Almost Product Structure On Differentiable Manifolds With Nijenhuis Tensor

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Abstract: In the present paper, we have discussed the almost product structure on differentiable manifold with Nijenhuis tensor. In section 1, we emphasized the introductory part of the almost product differentiable manifold and their applications. Again in section 2, we study the Nijenhuis tensor with bilinear function N and, explain corollaries, remarks and proof some theorems. Further, in section 3, we define the modified Nijenhuis tensor and solve some theorems and corollaries. In the end, we discussed about the almost product manifold and the application of the Nijenhuis Tensor.

Index Terms: Nijenhuis tensor, GF-structure, differentiable manifold, H-structure, Riemannian connexion

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1 Introduction

Let us consider n-dimensional almost product on differentiable manifold M^n of class C^∞ in which there exist a vector valued linear function F such that

(1.1)a $\overline{\overline{X}} = X$ where $\overline{X} = FX$

Let us agree to say that F gives to M^n, a differentiable manifold, in brief almost product structure.

Agreement 1.1. All the equations which follow hold for arbitrary vector field X, Υ, Z...., etc. If the given almost product structure on differentiable manifold is endowed with a hermite metric g, such that

(1.2) $g(\overline{X}, \Upsilon) = g(X, \overline{\Upsilon})$

Then we say that (F, g) gives to M^n, a Hermite structure briefly H-structure, subordinate to almost product structure on differentiable manifold. Let M^n, equipped with H-structure, a tensor f of the type (0, 2), such that

(1.2)a $f(X,\Upsilon) \overset{def}{=} g(\overline{X},\Upsilon) = g(X,\overline{\Upsilon})$

Then the following results hold:

(1.2)b $f(\overline{X},\Upsilon) = g(X,\Upsilon) = f(X,\Upsilon)$

(1.2)c $f(\overline{X},\overline{\Upsilon}) = g(X,\overline{\Upsilon}) = g(\overline{X},\Upsilon) = f(X,\Upsilon)$

Since g is symmetric, equations (1.2)a, (1.2)b and (1.2)c imply that f is skew-symmetric. If for an H-structure on differentiable manifold

(1.3)a $(D_X F)Y = 0$, $(D_X F)(\overline{\Upsilon}) = 0$, $(D_\Upsilon F)(X) = 0$, $(D_\Upsilon F)(\overline{X}) = 0$

Is satisfied, then we say that M^n is a kähler manifold on almost product structure in vector valued function F given below

(1.3)b $(D_X F)(\Upsilon) + (D_\Upsilon F)(X) = 0$

Is satisfied, then we say that M^n is an almost Tachibana manifold in the broad sense. A bilinear function ϕ is said to be pure in the two slots, if

(1.4)a $\phi(\overline{X},\overline{\Upsilon}) - \phi(X,\Upsilon) = 0$

(1.4)b $\phi(\overline{X},\overline{\Upsilon}) - \phi(\overline{X},\Upsilon) = 0$, and $\phi(\overline{X},\overline{\Upsilon}) - \phi(X,\overline{\Upsilon}) = 0$

2 Nijenhuis Tensor

Nijenhuis tensor on almost product structure equipped with respect to F is a vector valued bilinear function N given by

(2.1) $$N(X,\Upsilon) = [\overline{X},\overline{\Upsilon}] + \overline{\overline{[X,\Upsilon]}} - \overline{[X,\overline{\Upsilon}]} - \overline{[\overline{X},\Upsilon]}$$
$$= [\overline{X},\overline{\Upsilon}] + [X,\Upsilon] - \overline{[X,\overline{\Upsilon}]} - \overline{[\overline{X},\Upsilon]}$$

Where, $[X,\Upsilon] = D_X\Upsilon - D_\Upsilon X$, and D is Riemannian connexion.

(2.2)a $N(X,\Upsilon) = -N(\Upsilon,X)$

(2.2)b $N(\overline{X},\Upsilon) = N(X,\overline{\Upsilon}) = -\overline{N(X,\Upsilon)}$

(2.2)c $N(\overline{X},\overline{\Upsilon}) = N(X,\Upsilon) = -\overline{N(X,\overline{\Upsilon})} = -\overline{N(\overline{X},\Upsilon)}$

From (2.2)c it is clear that N (X , Υ) is pure in X and Υ . Also, If V^n is equipped with an almost tangent structure, than

(2.2)d $N(\overline{X},\overline{\Upsilon}) + \overline{N(X,\overline{\Upsilon})} = N(\overline{X},\Upsilon) + \overline{N(X,\Upsilon)} = 0$

Theorem 2.1. Let us put:

(2.3) $P(X,\Upsilon) \overset{def}{=} [\overline{X},\overline{\Upsilon}] - \overline{[\overline{X},\Upsilon]}$

(2.4)b $\overline{P(\overline{X},\overline{\Upsilon})} = -P(\overline{X},\overline{\Upsilon}) = (\overline{[X,\Upsilon]} - [X,\overline{\Upsilon}])$

(2.4)c $\overline{P(X,\Upsilon)} = -P(X,\overline{\Upsilon}) = (\overline{[\overline{X},\overline{\Upsilon}]} - [\overline{X},\Upsilon])$

(2.4)d $P(\overline{X},\overline{\Upsilon}) = -\overline{P(\overline{X},\Upsilon)} = [X,\Upsilon] - \overline{[X,\overline{\Upsilon}]}$

Consequently

(2.5)a $\overline{P(X,\overline{\Upsilon})} + \overline{P(\overline{X},\Upsilon)} = -N(\overline{X},\overline{\Upsilon}) = -N(X,\Upsilon)$

(2.5)b $P(\overline{X},\overline{\Upsilon}) + P(X,\Upsilon) = -\overline{N(\overline{X},\Upsilon)} = N(X,\Upsilon)$

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(2.5)c $\overline{P(X,\Upsilon)} + \overline{P(\bar{X},\bar{\Upsilon})} = -N(X,\bar{\Upsilon})$

(2.5)d $P(\bar{X},\Upsilon) + P(X,\bar{\Upsilon}) = -\overline{N(\bar{X},\bar{\Upsilon})}$

Proof. Barring (2.3) throughout or different vectors in it and using (1.1)a we get (2.4)a - (2.4)d. Again using (1.1)a, (2.2) and (2.4) in the following equations :

(2.6)a $N(X,\Upsilon) = [\bar{X},\bar{\Upsilon}] + [X,\Upsilon] - \overline{[\bar{X},\Upsilon]} - \overline{[X,\bar{\Upsilon}]}$

(2.6)b $\overline{N(X,\Upsilon)} = \overline{[\bar{X},\bar{\Upsilon}]} + \overline{[X,\Upsilon]} - [\bar{X},\Upsilon] - [X,\bar{\Upsilon}]$

(2.6)c $N(\bar{X},\Upsilon) = [X,\bar{\Upsilon}] + [\bar{X},\Upsilon] - \overline{[\bar{X},\bar{\Upsilon}]} - \overline{[X,\Upsilon]}$

(2.6)d $\overline{N(\bar{X},\Upsilon)} = \overline{[X,\bar{\Upsilon}]} + \overline{[\bar{X},\Upsilon]} - [\bar{X},\bar{\Upsilon}] - [X,\Upsilon]$

(2.6)e $N(X,\bar{\Upsilon}) = [\bar{X},\Upsilon] + [X,\bar{\Upsilon}] - \overline{[X,\Upsilon]} - \overline{[\bar{X},\bar{\Upsilon}]}$

(2.6)f $\overline{N(X,\bar{\Upsilon})} = \overline{[\bar{X},\Upsilon]} + \overline{[X,\bar{\Upsilon}]}$

(2.6)g $N(\bar{X},\bar{\Upsilon}) = [X,\Upsilon] + [\bar{X},\bar{\Upsilon}] - \overline{[\bar{X},\Upsilon]} - \overline{[X,\bar{\Upsilon}]}$

(2.6)h $\overline{N(\bar{X},\bar{\Upsilon})} = \overline{[X,\Upsilon]} + \overline{[\bar{X},\bar{\Upsilon}]} - [\bar{X},\Upsilon] - [X,\bar{\Upsilon}]$

We get (2.5)a – (2.5)d

Note 2.1. Some more relations of the type (2.4)a, b, c, d and (2.5) a, b, c, d.

Remark 2.1. If almost product structure on manifold M^n is equipped With an almost tangent structure then from (1.4)b and (2.5)b, it follows that P(X, Υ) is hybrid in X and Υ .

Theorem 2.2. The proof of these equations follows the pattern of the proof of the theorem 2.1.

(2.9)a $P(X,\bar{\Upsilon}) = Q(\bar{X},\Upsilon) = -\overline{P(X,\Upsilon)}$ or $P(\bar{X},\Upsilon) = -\overline{Q[X,\Upsilon]} = -Q(X,\bar{\Upsilon})$

(2.9)b $\overline{P(X,\bar{\Upsilon})} = -\overline{Q(\bar{X},\Upsilon)} = P(X,\Upsilon)$ or $P(\bar{X},\bar{\Upsilon}) = Q(X,\Upsilon)$

Proof. The statement follows of these equations follows from (2.4), (2.7)a-(2.7)d and (1.1)a

Corollary 2.2. We have in GF-Structure

(2.10)a $N(X,\Upsilon) = P(X,\Upsilon) + Q(X,\Upsilon)$

(2.10)b $N(\bar{X},\Upsilon) = P(X,\bar{\Upsilon}) - \overline{Q(X,\Upsilon)}$

(2.10)c $N(\bar{X},\bar{\Upsilon}) = -\overline{P(X,\bar{\Upsilon})} + Q(X,\Upsilon)$

(2.10)d $\overline{N(\bar{X},\bar{\Upsilon})} = -P(X,\bar{\Upsilon}) - Q(X,\bar{\Upsilon})$

Proof. Equation (2.10)a is the consequence of the equations (2.1)a, (2.3) and (2.7). The relation (2.10)b is obtained by (2.8)b and (2.9)a. We get (2.10)c by using (2.7)c and (2.9)c in (2.8)a. Barring (2.10)c throughout, using (1.1)a and (2.7)a, we get (2.10)d.

Remark 2.2. If almost product structure on manifold M^n is equipped with an almost tangent structure then due to (1.4)b equation (2.8)a shows that Q (X , Υ) is hybrid in X and Υ .

Theorem 2.3. If we put

(2.11) $R(X,\Upsilon) \overset{def}{=} [X,\Upsilon] - \overline{[\bar{X},\Upsilon]}$ Then

(2.12) $\overline{R[\bar{X},\Upsilon]} - R(X,\bar{\Upsilon}) = [\bar{X},\bar{\Upsilon}] - [X,\bar{\Upsilon}]$ Consequently

(2.13) $R(X,\bar{\Upsilon}) - \overline{R(X,\Upsilon)} = N(\bar{X},\Upsilon)$

Proof. Barring X, Υ in (2.11) and then throughout there is solving equation obtained and using (1.1)a, we get (2.12). Barring X in (2.1)a and using (1.1)a we get (2.12). Barring X in (2.1)a and using (1.1)a, we have

(2.14) $N(\bar{X},\Upsilon) = [X,\bar{\Upsilon}] + [\bar{X},\Upsilon] - \overline{[\bar{X},\bar{\Upsilon}]} - \overline{[X,\Upsilon]}$

$$= \left[[X,\bar{\Upsilon}] - \overline{[\bar{X},\bar{\Upsilon}]}\right] - \left[\overline{[X,\Upsilon]} - [\bar{X},\Upsilon]\right] = R(X,\bar{\Upsilon}) - \overline{R(X,\Upsilon)}$$

Which are (2.13).

Theorem 2.4. Let us put

(2.15) $S(X,\Upsilon) \overset{def}{=} \overline{[\bar{X},\Upsilon]} + \overline{[X,\bar{\Upsilon}]}$ Then

(2.16) $\overline{S(\bar{X},\Upsilon)} = \overline{S(\bar{X},\Upsilon)} = [X,\Upsilon] + [\bar{X},\bar{\Upsilon}]$ Consequently

(2.17) $\overline{S(\bar{X},\Upsilon)} - S(\bar{X},\Upsilon) = N(X,\Upsilon)$

Proof. The statement follows the pattern of the Theorem 2.3.

Note 2.3. Other relations for R(X , Υ) and S(X , Υ) can also be established as for P(X , Υ) and Q(X , Υ).

Remark 2.3. If M^n is equipped with an almost tangent structure then R(X , Υ) is also hybrid in X and Υ . For the following discussions, we suppose almost product structure on manifold M^n to be equipped with an H-structure subordinate to GF-Structure unless stated otherwise. We know that the Nijenhuis tensor for F, with a suitable connection D with respect to g is given by

(2.18) $N(X,\Upsilon) = [\bar{X},\bar{\Upsilon}] + [X,\Upsilon] - \overline{[X,\bar{\Upsilon}]} - \overline{[\bar{X},\Upsilon]}$ if we put

(2.19) $'N(X,\Upsilon,Z) \overset{def}{=} -g(N(X,\Upsilon),Z) = -g(N(\bar{X},\bar{\Upsilon}),Z)$

Then $'N(X,\Upsilon,Z)$ is skew-symmetric in X and Υ , i.e.

(2.20) $'N(X,\Upsilon,Z) = -'N(\Upsilon,X,Z) = 'N(\Upsilon,Z,X)$

(2.21) $'N(\bar{X},\bar{\Upsilon},Z) = 'N(\bar{X},\Upsilon,\bar{Z}) = 'N(X,\bar{\Upsilon},\bar{Z}) = 'N(X,\Upsilon,Z)$

Corollary 2.3. Let us define

(2.21)a $'P(X,\Upsilon,Z) \overset{def}{=} g(P(X,\Upsilon),Z)$

(2.21)b $'Q(X,\Upsilon,Z) \overset{def}{=} g(Q(X,\Upsilon),Z)$

(2.21)c $'R(X,\Upsilon,Z) \overset{def}{=} g(R(X,\Upsilon),Z)$

(2.21)d $'S(X,\Upsilon,Z) \overset{def}{=} g(S(X,\Upsilon),Z)$

Then $'N(X,\Upsilon,Z)$ can be put in the form

(2.22)a $'N(X,\Upsilon,Z) = 'P(X,\Upsilon,Z) + 'P(\bar{X},\bar{\Upsilon},Z)$

(2.22)b $'N(X,\Upsilon,Z) = 'Q(X,\Upsilon,Z) + 'Q(\bar{X},\bar{\Upsilon},Z)$

(2.22)c $'N(X,\Upsilon,Z) = 'R(X,\Upsilon,Z) + 'R(\bar{X},\bar{\Upsilon},Z)$

(2.22)d $'N(X,\Upsilon,Z) = 'S(X,\Upsilon,Z) + 'S(\bar{X},\bar{\Upsilon},Z)$

Proof. The equation (2.22)a follows from (2.2)c, (2.5)b and (2.21)a by using (2.2)c, (2.8)a and (2.21)a we get (2.22)b. The remaining two can be proved similarly

3 Modified Nijenhuis Tensor

Now we studies the modified Nijenhuis tensor on the differentiable manifold equipped with two or three variables, such that

(3.1) $S(\bar{X},\bar{\Upsilon}) = S(X,\Upsilon) = \overline{[X,\bar{\Upsilon}]} + \overline{[\bar{X},\Upsilon]}$ Then

(3.2) $'S(\bar{X},\bar{\Upsilon},Z) = 'S(X,\Upsilon,Z)$

Consequently $'S(X,\Upsilon,Z)$ is pure in X and Υ.

Proof. using (2.21)d in (3.1) we have (3.2). The equation (3.2) together with (1.4)a implies that $'S(X,\Upsilon,Z)$ is pure in X, Υ.

Note 3.1. If the given H-structure is subordinate to an almost tangent structure subordinate then equations (2.22)a – (2.22)d imply that

(3.3) $'P(\bar{X},\bar{\Upsilon},Z) = 'Q(\bar{X},\bar{\Upsilon},Z) = 'R(\bar{X},\bar{\Upsilon},Z) = 'S(\bar{X},\bar{\Upsilon},Z) = 0$

Theorem 2.5. The necessary and sufficient condition for almost product structure on differentiable manifold with an H-structure subordinate to the almost product structure subordinate to be a kahler manifold is

(3.4)a $D_X \Upsilon = \overline{D_X \bar{\Upsilon}}$ equivalently

(3.4)b $D_{\bar{X}} \Upsilon = \overline{D_{\bar{X}} \bar{\Upsilon}}$ c. $\overline{D_{\bar{X}} \Upsilon} = D_{\bar{X}} \bar{\Upsilon}$ d. $\overline{D_X \Upsilon} = D_X \bar{\Upsilon}$

Proof. we know that

(3.5)a $(D_X F)(\Upsilon) + F(D_X \Upsilon) = D_X \bar{\Upsilon}$

(3.5)b $(D_X F)(\Upsilon) + \overline{D_X \Upsilon} = D_X \bar{\Upsilon}$

Substituting from (1.3)a, we have

(3.6) $\overline{D_X \Upsilon} = D_X \bar{\Upsilon}$

Barring and using (1.1)a in this equation, we get (3.4)a. Barring X in (3.4)c we obtain (3.4)b. The equation (3.4)c follows from barring (3.4)b and using (1.1)a and (3.4)d can be had by barring X in (3.4)c after using (1.1)a.

Corollary 2.5. For a kahler manifold, we have

(3.7)a $[X,\Upsilon] = \overline{[X,\bar{\Upsilon}]}$

(3.7)b $[\bar{X},\Upsilon] = [\bar{X},\bar{\Upsilon}]$

(3.7)c $[\bar{X},\bar{\Upsilon}] = \overline{[\bar{X},\Upsilon]}$

(3.7)d $[X,\bar{\Upsilon}] = \overline{[X,\Upsilon]}$

Proof. Interchanging X and Υ in (3.4)a and subtracting the resulting equation obtained from (3.4)a, we get (3.7)a. Barring X in (3.7)a, we get (3.7)b. By barring (3.7)b throughout and using (1.1)a, we obtained (3.7)c and (3.7)d follows from barring X in (3.7)c and using (1.1)a.

Remark 2.5. Since for a Kahler manifold, Nijenhuis tensor vanishes, we have

(3.8)a $P(X,\Upsilon) = -P(\bar{X},\bar{\Upsilon})$ or $Q(X,\Upsilon) = -Q(\bar{X},\bar{\Upsilon})$

(3.8)b $R(X,\Upsilon) = -R(\bar{X},\bar{\Upsilon})$ or $R(X,\Upsilon) = \overline{S(X,\bar{\Upsilon})}$

Theorem 2.6. The necessary and sufficient condition for almost product structure on differentiable manifold with an H-structure subordinate to the almost product structure to be an almost techibana manifold is

(3.9)a $D_X \Upsilon + D_\Upsilon X = \overline{D_X \bar{\Upsilon}} + \overline{D_\Upsilon \bar{X}}$ Equivalent to

(3.9)b $D_{\bar{X}} \Upsilon + \overline{D_\Upsilon X} = \overline{D_{\bar{X}} \bar{\Upsilon}} + \overline{D_\Upsilon X}$

(3.9)c $\overline{D_{\bar{X}} \bar{\Upsilon}} + \overline{D_{\bar{\Upsilon}} \bar{X}} = D_{\bar{X}} \Upsilon + D_{\bar{\Upsilon}} X$

(3.9)d $D_{\bar{X}} \bar{\Upsilon} + D_{\bar{\Upsilon}} \bar{X} = \overline{D_{\bar{X}} \Upsilon} + \overline{D_{\bar{\Upsilon}} X}$

Proof. We have

(3.10) $(D_X F)(\Upsilon) + F(D_X \Upsilon) = D_X \bar{\Upsilon}$

Interchanging X and Υ in (3.10) and adding the resulting equation obtained in (3.10), we have

(3.11) $(D_X F)(\Upsilon) + (D_\Upsilon F)(X) + \overline{D_X \Upsilon} + \overline{D_\Upsilon X} = D_X \bar{\Upsilon} + D_\Upsilon \bar{X}$

Subtracting from (1.3)b, we get

(3.12) $\overline{D_X \Upsilon} + \overline{D_\Upsilon X} = D_X \bar{\Upsilon} + D_\Upsilon \bar{X}$

Barring throughout the above equation and using (1.1)a, we obtain (3.9)a. Other relation follow from (3.9)a by barring different vectors or throughout the equation and using (1.1)a.

Theorem 2.7. The necessary and sufficient condition that $'N(X,\Upsilon,Z)$ is completely skew-symmetric in an X, Υ, Z in an Almost Techibana manifold is

(3.13) $\overline{D_{\bar{X}} \bar{\Upsilon}} + \overline{D_{\bar{\Upsilon}} \bar{X}} = D_{\bar{X}} \Upsilon + D_{\bar{\Upsilon}} X$

Proof. We know that a necessary and sufficient condition for $'N(X,\Upsilon,Z)$ to be completely skew symmetric in an H-structure is

(3.14)a $(D_{\bar{X}} F)(\Upsilon) + (D_{\bar{\Upsilon}} F)(X) = \overline{(D_X F)(\Upsilon)} + \overline{(D_\Upsilon F)(X)}$

(3.14)b $\overline{(D_{\bar{X}} F)(\Upsilon)} + \overline{(D_{\bar{\Upsilon}} F)(X)} = (D_X F)(\Upsilon) + (D_\Upsilon F)(X)$

But when the almost product manifold is an almost Techibana, substituting from (1.3)a, we have

(3.15)a $\overline{(D_{\bar{X}} F)(\Upsilon) + (D_{\bar{\Upsilon}} F)(X)} = 0$

(3.15)b $D_{\bar{X}} \bar{\Upsilon} - \overline{D_{\bar{X}} \Upsilon} + D_{\bar{\Upsilon}} X - \overline{D_{\bar{\Upsilon}} X} = 0$

(3.15)c $\overline{D_{\bar{X}} \bar{\Upsilon}} + \overline{D_{\bar{\Upsilon}} \bar{X}} = D_{\bar{X}} \Upsilon + D_{\bar{\Upsilon}} X$

This is the required result.

4 Discussion

The main aim of the present paper is to study Nijenhuis tensor and connexion with respect to which an almost product structure given globally is parallel. Almost product structure is a backbone of the manifold also observer that we work out on the differentiable manifold of the n-dimensional with the application of the Nijenhuis tensor. We have discussed the almost product manifold are the important role of dealing with n-dimensional space by using the Nijenhuis tensor and H-structure. Almost product manifold are important role of dealing the extended of n-dimensional space heavenly body, whose shape and size are not fixed but we can take some covering area.

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