An EPQ Model For Variable Production Rate With Exponential Demand And Unsteady Deterioration

Shital S. Patel

Abstract: The focus of research is to study the nature of economic production quantity model with unsteady deterioration rate is considered. In practical situation it is discover that the starting of any production activity, the rate of production in many firm remain less up to certain time, but after some time the production rate increase with time. Due to this reason the model consider variable production rate. Demand rate is associated with exponential function depend on time. Under this situation, mathematical form is formulated to enlarge the total profit. Numerical part is provide for respective solution to support the model and sensitivity analysis shows changes in some parameters.

Key Words: EPQ model, unsteady deterioration, exponential demand, variable production rate

1. INTRODUCTION:

One common misconception in industry is that production and inventory control are separate functions. Inventory control writes the orders while production control gets them made in the plant. However, the basic truth is that inventories in a manufacturing plant are maintained to support production or are themselves the result of production. Only where inventories are purchased and then resold without requiring further work can inventory control have meaning apart from production control. Wide range of literature written on the EOQ and EPQ model with deterioration studied in past. For making best inventory policy demand play crucial role in main part of inventory structure like production, distribution of items and retailing. Researchers are busy to develop the inventory models imagine the different types of demand like deterministic and probabilistic. Deteriorating commodity are commonly characterized in duration of their existence or usefulness are correlate with time as available stock. Originally inventory models for deteriorating commodity was attention by Whitin(1957). Ghare and Schrader (1963) initially create a form with equable rate of deterioration Balkhi [2001] develop a model on finite horizon production lot size. Goyal (2003) provide solution for production inventory about time varying demand and deterioration rates. A production inventory model with price and stock dependent demand was studied by Teng and Chang(2005) developed. Production model given by Bansal (2012) was based on assumption that demand as a form of price dependent. Ghasemi (2015) developed EPQ models for rapid deteriorating items. Raafat (1991) and Ruxian (2010) present study on deteriorating items inventory models. Roy and Chaudhary (2009) develop production inventory model over stock dependent demand. Samanta and Roy (2004) established a production inventory model with deteriorating commodity and shortages. Patel and Patel (2013) analyzed production inventory model for Weibull deteriorating item on price and quantity based demand under differ holding cost. Sheikh and Patel (2017) provide a production model with desperate deterioration rates. Saha and Chakrabarti (2018) study an EPQ model for deteriorating commodity among probabilistic demand and movable production rate. Production inventory model using varying deterioration rates under inflation and delay in payments was develop by Patel (2017). This study is relate with EPQ model with unsteady deterioration rate is taken. Demand function should be consider as an exponential with time. Due to production escalate rate of variable production taken. Under this situation, a total profit of the mathematical form has been elaborated. Numerical case given to represent the model. Affectability investigation is likewise done for parameters.

2. ASSUMPTIONS AND SYLLABARY:

The subsequent series of written symbols are used to develop the mathematical form: d(t : Demand is a growing form of time i.e. exponential (ae^{bt} , a>0, 0<b<1)k_o : Manufacturing rate per unit time A: Manufacturing setup charge c : Pickup charge per unit p : Selling price per unit h : carrying charge per unit time T : Total period of inventory I(t): Inventory measure at time t Q_1: Inventory measure initially Q_2 : inventory measure at time t_2 θt: Deterioration rate P : Total profit Below belief are taken as expansion of the model. The demand consider to be exponential function based on time. Replenishment rate is extent and immediate. Initially starting time is nil. Shortages are not taken into consideration. Deteriorated units neither be renovate nor put away throughout the period of inventory. Variable production rate K is taken as;

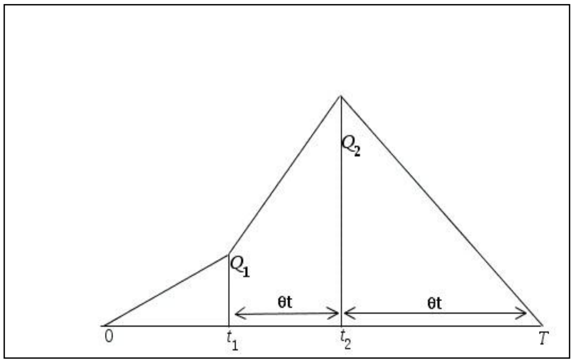
$$K = \begin{cases} k_0, & 0 \le t \le t_1 \\ k_0 e^{\mu(t-t_1)}, & t_1 \le t \le T \end{cases}$$

Where μ is a constant (0 < μ < 1) (stated by Patra and Mondal (2015). In this paper production rate assumed to be variable. Starting of any production process first the process is in slow nature but after some time production increase due to some improvement factors i.e. some efficient parameter goes to production gradually increase. Based on this concept manufacture maintain the process due to some extra cost consider as development cost i.e. α k_0. (α is a constant)

- *Shital S Patel, Department of Statistics,*
- *Veer Narmad South Gujarat University, Surat-395007, Gujarat, Imdia*
  Email: sheetal.patel003@gmail.com

3. CONCEPT OF THE MATHEMATICAL ANALYSIS:

Below diagram shows behaviour of the inventory at time 0



To T. The production rate k_0 remains fixed at $0 \le t \le t_1$, then production rate increase due to some improvement production method in interval $t_1 \le t \le t_2$. The inventory sets are expressed as differential equations. $\frac{d}{dt}I_1(t) = k_0 - d(t)$

$$0 \le t \le t_1 \qquad (1)$$

$$\frac{d}{dt}I_2(t) + \theta t I_2(t) = e^{\mu(t-t_1)}k_0 - d(t), \quad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{d}{dt}I_3(t) + \theta t I_3(t) = -d(t), \quad t_1 \le t \le T \qquad (3)$$

with initial orders I(0) = 0, I(t_1) = Q_1, I(t_2) = Q_2, I(T) = 0. Solving differential equations (1) to (3) we have, $I_1(t) = k_0 t - \frac{ae^{bt}}{b} + \frac{a}{b}$

(4)

$$\begin{aligned} I_2(t) = {} & \left(\frac{t^2}{8} + \frac{1}{2}\right)(-\mu k_0 - ab)t^2 + \frac{1}{6}\left((1+\mu t_1)k_0 - a\right)\theta t^3 \\ & + \left((1+\mu t_1)t - t_1\right)k_0 - (t - t_1)a + Q_1 + \frac{1}{2}(Q_1\theta - \mu k_0 + ab)t_1^2 \\ & - \left(\frac{1}{24}\theta\mu k_0 - \frac{1}{8}\theta ab\right)t_1^4 - \frac{1}{6}(k_0 - a)\theta t_1^3 \\ & - \frac{1}{2}\theta t^2 \left(\begin{aligned} & \frac{1}{2}(-\mu k_0 - ab)t^2 + (1+\mu t_1)k_0 t \\ & -a(t - t_1) + Q_1 - \frac{1}{2}(\mu k_0 - ab)t_1^2 - k_0 t_1 \end{aligned}\right) \qquad (5) \end{aligned}$$

$$\begin{aligned} I_3(t) = {} & -a\left(t + \frac{1}{8}b\theta t^4 + \frac{1}{6}\theta t^3 + \frac{1}{2}bt^2\right) \\ & + \frac{1}{24}aT\left(24 + 3b\theta T^3 + 4\theta T^2 + 12bT\right) \\ & - \frac{1}{2}\theta t^2\left(-a\left(t + \frac{1}{2}bt^2\right) + \frac{1}{24}aT(24 + 12bT)\right) \qquad (6) \end{aligned}$$

Putting t = t_1 in solution of differential equation (4), we obtain

$$Q_1 = k_0 t_1 - \frac{ae^{bt_1}}{b} + \frac{a}{b} \qquad (7)$$

Putting t = t_2 in solution of differential equation (5) and (6), we acquire

$$\begin{aligned} I_2(t_2) = {} & \left(\frac{t_2^2}{8} + \frac{1}{2}\right)(-\mu k_0 - ab)t_2^2 + \frac{1}{6}\left((1+\mu t_1)k_0 - a\right)\theta t_2^3 \\ & + \left((1+\mu t_1)t_2 - t_1\right)k_0 - (t_2 - t_1)a + Q_1 + \frac{1}{2}(Q_1\theta - \mu k_0 + ab)t_1^2 \\ & - \left(\frac{1}{24}\theta\mu k_0 - \frac{1}{8}\theta ab\right)t_1^4 - \frac{1}{6}(k_0 - a)\theta t_1^3 \\ & - \frac{1}{2}\theta t_2^2 \left(\begin{aligned} & \frac{1}{2}(-\mu k_0 - ab)t_2^2 + (1+\mu t_1)k_0 t_2 \\ & -a(t_2 - t_1) + Q_1 - \frac{1}{2}(\mu k_0 - ab)t_1^2 - k_0 t_1 \end{aligned}\right) \qquad (8) \end{aligned}$$

$$\begin{aligned} I_3(t_2) = {} & -a\left(t_2 + \frac{1}{8}b\theta t_2^4 + \frac{1}{6}\theta t_2^3 + \frac{1}{2}bt_2^2\right) \\ & + \frac{1}{24}aT\left(24 + 3b\theta T^3 + 4\theta T^2 + 12bT\right) \\ & - \frac{1}{2}\theta t_2^2\left(-a\left(t_2 + \frac{1}{2}bt_2^2\right) + \frac{1}{24}aT(24 + 12bT)\right) \qquad (9) \end{aligned}$$

So from result (8) and (9), we have $t_2 = \frac{aT}{k_0(1+\mu t_1)}$

(10) The answer of result (10) produced that t_2 is in form of t_1 and T, so t_2 is not a determination variable.Similarly, putting value of Q_1 in result (5), we ge

$$\begin{aligned} I_{2.1}(t) = {} & \frac{1}{8}(\mu k_0 + ab)\theta t^4 + \frac{1}{3}\left(-(1+\mu t_1)k_0 + a\right)\theta t^3 \\ & + \frac{1}{24}\left(-6t_1^2(ab - \mu k_0)\theta - 12(ab + \mu k_0)\right)t^2 \\ & + \frac{1}{24}\left(24k_0 - 24a + 24k_0\mu t_1\right)t \\ & + \frac{1}{8}\left(\left(\begin{aligned} & \left(-\frac{1}{3}\mu t_1 + \frac{8}{3}\right)k_0 \\ & + a\left(bt_1 - \frac{8}{3}\right) \end{aligned}\right)t_1\theta + 4ab - 4\mu k_0\right)t_1^2 \qquad (11) \end{aligned}$$

Putting t = t_2 in result (6), we have

$$Q_2 = \left[\begin{aligned} & -a\left(t_2 + \frac{1}{8}b\theta t_2^4 + \frac{1}{6}\theta t_2^3 + \frac{1}{2}bt_2^2\right) \\ & + \frac{1}{24}aT\left(24 + 3b\theta T^3 + 4\theta T^2 + 12bT\right) \\ & - \frac{1}{2}\theta t_2^2\left(\begin{aligned} & -a\left(t_2 + \frac{1}{2}bt_2^2\right) \\ & + \frac{1}{24}aT(24 + 12bT) \end{aligned}\right) \end{aligned}\right] \qquad (12)$$

(not consider the higher powers of θ) The EPQ model have the following cost which are the part of the total profit (P) function: (a) Set-up cost (SeC) = A

(13) b) $HC = h\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{t_2} I_{2.1}(t)dt + \int_{t_2}^{T} I_3(t)dt\right]$ (14)

(c) $DC = c\left(\int_{t_1}^{t_2} \theta t I_{2.1}(t)dt + \int_{t_2}^{T} \theta t I_3(t)dt\right)$ (15)

(d) $$SR = p\left(\int_0^T ae^{bt}dt\right) \quad (16)$$

(e) Development Cost = αk_0 (17)

The total profit (P), through a inventory cycle made up the following: $$P(T,t_2) = \frac{1}{T}[SR - SeC - HC - DC - \alpha k_0] \quad (18)$$

Fill result (13) to (17) in result (18), we get overall profit per unit in terms of t_1 and T. Putting $t_1 = v_1t_2$, (0< v < 1) and value of t_2 from result (10) and (18) we get profit in expression of T.

$$P(T) = \frac{1}{T}[SR - SeC - HC - DC - \alpha k_0] \quad (19)$$

Differentiating equation (19) *regarding* T and equate it to zero, we get optimal value of T.

$$\text{i.e.} \frac{\partial P(T)}{\partial T} = 0 \quad (20)$$

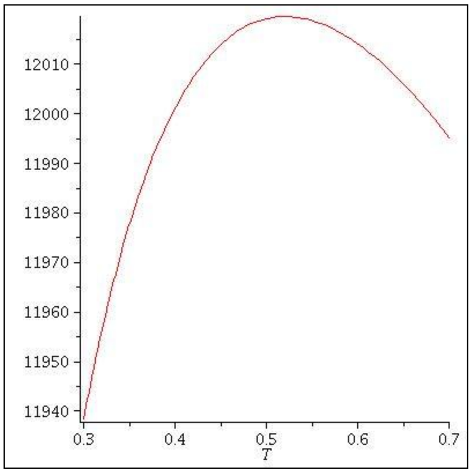
Optimal value of T satisfies the second order condition given below. $$\frac{\partial^2 P(T)}{\partial T^2} \geq 0 \quad (21)$$

4. NUMERICAL EXAMPLE:

Examine model for the below given values in appropriate units, we present a numerical example to support the model.

a	b	v	B	c	h
250	0.01	0.10	80	30	7.5
α	p	μ	k_0	θ	α
0.10	50	0.04	500	0.07	0.10

Optimal cycle length $T^* = 0.5202$ and Profit(P^*) =Rs. 12019.6541.The concavity of the profit function is shown graphically.



5. PARAMETRIC ANALYSIS:

Now we observe the achievement of the parameters on profit function. Moving included parameter by ±20% and ±10% hold one parameter at once and keep rest of parameters are same.

Table
Parametric Analysis

Parameter	%change	T	Profit(P)
a	+20	0.5356	14535.6204
	+10	0.5249	13274.9202
	-10	0.5209	10769.6376
	-20	0.5171	9524.8663
θ	+20	0.5133	12016.0823
	+10	0.5164	12017.8569
	-10	0.5238	12021.4755
	-20	0.5275	12023.3224
h	+20	0.478	11972.7117
	+10	0.4978	11995.6842
	-10	0.5456	12044.7535
	-20	0.5748	12071.1445
B	+20	0.5488	11989.7242
	+10	0.5347	12004.4885
	-10	0.5051	12035.2586
	-20	0.4894	12051.3459
p	+20	0.5265	14526.1964
	+10	0.5233	13272.9152
	-10	0.5171	10766.4125
	-20	0.5140	9513.1902
k_0	+20	0.4986	11958.3188
	+10	0.5068	11986.9377
	-10	0.5421	12058.2707
	-20	0.5803	12106.0258

From the conclusion of the above parametric analysis, the following consideration can be made. For the parameter a and p when we change decrease to increase the profit will increase. For other parameters θ, h, B and k_0 are less sensitive as increase to decrease the profit will decrease.

6. CONCLUSION:

In this study of EPQ model with unsteady deterioration rate is examine. The model justly taken as demand as exponential function of time. Due to some circumstance during production process, the rate of production is less up to certain time, but after some time the production rate increase with time. Due to this reason the model consider variable production rate. Numerical part proves the applicability of the derive model and parametric analysis shows the effect in some parameter. The derived model can expand for items having different types of demand and deterioration.

REFERENCES

[1] Balkhi, Z.T. (2001): On a finite horizon production lot size inventory model for deteriorating items: An optimal solution; Euro. J. Oper. Res., Vol. 132, pp. 210-223.
[2] Bansal, K.K. (2012): Production inventory model with price dependent demand and deterioration; International J. of Soft Computing and Engineering, Vol. 2, pp. 447-451.
[3] Ghare, P. N. and Schrader, G. F. (1963): A model for exponentially decaying inventory items; J. Indus. Engg., Vol. 14, pp. 238-243.
[4] Ghasemi, N. (2015): Developing EPQ models for non- instantaneous deteriorating items; J. Ind. Eng. Int., Vol. 11, pp. 427-437.
[5] Goyal, S.K. and Giri, B.C. (2003): The production inventory problem of a product with time varying

demand, production and deterioration rates; Euro. J. Oper. Res., Vol. 147, pp. 549-557.

[6] Patel Raman and Shital Patel(2013); Production inventory model for weibull deteriorating Items with price and quantity dependent demand with time varying holding cost; International Journal of Pure and Applied Mathematical Sciences, Vol. 6(5), pp. 425-435.

[7] Patel, S. S. (2017): Production inventory model for deteriorating items with different deterioration rates under stock and price dependent demand and shortages under inflation and permissible delay in payments; Global Journal of Pure and Applied Mathematics, Vol 13 No. 7, pp 3687-3701.

[8] Patra K, Mondal S. K.(2015); Risk analysis in a production inventory model with fuzzy demand, variable production rate and production time dependent selling price. OPSEARCH, Vol. 52, 505-529.

[9] Raafat, F. (1991): Survey of literature on continuous deteriorating inventory model, J. of O.R. Soc., Vol. 42, pp. 27-37.

[10] Roy, T. and Chaudhary, K.S. (2009): A production inventory model under stock dependent demand, Weibull distribution deterioration and shortages; International Transactions in Operations Research, Vol. 16, pp. 325-346.

[11] Ruxian, L., Hongjie, L. and Mawhinney, J.R. (2010): A review on deteriorating inventory study; J. Service Sci. and management; Vol. 3, pp. 117-129

[12] Saha and Chakrabarti (2018): An EPQ model for deteriorating item with probabilistic demand and variable production rate; American Journal of Engineering Research, Vol. 7, pp. 153-159.

[13] Samanta, G.P. and Roy, A. (2004): A production inventory model with deteriorating items and shortages; Yugoslav J. of Operations Research, Vol. 14, pp. 219-230.

[14] Sheikh, S. R. and Raman Patel (2017): Deteriorating items production inventory model with different deterioration rates under stock and price dependent demand; International Journal of Statistics and Systems, Vol. 12, No.3, pp. 607-618.

[15] Teng, J.T. and Chang, H.T. (2005): Economic production quantity model for deteriorating items with price and stock dependent demand; Computers and Oper. Res., Vol. 32, pp. 279-308.

[16] Whitin, T. M. (1957): Theory of inventory management, Princeton Univ. Press, Princeton, NJ.