Image Approximation Based Image Compression System

Taiwo Samuel Aina, Oluwaseun Olanrewaju Akinte, Babatunde Ademola, Iyaomolere, Innocent Iriaghuan Abode

Abstract - The purpose of this work is to design an efficient image compression system using wavelet transforms and image approximation by modifying the wavelet coefficient. The efficiency of the system will be tested using a test image and determining the Mean Square Error (MSE). 2D-daubechies wavelet transformation with global threshold for wavelet coefficients and numerical presentation utilizing Matlab programming are the techniques used. The Discrete Wavelet Transform (DWT) has a basic principle of splitting signals into two parts namely; the high frequencies and low frequencies. For a number of repetitions, the low frequency section is further divided into high and low frequency parts, which are generally chosen by the application. The performance of an image compression system is commonly measured by calculating the MSE and the Peak Signal to Noise Ratio (PSNR).

Keywords *: Image compression, wavelet transform, Image approximation, wavelet coefficient,* Discrete Wavelet Transform.

I. INTRODUCTION

In computer vision, image compression has been a major area of study strength [2]. Capturing several images with the help of camera is done by every computer vision application [7]. Compression of images eliminate unnecessary or non-essential information from a digital image to reduce the amount of data required to depict it [1]. Many of compression techniques like JPEG and JPEG2000 are used recently [6]. Compression of image can be done by different methods such as Discrete Cosine Transform (DCT) and Wavelet Algorithm Transform(DWT). DCT functionality is to separate images into differing frequencies part [4]. In image coding and compression applications, discrete wavelet transform (or DWT) is one of the adaptable techniques offered to increase the performance of the compression scheme [3]. The Discrete Wavelet Transform (DWT) has a basic principle of splitting signals into two parts namely; the high frequencies and low frequencies. The low frequency part is further split into high and low frequency parts for a number of iterations often determined by the application. The original signal is usually recovered or reconstructed at the decoder by employing the inverse DWT . The wavelet functions used to implement the multi-level decomposition of an image include the following but not limited to Haar wavelet, Symlet wavelet, and Daubechies wavelets. The fact that neighbouring pixels are connected and hence store redundant information is a common feature of most image [5]. [8][9][10][11][12] [13] added directionality into existing 2D wavelet functions, resulting in only three orientations in the decomposition subbands for Separable 2D wavelets. These orientations are vertical, horizontal and diagonal [13]. In the transmission and storage of data, there is a challenge of limited bandwidth and memory space. To overcome this problem, image compression techniques are employed to reduce the data size. However, the compressed image needs to be reconstructed with minimal loss [4]. II. Implementation of a single level decomposition of a W × H image size using the 2D DWTThe implementation of a single level decomposition of a W × H image size using the 2D DWT is shown in Fig. 1. The analysis stage involves the use of low pass filter and high pass filter to filter the 2D image along the rows and columns to produce four sub-band images of size W/2 × H/2. In the schematic diagram of Fig. 1, the sub-bands are identified as HH, HL, LH and LL. The LL sub-band is the approximation image or lowest frequency subband while HH, HL, and LH are diagonal, horizontal and vertical component of the image transformation respectively. Down sampling has a scaling factor of 2.

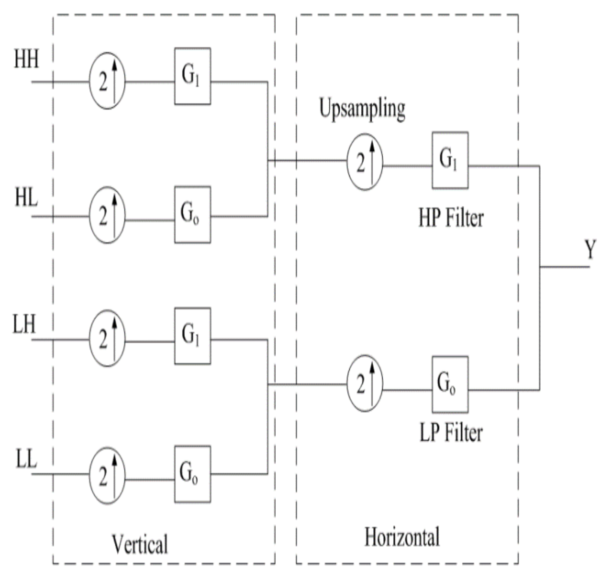

Fig. 1: A schematic diagram of a single level wavelet decomposition of 2D image

III. Inverse Wavelet Transform

In order to recover the original image from the single level decomposition, the inverse DWT was employed to carry out a single level reconstruction of the compressed image. The synthesis stage of the reconstruction also involves the use of low pass filters and high pass filters with the outputs of the decomposition used as its inputs as shown in Fig. 2. The filtering is also done along the columns and rows and upsampling of scaling factor of 2 is applied in this synthesis process.

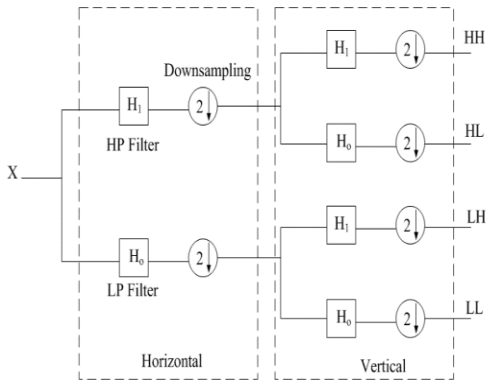

Fig. 2: A schematic diagram of a single level wavelet reconstruction of 2D image

IV. Wavelet transform deployment with 3 Iterations to Images

To achieve better compression, multi-resolution decomposition using a 2D-DWT on k levels can be employed to produce a set of 3k + 1 sub-bands. For instance in Fig. 3, the three-level wavelet decomposition of 2D image using a single level 2D-DWT in a row-column approach is presented. If the original image is of a size of 512 × 512 pixels, the first level of decomposition will yield four sub-band images of size 256 × 256 pixels, then the approximation image of the first level acts as input to the second level of decomposition and produces another four sub-band images of size 128 × 128 pixels. Similarly, the third level of decomposition produces another four sub-band images with the size of 64 × 64 pixels.

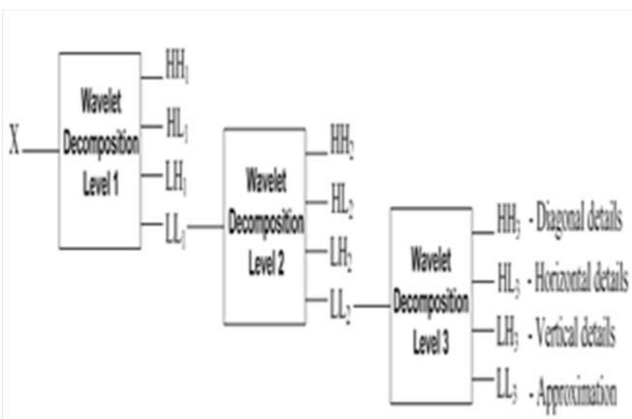

Fig. 3: A schematic diagram of a 3-level wavelet decomposition of 2D image

V. Reconstructing images using 3 iterations (decomposition levels)

The reconstruction of the original image is done by employing the inverse DWT to the outputs (sub-band images) from the third level decomposition. As shown in Fig. 4, the sub-band images are passed through three stages of synthesis with an upsampling of scale factor 2. The inverse DWT of 2D sub-band images also follows the row-column approach.

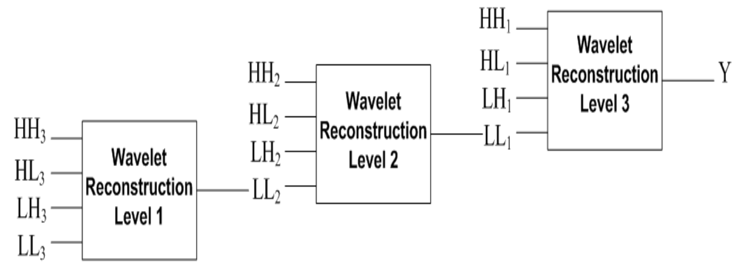

Fig. 4: A schematic diagram of a 3-level wavelet reconstruction of 2D image

VI. Image approximation

In digital image processing, the sub-band images for each level of decomposition have four wavelet coefficients in the LL, LH, HL and HH sub-band. These wavelet coefficients need to undergo further coding to enhance the reconstruction of the original image. There are several methods to achieve this goal which includes the thresholding, vector or scalar quantization and the zero-tree based approximation. In this design, the zero-tree based approximation is employed to set the coefficients that does not belong to the lowest pass band (i.e. LL) to zero or white colour. The schematic diagram of the system that implements the modifications to the wavelet coefficients is shown in Fig. 5.

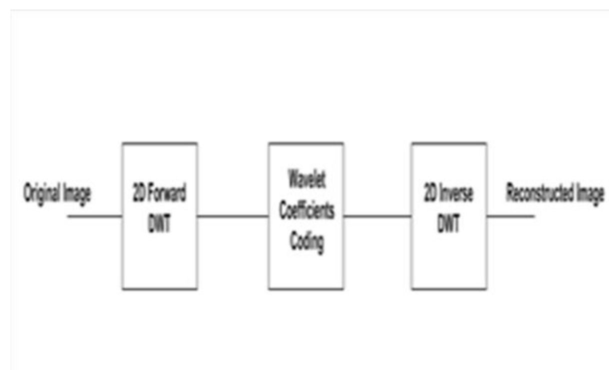

Fig. 5: A schematic diagram of a 3-level wavelet reconstruction of 2D image

The working operation of the image approximation system is based on multiply-accumulate principle. Considering the filter components in Fig. 1;

VII. Calculation

The Convolution theorem is used to represent the mathematical operations performed in the cascaded low-pass and high-pass filters. At the first level of wavelet decomposition, the coefficients at the sub-bands are given as;

LL:

$$a_{i+1}(n_1, n_2) = \sum_{k_1=0}^{L-1} \sum_{k_2=0}^{L-1} H_0(k_1) H_0(k_2) X(2n_1 - k_1, 2n_2 - k_2) \quad (1)$$

LH:

$$d_{i+1}(n_1,n_2)=\sum_{k_1=0}^{L-1}\sum_{k_2=0}^{L-1}H_0(k_1)H_1(k_2)X(2n_1-k_1,2n_2-k_2) \quad (2)$$

HL:

$$d_{i+1}(n_1,n_2)=\sum_{k_1=0}^{L-1}\sum_{k_2=0}^{L-1}H_1(k_1)H_0(k_2)X(2n_1-k_1,2n_2-k_2) \quad (3)$$

HH:

$$d_{i+1}(n_1,n_2)=\sum_{k_1=0}^{L-1}\sum_{k_2=0}^{L-1}H_1(k_1)H_1(k_2)X(2n_1-k_1,2n_2-k_2) \quad (4)$$

Where k1 and k2 stands for the rows and columns of the image, L is the filter size

In the implementation of a three level decomposition of 2D images, the LL coefficient above is further split into four sub-band images. This way the number of addition and multiplication operation needed can be determined.

The implementation of the proposed image compression was done on MATLAB using the Wavelet commands. The algorithm used is as follows:

Step 1: Load or Read image using the imread command, the image used is the 'lena.tiff' of size 512 x 512 pixels.

Step 2: Choose the wavelet function and number of decomposition level which is 'db2' and 3 respectively

Step 3: Apply the multi-level 2D wavelet decomposition using the wavedec2 function

Step 4: Implement the reconstruction of original image from the approximation coefficients using the wrcoef2 function

Step 5: Calculate the MSE value between the reconstructed image and the original image.

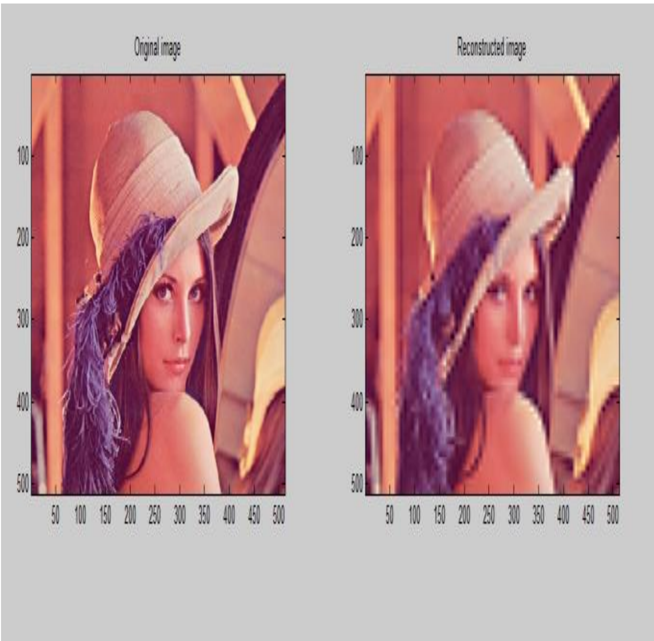

Fig. 6: The diagram showing the original and reconstructed image

The pictorial result of the MATLAB simulation is shown in Fig. 6. From the experiment carried out, the MSE value is 9.7656e-004 or 0.0009756.

VIII. Conclusion

The challenges of limited bandwidth and storage space faced in image-based applications are being overcome by image compression techniques like DWT. The DWT has the advantage of achieving quality compression ratio with minimal loss of information. This work presented the system architecture of the multi-resolution analysis approach to image compression and showed how a 2D image can be split into four sub-band images including the approximation image and details image. These wavelet coefficients are further split into another four sub-band images. The image approximation was implemented by employing the 2D – DWT multi-resolution decomposition. The reconstruction of the original image was done by extracting the lowest frequency sub-band image (i.e. LL) from the three-level decomposition output. The modification to the wavelet coefficients was done by using the approximation image alone to reconstruct the original image. The result of the experiment performed in MATLAB environment showed a minimal error of 0.0009756. Hence, the two-dimension DWT approach is highly efficient for achieving a good quality of image approximation.

IX. References

[1] A. Al-hamid, A. Yahya and R. El-Khoribi, "Optimized Image Compression Techniques for the Embedded Processors", International Journalof Hybrid Information Technology, vol. 9, no. 1, pp. 319-328, 2016.

[2] S. Sharma and U. Bhat, "Image Compression using an efficient hybrid algorithm", Journal on Today's Ideas-Tomorrow's Technologies, vol. 1, no. 1, pp. 45-50, 2013.

[3] P. B and S. M P, "Comparative Analysis of JPEG2000 Image Compression with Other Image Compression Standards using Discrete Wavelet Transforms Technique", International Journal of Engineering Trends and Technology, vol. 13, no. 3, pp. 107-110, 2014.

[4] N. Taujuddin and N. Lockman, " Image compression using wavelet algorithm ", Mathemat Ics, pp. 1–8. 2011.

[5] Y. Lai and C. Kuo, "A Haar Wavelet Approach to Compressed Image Quality Measurement", Journal of Visual Communication and Image Representation, vol. 11, no. 1, pp. 17-40, 2000.

[6]A. Hussain, G. AL-Khafaji and M. Siddeq, "Developed JPEG Algorithm Applied in Image Compression", IOP Conference Series: Materials Science and Engineering, vol. 928, p. 032006, 2020.

[7]S. Shelke, S. Sinha and G. Patel, "Development of complete image processing system including image filtering, image compression & image security", Materials Today: Proceedings, 2021.

[8] M. N. Do and M. Vetterli, "Contourlets: A directional multiresolution image representation," in Proc. IEEE Inter. Conf. Image Process., Sept 2002.

[9]M. Do and M.Vetterli, "The contourlet transform: an efficient directional multiresolution image representation," IEEE Trans. Image Process., vol. 14, no. 12, pp. 2091–2106, Dec. 2005.

[10]E. J. Cande`s and D. L. Donoho, "Ridgelets: a key to higher dimensional intermittency?" in Philos. Trans. Roy. Soc. London Ser., Sept 1999, vol. 357, no. 1760, pp. 2495–2509.

[11]E. L. Pennec and S. Mallat, “Image compression with geometrical wavelets,” in Proc. IEEE Inter. Conf. Image Process., Sept 2000, vol. 1, pp. 661–664.

[12] “Sparse geometric image representations with bandelets,” IEEE Trans. Image Process., vol. 14, no. 4, pp. 423–438, Apr. 2005.

[13]W. Lee and A. Kassim, "Signal and Image Approximation Using Interval Wavelet Transform", IEEE Transactions on Image Processing, vol. 16, no. 1, pp. 46-56, 2007.