Anticipating Failure Of Students' Productive Connective Thinking Transformation In Solving Mathematical Problems

Nurfaida Tasni, Toto Nusantara, Erry Hidayanto, Sisworo

Abstract: This case study investigates how students do strategy transformation in connective thinking through a process of reflection. This paper aims to anticipate students' failures in connective thinking when solving mathematical problems. Strategy transformation has a positive effect on student success in solving problems, especially mathematical problems. The results of the study show 5 transformation strategies that must be done by students in anticipating failure in productive connective thinking, namely 1) students must be able to develop their ideas to develop more mature planning in order to find several resolution strategies. 2) Students must identify errors made in the calculation process through the data verification process. 3) Students must have a good understanding of the basic concepts in constructing problem solutions. 4) students must carry out a systematic and comprehensive verification of the problem solving process. 5) students must have positive motivation and confidence in themselves to develop connection ideas based on their reflection and experience in solving problems. Therefore, the findings in this study can be used as the basis for each student to anticipate failure in solving mathematical problems that ultimately make students able to achieve success in the learning process.

Index Terms: Process of Thinking, mathematical connection, connective thinking, productive connective, Thinking Strategy, Reflection, Transformation, Problem Solving.

1 INTRODUCTION

Mathematical connections are needed in the problem solving process as an effort to find solutions based on the knowledge they have (Olving, Etacognition, & Schoenfeld, 1992). This indicates that a person's experience in solving problems is certainly not separated from the connection of mathematical ideas. Anthony & Walshaw, (2009) state that through connection of mathematical ideas, one can develop his understanding of concepts or procedures that are interrelated to be used in solving problems. Therefore, it is very necessary for students' ability to associate mathematical ideas when solving mathematical problems. The cognitive process to make a connection between mathematical ideas in solving mathematical problems is a process of connective thinking (Elly Susanti, 2018). Students who cannot build new ideas by processing information linkages with their initial knowledge have a tendency to simple connective thinking. Whereas students who are able to build new ideas based on the relevance of information with their initial knowledge are able to develop appropriate strategies until they find the right solution to the problem so that they have the tendency to think productively. The more connection ideas that can be constructed by students, the more productive their connectivity thinking abilities (Elly Susanti, 2018). Singh, Yager, Yutakom, Yager, & Ali, (2012) explain that constructivist learning theory emphasizes the process (1) constructs meaningful knowledge, (2) links between ideas and information provided, (3) links between ideas in construct knowledge as a whole. When students actively fulfill the three criteria, students will be able to independently construct productive connective thinking networks that can be interpreted in the problem solving process. Holyoak, K.J and Morisson, (2012) explain that the connection between ideas can develop through the relationship between concrete experience, understanding of language, and images and mathematical symbols that students have. Therefore, students who still have a simple connective thinking tendency can be trained to develop their connective thinking skills in the process of reflection.

Several studies have shown the importance of constructing mathematical ideas in solving mathematical problems (Tray, Çatlioğlu, Coştu, & Birgin, 2009; De Corte, Verschaffel, & Greer, 2000; Eli, Mohr-schroeder, & Lee, 2011; Hendriana, Slamet , & Sumarmo, 2014; L. Suominen, 2015; Mhlolo, Venkat, & Schfer, 2012; Slaney & Garcia, 2015; Stylianou, 2013; Susanti, 2015; Tasni & Susanti, 2016). Cognitive processes in relating mathematical ideas or connective thinking can be achieved maximally by applying appropriate thinking strategies. Tasni, (2017) has conducted a study that shows the existence of barriers to productive connective thinking of students in solving mathematical problems. This is due to the inability of students to build conceptual connections at the stage of cognition, incompleteness in formulating logical data in the stage of inference, inability to develop settlement strategies based on connections are built, and there is no connection of justification and representation at the reconstruction stage. Based on previous research findings, it is necessary to hold more in-depth observations about changes in thinking strategies in this case the strategy transformation used by students to anticipate students' failures in productive connective thinking. This is important, because productive connective thinking is a form of realization of the use of mathematical connections as an effort to find solutions to the appropriate problems. So that in the end it can train students at every level of education to become reliable problem solvers so they can contribute positively to improving students' thinking skills in achieving successful learning. As explained by (Meissner, Velikova, & Matsko, 2015) that realizing the relevance of concepts in mathematics through the application of mathematical connections in solving mathematical problems can help to absorb learning objectives. Suominen, (2015) describes mathematical connections as mathematical activity, this means that connections exist in every part of mathematics and that to succeed in mathematics learning

- *Nurfaida Tasni, graduate student at Malang State University, institution from STKIP YPUP Makassar, Indonesia, 085298921849, Email: nurfaida.tasni1503119@students.um.ac.id*
- *Prof. Dr. Toto Nusantara, M. Si, Postgraduate Lecturer at Malang State University, Indonesia, 081233694201, toto.nusantara.fmipa@um.ac.id*
- *Dr. Erry Hidayanto, M.Si, Postgraduate Lecturer at Malang State University, Indonesia, 085331305127, Email: erryhidayanto.fmipa@um.ac.id*
- *Dr. Sisworo, M.Si, Postgraduate Lecturer at Malang State University, Indonesia, 08113502699, Email: sisworo.fmipa@um.ac.id*

each student must be involved in constructing mathematical ideas. However, there has not been a study that has specifically examined the changes in students' thinking strategies as an effort to maximize their thinking ability in a more productive direction after the reflection process. Therefore, this paper will provide a detailed description of the transformation of students' thinking connectivity strategies in solving mathematical problems. The purpose of this study is to provide an overview of the transformation of thinking strategy as an effort by students to anticipate failure in productive connective thinking after being given the opportunity to reflect. The description of the transformation of thinking strategies undertaken by students is explained below.

2 MATERIALS AND METHODS

Transformation comes from the Latin language transformer which means changing form or metamorphosis. Montenegro, et al. (2018) define transformation as a transfer from one system to another. Duval (2006) identified two types of representation transformations, namely treatment and conversion. Treatment occurs in one representation system while conversion is a transformation of representation in one system into representation on another system. According to Martino (2007) the potential for transformation of thinking strategies occurs if someone tries to develop his thoughts continuously and practice catching up. This view is in line with the opinion of Dumestre (2016) which explains that transformation is an innovation that is used as a way of implementing change.

Dumestre (2016) explains that thinking strategies for transformation are strategic thinking for change through the Generalized Empirical Method (GEM), Strategic Planning Process (SPP), Strategic Thinking Actions (STA), and Desired Results (DREST) within a certain time frame. Dumestre (2016) developed the theory of thinking strategy for transformation based on Bernard Lonargen's cognitive theory called the empirical generalization method (GEM), which is a theory that introduces a set of teachings to explain the development of knowledge through dynamic processes involving experience, understanding, judgment, and action (Hall et al., 2017). The method of empirical generalization (GEM) provides a framework that can be applied in various disciplines including Science (Marroum, 2004). Lonargen explained that the method of empirical generalization (GEM) offers a way to reconcile constructivist learning theories, this is explained in the lonargen Theory of Cognition, Constructivism and Science Edcation (Roscoe, 2004).

Reflection acts as a tool that encourages thinkers during problem solving situations, because it provides an opportunity to step back and think about the best strategy to achieve goals (Yen et al., 2016). Reflection is needed in every problem solving process. In this study, in order to observe the transformation of strategies in connective thinking in an effort to anticipate students' failures in productive connective thinking, a change in the strategy applied was observed in building connection ideas in each construction hole after reflection. (Leung & Kember, 2010) explains that reflection involves active, persistent and careful consideration of any assumptions or beliefs based on one's awareness. Reflection is done, namely giving relevant questions in the form of Solution Tests Connection problems with additional information as a form of stimulation for students to open insights in transforming their thinking strategies to build connective thinking network schemes that can be interpreted in solving problems.

In this study observed how to anticipate student failure in productive connective thinking. Therefore this research is a qualitative research that provides detailed and systematic descriptions according to facts and phenomena about thinking strategies in anticipating students' failures in productive connective thinking. Subjects are high school students who have studied flat-build material, number patterns, arithmetical sequences, and algebraic functions. Observations on students' productive thinking strategies for solving mathematical problems using the interrelationships of concepts and procedures in the material are very important to observe. This is because each student must be able to maximize his or her thinking ability by using appropriate thinking strategies to become a reliable problem solver so as to gain success in the learning process.

The data used was obtained from the results of observations of the problem solving process that was carried out by students before and after reflection with aloud thinking. Students who observed the thought process were taken from 85 students who had been given initial tests to find out their understanding of connection problem solving material. Next, students who experience productive connective thinking fail to be selected for their thought processes to solve connection problems. In order to explore the thinking process of students conducted an interview process based on the results of think aloud before and after reflection. This activity is carried out to obtain in-depth and detailed information about observed phenomena (Creswell, 2012). The results of video recordings and subsequent interviews are presented in the form of transcripts and then coded for each student's thinking process to describe the structure of thinking. Furthermore, data analysis was performed using the Toshio scheme (Jaijan, 2008) which consisted of stages of cognition, inference, formulation and reconstruction. Therefore an overview of students' thinking strategies is obtained to anticipate failure in productive connective thinking, namely through transformation of thinking strategies in solving mathematical problems.

3. RESULTS

Failure to transform strategies in connective thinking occurs because the subject does not carry out an adequate transformation strategy after reflection. This condition occurs because the connection ideas that have been built have not been able to complete each construction hole. Therefore new ideas have not yet emerged that can form a connective thinking network scheme to be interpreted in the problem solving process.

3.1 Exposure to the Structure of Student Thinking Before Reflection

In this section describes the structure of student thinking before reflection taken from the fulfillment of thinking strategies for the occurrence of transformation in Toshio's scheme (Jaijan, 2008) when completing Task Connection problem solving one. At the cognition stage, students begin the completion process by reading the questions and observing origami ornament images 1 level and 2 levels in the question information, then students begin to observe the table and immediately complete the table in the first column to fifth with data showing the number of triangles with the length of one side units, triangles with side lengths of two units, rhombus which contains two unit triangles, isosceles trapezium containing three unit triangles, and hexagon polygons which contain six unit triangles in origami ornament 1 level up to 2 levels. In this activity students have carried out a strategy of empirical generalization methods but have not been fully implemented. This condition is known based on fragments of aloud think record from the following students:

"*It is known that wall hangings made from origami paper are called origami ornaments which are composed of triangles with side lengths of one unit affixed to the wall of the room as shown ... origami ornament is 1 level, length of side is one unit ... origami 2 level long the sides are two units ... means I immediately complete the table for origami ornament 1 level of the number of triangles with one side length of unit1, triangle with side lengths of two units 0, rhombus which contains two triangles unit 0, trapezium isosceles containing three satan triangles 0, and hexagon polygons that contain six unit triangles 0. Then the two-level origami ornament ...* "

Based on a fragment of the record, it is known that students do not apply the Full Method empirical generalization method in building connection ideas at the cognition stage. This can be observed from the initial steps students take to complete the table for each part of the question and do not check and evaluate the interpretation of the data collected.

In the inference stage, students begin to redraw origami ornament 1 level to 4 levels, because previously students have first completed the table for the number of each flat building that can be loaded on origami ornament 1 level up to 2 levels. Students immediately observe the 3-level origami ornament images they draw and begin to compile a number sequence based on their interpretation of the pattern of each flat build formed by origami ornament 1 level up to 3 levels. However, students begin to make mistakes to determine the number of trapezium flat legs that can be formed on three levels of origami ornament. Students in the beginning found that there were 5 flat trapezium shapes which could be formed on 3-level origami ornament, then changed their minds and looked again and then wrote that there were 8 trapezoidal shapes that could be formed on 3-level origami ornaments. However, students' interpretations remain wrong because basically there are still 2 trapezium patterns that are not identified. These activities can be seen from the fragments of interview transcripts conducted by the following students:

R : *How do you find the number of flat wake trapezium formed on origami ornament 3 levels?*

S : *Earlier I observed pattern can be formed trapezium number five mom, but I count back turned out to be eight mom*

R : *Are you sure you have found all the trapezium patterns formed?*

S : *Hmmm ... looks like yes ma'am.*

Based on the fragments of the interview transcript, it is known that students are sure of the correctness of the data collected, this shows that students' understanding of the characteristics of the trapezium pattern that is desired in the problem is not sufficient. Because students do not identify all the trapezium patterns that should have formed.

At the formulation stage. students try to do calculations and complete tables for the number of each waking up flat on origami ornament 4 levels up to 10 levels. Students begin by drawing origami ornament 4 levels up to 10 levels, then on the same origami ornament image, Students directly determine the number of flat triangles with side lengths of one unit, two units, rhombus, trapezium and hexagon polygons in the origami ornament 4 levels up to 10 levels. These activities can be observed from the results of student work presented in Figure 1 below:

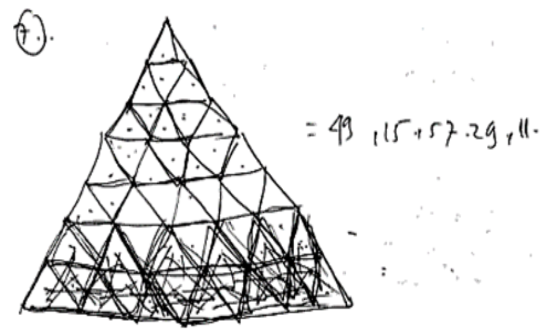

Fig 1. Student Work Results Before Reflection at the Formulation Stage for Each Flat Build.

Based on student work presented in Figure 1, it is known that students directly determine the number of flat triangles with unit side lengths, triangles with two sides, rhombus, foot trapezium and hexagon polygon in 7-level origami ornament. The absence of a mature strategy built by students causes it unable to obtain data that is appropriate for the number of flat builds contained in origami ornament 4 levels up to 7 levels.

In the reconstruction phase. After determining the number of flat shapes contained in origami ornament 1 level up to 10 levels, students try to determine the formula for each flat building that shows a flat wake formed on the origami ornament n level. But because students collect data that is not appropriate, it is difficult for them to determine the formula for the term ke-n of each flat wake. These activities can be observed from the results of student work presented in Figure 2 below:

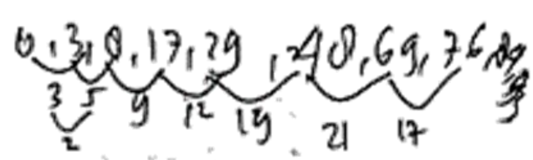

Fig 2. Student Work Results Before Reflection at the Reconstruction Stage to Build Flat-Footed Trapezoidal Trails.

Based on the work of the students presented in Figure 2, it is known that students compose a number sequence to determine the pattern of numbers formed, but students do not succeed in finding a suitable pattern to determine the formula of the flat trapezium leg build up containing three unit triangles in origami ornament n level .

3.2 Exposure to the Structure of Student Thinking After Reflection

In this section describes the structure of student thinking after the reflection taken from the fulfillment of thinking strategies for the occurrence of transformation in Toshio's scheme (2010) when completing the problem solving task connection two At the cognition stage, After reflection the students begin the same procedure when completing TPMK I, which is reading the question information and immediately completing the table. Students are aware of additional information about the problem. Based on the information about the questions, the students again pay attention to their initial interpretation of the pattern of each flat wake. After coordinating between the data in the table with the origami ornament images, the subject was finally able to identify the characteristics of each flat wake. These conditions can be observed from the fragments of Aloud II results from the following students:

"*Based on the information about the question, it is known that the origami ornament image is 1 level up to 3 levels, then in the table we know the number of triangles with one unit side length, triangle with two sides, rhombus which contains two unit triangles, isosceles trapezium which contains three unit triangle and hexagon polygon which contains six unit triangles in origami ornament 1 level up to 3 levels, ... hmmm means I just need to continue drawing origami ornament 4 levels to 10 levels then determine the number of flat shapes formed* ".

Based on the recording fragment of the second aloud think, it is known that students can build connection ideas so that they are able to identify the characteristics of each flat building that can be formed on origami ornament 1 level up to 3 levels.

In the inference stage, After reflection students begin drawing 4 levels of origami ornament to determine the number of each flat building formed on 4 levels of origami ornament. These activities can be observed from the results of student work presented in Figure 3 below:

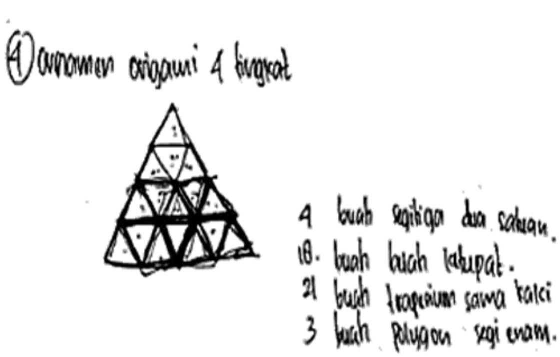

Fig 3. Student Work Results After Reflection at Stage Inference for Each Flat Build.

Based on the work results of the students presented in Figure 3 it is known that students still use the same strategy when completing the connection problem solving task before reflection. Students have not focused on collecting data for a flat pattern that is formed. This is also confirmed by the transcript of the interview conducted with the following students:

R : *What strategies do you use to design the solution for the two connection problem solving tasks?*
S : *Here I will draw the mom ...*
R : *Why did you decide to keep using the pictures?*
S : *Hmmm ... because I think this method is the easiest mom*
R : *What about the question information in the table, can you get a picture of the solution to this problem after observing the question information provided?*
S : *Hmmm… .that is mom*
R : *What do you mean by a solution plan?*
S : *Here for one unit triangle I can observe the pattern of numbers directly bu ... and I can get if the formula is the square of 1, 2 and 3*
R : *Then what about the other four flat builds?*
S : *To get up to the other flat, I will draw another origami ornament and determine the number of flat triangles, rhombus, trapezium and hexagon polymers contained in origami ornament images 4 levels up to 10 levels.*

Based on the fragments of interview transcripts conducted with the students above, it is known that students did not transform the strategy adequately in designing problem solving even though they were aware of additional information about the problem. Students can only find patterns for flat triangles with one side length, but it is difficult to identify drawing patterns and number patterns for the other four flat shapes. This happens because students still do not focus on collecting data for every waking up flat.

In the formulation stage, students make calculations on the number of flat wake patterns that are formed on each origami ornament based on the origami ornament images that they draw. Activities from students can be observed from the results of their work presented in Figure 4 below:

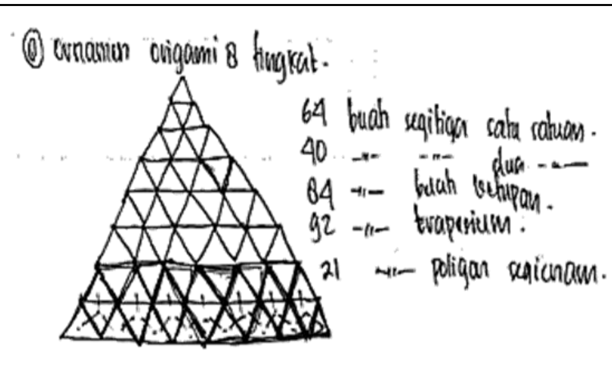

Fig 4. Student Work Results After Reflection at the Formulation Stage for Each Flat Build.

Based on the results of the work presented in Figure 4, it is known that students make mistakes in calculating a flat wake pattern formed on origami ornament 8 levels to construct a flat triangle with sides of two units, rhombus and trapezium.

During the reconstruction phase, students do not succeed in finding the nth term formula to construct a triangular flat with two unit sides, rhombus, isosceles trapezium and hexagon polygons. By writing data that is not appropriate for the number of each flat build in the table, students have difficulty finding the formula for the number of each flat build that can be formed on the origami ornament n level.

4. DISCUSSIONS

When completing the task of solving one connection problem before reflection, students are only able to apply empirical generalization methods. However, the concept of construction that is built is not sufficient to form a network of connective thinking that can be interpreted to design problem solving strategies. After being given the opportunity to reflect on students are not able to transform the thinking strategy and cannot build appropriate connection ideas for each construction hole. Therefore students do not experience adequate transformation and remain unable to find logical and reasonable information at the stage of cognition. Students are unable to design problem solving strategies through focused data collection and coherent data interpretation. Students are not able to verify data to make the appropriate calculations at the formulation stage. In addition, students also do not carry out a comprehensive evaluation of the effectiveness of the strategies used in the reconstruction phase so that they cannot find the nth term formula of any flat building that can be generalized to the other domains.

The description of the failure of strategy transformation in the students' thinking connective identified from the absence of an adequate transformation of strategies to build connection ideas in each construction hole after reflection is as follows:

Stage of Cognition Before Reflection

Students use empirical generalization methods to understand problem situations. However, Students only do one of the four components in the strategy. that is, attention to experience by connecting information with the initial knowledge possessed. While for the other three components, which are intelligent in understanding something, logical in making judgments, and evaluation of generalizations decided based on observations have not been applied by the subject. This condition is known because students do not observe and pay attention to each data obtained to be adjusted again with each part of the question in the matter of constructing a flat triangle, rhombus, trapezium and hexagon polygon in the origami ornament image. Therefore, students are only able to build simple initial connections that have not been able to form connective thinking networks to be interpreted in the process of drafting a problem solving plan.

Stage of Cognition After Reflection

Students do not carry out sufficient transformation strategies to construct connection ideas in each construction hole in the cognition stage. Even though students are aware of the additional information, students do not re-coordinate the data in the table with each flat wake pattern formed by origami ornament 1 level up to 3 levels. Therefore students still build simple connection ideas so that they have not been able to build an understanding to identify each flat pattern that can be formed on origami ornament 1 level up to 3 levels based on question information.

Stage Inference Before Reflection

In the inference stage students use the strategic planning process strategy to find the appropriate information that will be used in planning problem solving. However, students do not apply every component in this strategy adequately. Students only do one of the four components in this strategy that is coherent data interpretation. Students provide appropriate data interpretation to develop a settlement strategy. Even so, he did it to construct a flat triangle with just one side length. But for a flat triangle with two sides, rhombus, hexagon and polygon hexagonal sides students still make mistakes in providing coherent data interpretation. In addition, students do not apply the first component, namely data collection focused on the strategy. This condition is known because students do not focus on completing each part of the question and immediately identify the five flat wake patterns namely one unit triangle, triangle with two sides, rhombus which contains two unit triangles, isosceles trapezium which contains three unit triangles and hexagon polygons which contains six unit triangles at each level of orgasm ornament. This causes students not to make reliable decisions, where students do not find suitable data that shows the number of flat patterns that can be formed at each level of origami ornaments.

Stage Inference After Reflection

Students do not transform strategies to build connection ideas for each construction hole so students do not find suitable information to use in planning problem solving. This condition is known from the activities of students who still do not focus on collecting data by identifying the characteristics of each flat wake pattern one by one. So that students still do not meet the first component in this strategy, namely focused data collection. Second, students still do not interpret the data in a coherent manner as a whole, that is the interpretation of data that is appropriate for developing a settlement strategy. This condition is known from the inability of students to provide interpretation of data that is appropriate for the number of flat triangles with two sides, rhombus, and trapezium with isosceles. Third, students have not made a reliable decision on the data used, students have not used the appropriate data based on the connection of mathematical ideas that arise from the question information and the initial knowledge they have. This condition is known from the completion strategy designed by students who still use the drawing method to determine the number of each flat wake on origami ornament 1 level up to 10 levels. Therefore the connection ideas that are built by the subject at the inference stage are not sufficient enough to find the final solution to the problem.

Formulation Level Before Reflection

In the formulation stage before reflection, students do not use strategic thinking actions, namely the appropriate problem solving actions based on the right thinking design in obtaining solutions to problems. Students only meet the first component of the strategy, which is to coordinate between internal and external conditions, namely coordinating between the initial knowledge possessed with the information given, but not yet applied to any desired flat build in the problem. Students match the data for the number of flat triangle shapes with the sides of one unit so that they can only find data that is appropriate for the number of flat shapes.

Students make a mistake in identifying a flat triangle pattern with two side lengths, a flat rhombus pattern that contains two unit triangles and a flat trapezium foot-shaped pattern that contains three unit triangles. Therefore students are only able to establish a simple connection in verifying every data obtained about the number of each waking up flat on origami ornament 1 level up to 10 levels.

Stage of Formulation After Reflection

After reflection, students do not carry out adequate strategy transformation to build connection ideas for each construction hole. This condition is known from the inability of students to find each flat wake pattern that is formed on origami ornament 1 level up to 10 levels drawn. In this condition students have not coordinated between the initial knowledge they have and the information about the questions given to build the appropriate network of connective thinking. Students are only able to find the right data for the number of triangular flat shapes with two-unit side lengths and hexagonal polygons. Whereas for a flat triangle with two sides, rhombus, foot trapezium and hexagon polygons students still make mistakes.

Students have not made an appropriate assessment of the choice of the next step of completion, namely students do not verify the suitability of the connective thinking network built to compile a new connective thinking network. This condition is known from the inability of students to identify mistakes made in collecting initial data obtained to develop strategies in processing data. Third, students have not developed several problem solving steps. This condition is known from the ability of students who only use one strategy to determine the number of each flat building formed on origami ornament 1 level up to 10 levels, namely using only origami ornament images. However, because previously the students did not carry out the verification process on the data which showed the number of flat triangles with two sides, rhombus and trapezoidal sides. So, it cannot complete the table with the correct data, that is data that shows the number of each flat build that can be formed on origami ornament 1 level up to 10 levels. Students have not used the appropriate completion steps to find part of the solution to the problem. This condition is known because students only use origami ornament images to complete the table of the number of flat triangles with unit side lengths, triangles with two unit lengths, rhombus, trapezium and hexagon polygons in origami ornament 1 level up to 10 levels.

Reconstruction Stage Before Reflection

Before reflection students are only able to build simple connection ideas that have not been able to form a suitable connective thinking network that can be interpreted to design problem solving strategies to find a formula that shows the number of flat triangles with one side length, two sides of a triangle , rhombus which contains two unit triangles, trapezium with legs which contains three unit triangles and hexagon polygons which contain six unit triangles in the origami ornament n level which is the final solution to the problem given.

Reconstruction Stage After Reflection

After reflection students do not transform strategies to build appropriate connection ideas for each construction hole. Students still do not apply thinking strategies to achieve the desired results. This condition is known because students do not look back and evaluate the entire problem solving process so that they are not able to detect connection errors that cause the emergence of construction holes in the connective thinking network. Although students have been able to establish connections at the reconstruction stage, they are not yet adequate because students have not been able to reconstruct the entire problem solving process until they find a suitable general formula that can be applied to determine the number of flat shapes formed on origami ornament n level. This happens because students only use image and number patterns, but because the data arranged in a row of numbers, it is data that does not match students having difficulty identifying their number patterns and not succeeding in finding a formula for each flat build.

Based on the thinking process of students at the stage of cognition, before reflection students in this group are only able to build simple connection ideas. Students have not been able to understand the problem situation. This condition is known from the inability of students to identify each pattern from a flat triangular, rhombic, isosceles trapezium and hexagon polygon. The incompleteness of connections built by students in an effort to understand the problem situation results in students being unable to collect logical data to design a problem solving strategy. As stated by Raif (2013) that establishing a connection between important visualization in the process of finding a solution to a problem. In this process the subject does not carry out initial verification of the data that has been collected. Papadopoulos and Dagdilelis (2008) describe verification as an alternative calculation method for checking the correctness of solutions. The same thing was explained by Eizenberg, et al (2015) that verification contributes to the success of problem solving. So that even though data processing is done at the formulation stage, the strategies implemented by students are not sufficient to identify errors made in collecting initial data. Therefore, he cannot implement the appropriate resolution strategy to find a solution to the problem.

After reflection, students only transform strategies using empirical generalization methods based on the information provided. Students have been able to identify the characteristics of each flat wake pattern in origami ornament 1 level up to 3 levels based on the initial data presented in the problem. Therefore the structure of thinking of students is formed based on information on the problem given. As explained by Papadopoulos and Iatridou, (2010), there are two empirical strategies, namely visual verification, namely justifying an idea based on what is observed and verification based on adaptation, namely building ideas based on the relationship between what is thought and what is observed so as to provide encouragement or action to find the correct solution. In these activities the students connect the initial understanding of each flat wake pattern with his observations of the images and data presented in the table which is a matter of information.

Furthermore, for the inference stage students have not been able to build connection ideas. Students have not focused on data collection and still use images as the only way to determine the number of each flat building that is formed on origami ornament 4 levels to 10 levels. As explained by Olving, et al (1992) that the lack of a plan evaluation and the absence of a review is a significant factor that contributes to failure in solving problems. Students are not able to arrange a more mature plan by using several resolution strategies, namely using number patterns, and algebraic processes, it only uses images. Although in the research of Wilkie (2019), there was a strong role for visualization in the tasks of figural patterns. However, because only using origami ornament images, the subject is unable to construct the appropriate formula. As explained by Williams, et al. (2011) that linking arithmetic and algebraic reasoning will give students access to understand problems and find solution paths that lead to greater success in finding solutions to problems.

Students do not check again when calculating the number of image patterns formed at each level of origami ornaments. Therefore students do not identify errors made when not taking into account some flat wake patterns that should have formed at each level of origami ornaments. With data that is not suitable students have difficulty identifying patterns of numbers formed. In this situation students do not verify. According to Papadopoulos and Dagdilelis, (2008) verification as a way of calculating alternatives to examine the correctness of solutions or as a way for students to examine the reasonableness of their choices. Students are only able to collect data that is suitable for the number of triangular flat shapes with one side length and find the bilagan pattern. However, he still made a mistake for the other four flat builds.

Montenegro, et al. (2018) explained that errors are always possible in the problem solving process so verification is needed especially if there are several fast and intuitive procedures as a way to test the suitability of the results or arguments obtained. From the interview process it is known that students can only determine the formula of the number of triangular flat shapes with one side length, because the pattern of numbers is easy to guess, but to construct a triangular flat with two unit sides, rhombus, isosceles trapezium and hexagon polygons students claim no can find the number pattern. In this situation students also showed a lack of understanding of second-level arithmetic concepts, so they were unable to present different forms of representation from the number of flat-drawn image patterns formed in origami ornament images into general rules through algebraic processes. Callejo and Zapatera (2014) explain the importance of student knowledge about general rules in the process of finding solutions to problems through algebraic processes. Kayhan, et al (2014); Mouhayar and Jurdak (2016) in their research on the ability of students to complete the number pattern found the main problem faced by students was how to generalize which patterns, strategies, and representations they chose with the aim of finding the relationship between the adopted strategy and the representation used.

Students do not realize their mistakes in determining the number of each flat build formed on origami ornament 4 levels to 10 levels. This condition occurs, because He does not carry out a comprehensive evaluation process. The findings in this study are in line with the findings of Papadopoulos and Dagdilelis (2008) which explain the importance of verification in problem solving. He found one of the characteristics of students in verifying that students did not systematically verify solutions to problems. Although the development of verification skills is very important for improving problem-solving abilities, students usually do not verify the accuracy of their final answers and when they do, it often takes the form of incomplete checks or just repetitions of the reassessment of what they have just completed (Olving et al., 1992 ; Pugalee, 2004).

Furthermore, students choose not to complete their work when experiencing difficulties in identifying number patterns from data showing the number of each flat building that can be formed in origami ornaments 1 level to 10 levels. This condition shows students do not have motivation in themselves to develop connection ideas that are built and find solutions to problems. As explained by Francisco (2005) that epistemological beliefs and beliefs are very important for students to develop ideas and mathematical reasoning based on reflection on their learning experiences. Ozturk and Guven (2016) in their research evaluating students 'beliefs in the problem solving process also found that beliefs have a strong effect on students' thoughts and behavior in solving problems. Further explained that there are two beliefs in students, namely positive beliefs and negative beliefs. Positive beliefs motivate students to build more ideas to make the problem solving process, while negative beliefs lead to lack of determination to solve problems and reduce students' success in finding solutions to problems. These conditions indicate a lack of motivation and the presence of negative beliefs held by students. Therefore by observing all the activities shown by students, it can be concluded that students are only able to build simple connection ideas at the stage of cognition. Students apply empirical generalization methods based on their observations of information about the questions given that are not connected with initial knowledge to form new connective thinking networks. The established connective thinking network is not sufficient to be used to design problem solving strategies so that students do not experience a strategy transformation in adequate connective thinking.

5. CONCLUSIONS

Based on the problem solving process carried out by students before and after reflection it is known that students do not transform adequate thinking strategies, thus causing students to fail in productive connective thinking. Therefore, it takes five transformation strategies in solving mathematical problems to anticipate students' failures in productive connective thinking, namely: 1) students must be able to develop their ideas to develop more mature plans in order to find several resolution strategies. 2) Students must identify errors made in the calculation process through the data verification process. 3) Students must have a good understanding of the basic concepts in constructing problem solutions. 4) students must carry out a systematic and comprehensive verification of the problem solving process. 5) students must have positive motivation and confidence in themselves to develop connection ideas based on their reflection and experience in solving problems.

ACKNOWLEDGEMENTS

The author would like to thank DP2M Dikti as a research funder. Furthermore, the author would like to thank all the UPT units of the education unit in Bulukumba, South Sulawesi, for giving research permission to conduct research

REFERENCES

[1] Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the West. … of Mathematics …, 2(2), 147–164. https://doi.org/10.21831/JPE.V1I2.2633

[2] Baki, A., Çatlioğlu, H., Coştu, S., & Birgin, O. (2009). Conceptions of high school students about mathematical connections to the real-life. Procedia - Social and Behavioral Sciences, 1(1), 1402–1407. https://doi.org/10.1016/j.sbspro.2009.01.247

[3] Callejo, M. L., & Zapatera, A. (2014). Flexibilidad en la Resolución de Problemas de Identificación de Patrones Lineales en Estudiantes de Educación Secundaria Secondary School Students ' Flexibility when Solving Problems of Recognition of Lineal Patterns, 64–88.

[4] Creswell, J. W. (2012). Educational research: Planning, conducting, and evaluating quantitative and qualitative research. Educational Research (Vol. 4). https://doi.org/10.1017/CBO9781107415324.004

[5] De Corte, E., Verschaffel, L., & Greer, B. (2000). Connecting Mathematics Problem Solving to the Real World. International Conference on Mathematics Education into the 21st Century: Mathematics for Living, 66–73.

[6] Dumestre, M. . (2016). Strategic thinking for transformation. In Chapter 8 (pp. 99–116). Financial Sustainability in US Higher Education. https://doi.org/10.1057/978-1-349-94983-0_8

[7] Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics, 103–131. https://doi.org/10.1007/s10649-006-0400-z

[8] Eizenberg, M. M., Zaslavsky, O., & Eizenberg, M. M. (2015). Students ' Verification Strategies for Combinatorial Problems Students ' Verification Strategies for Combinatorial Problems, 6065(November). https://doi.org/10.1207/s15327833mtl0601

[9] Eli, J. A., Mohr-schroeder, M. J., & Lee, C. W. (2011). Exploring mathematical connections of prospective middle-grades teachers through card-sorting tasks, 297–319. https://doi.org/10.1007/s13394-011-0017-0

[10] Francisco, J. M. (2005). Students ' reflections on their learning experiences : lessons from a longitudinal study on the development of mathematical ideas and reasoning, 24, 51–71. https://doi.org/10.1016/j.jmathb.2004.12.005

[11] Hall, S., Rose-wiles, L. M., & Stiskal, D. (2017). Enhancing information literacy using Bernard Lonergan ' s Generalized Empirical Method : A three-year case study in a first year biology course .

[12] Hendriana, H., Slamet, U. rahmat, & Sumarmo, U. (2014). Mathematical connection ability and self-confidence (an experiment on Hunior High School students through Contextual Teaching and learning with Mathematical Manipulative). International Journal of Education, 8(1), 1–11. https://doi.org/10.17509/IJE.V8I1.1726

[13] Holyoak, K.J and Morisson, R. . (2012). Thinking and Reasoning: A reader's guide. New York (NY): Oxford University Press.

[14] Jaijan, W. (2008). Mathematics Connection is a major factor specifying in the objective of Mathematics curriculum in school (NCTM , 1989 ; NCTM , 2000) and core curriculum of Basic Education 2008 determining Mathematics Learning Substance focusing on every young student t.

[15] Kayhan, M., Özdemir, E., & Kurt, G. (2014). A study of middle grade students ' performances in mathematical pattern tasks according to their grade level and pattern presentation context. Procedia - Social and Behavioral Sciences, 116, 4542–4546. https://doi.org/10.1016/j.sbspro.2014.01.982

[16] L. Suominen. (2015). Abstract Algebra and Secondary School Mathematics: Identifying and Classifying Mathematical Connections. The University of Georgia.

[17] Leung, D. Y. P., & Kember, D. (2010). Educational Psychology : An International Journal of Experimental The Relationship Between Approaches to Learning and Reflection Upon Practice, (October 2014), 37–41. https://doi.org/10.1080/01443410303221

[18] Marroum, R. (2004). The Role of Insight in Science Education : An Introduction to the Cognitional Theory of Bernard, (Lonergan 1993), 519–540.

[19] Martino, D. J. (2007). staying away of, 38, 128–136. https://doi.org/10.1163/156916207X190274

[20] Meissner, H., Velikova, E., & Matsko, V. (2015). Creativity in Mathematics Education, 591–592. https://doi.org/10.1007/978-3-319-12688-3

[21] Mhlolo, M. K., Venkat, H., & Schfer, M. (2012). The nature and quality of the mathematical connections teachers make. Pythagoras, 33(1). https://doi.org/10.4102/pythagoras.v33i1.22

[22] Montenegro, P., Costa, C., & Lopes, B. (2018). Transformations in the Visual Representation of a Figural Pattern Transformations in the Visual Representation of a Figural Pattern. Mathematical Thinking and Learning, 20(2), 91–107. https://doi.org/10.1080/10986065.2018.1441599

[23] Mouhayar, R. El, & Jurdak, M. (2016). International Journal of Mathematical Education in Variation of student numerical and figural reasoning approaches by pattern generalization type , strategy use and grade level, 5211(February). https://doi.org/10.1080/0020739X.2015.1068391

[24] Olving, P. R. S., Etacognition, M., & Schoenfeld, A. H. (1992). L EARNING TO T HINK M ATHEMATICALLY : S ENSE -M AKING IN M ATHEMATICS, 334–370.

[25] Ozturk, T., & Guven, B. (2016). Evaluating students' beliefs in problem solving process: A case study. Eurasia Journal of Mathematics, Science and Technology Education, 12(3), 411–429. https://doi.org/10.12973/eurasia.2016.1208a

[26] Papadopoulos, I., & Dagdilelis, V. (2008). The Journal of Mathematical Behavior Students ' use of technological tools for verification purposes in geometry problem solving, 27, 311–325. https://doi.org/10.1016/j.jmathb.2008.11.001

[27] Papadopoulos, I., & Iatridou, M. (2010). Modelling problem-solving situations into number theory tasks: The route towards generalisation. Mathematics Education Research Journal, 22(3), 85–110. https://doi.org/10.1007/BF03219779

[28] Pugalee, D. K. (2004). A comparison of verbal and written

descriptions of students' problem solving processes, 27–47.

[29] Raif, R., Mouhayar, E., & Jurdak, M. E. (2013). Teachers ' ability to identify and explain students ' actions in near and far figural pattern generalization tasks, 379–396. https://doi.org/10.1007/s10649-012-9434-6

[30] Roscoe, K. (2004). Lonergan ' s Theory of Cognition , Constructivism and Science Education, 541–551.

[31] Singh, A., Yager, S. O., Yutakom, N., Yager, R. E., & Ali, M. M. (2012). Constructivist teaching practices used by five teacher leaders for the Iowa chautauqua professional development program, 7(2), 197–216.

[32] Slaney, K. L., & Garcia, D. A. (2015). Constructing psychological objects: The rhetoric of constructs. Journal of Theoretical and Philosophical Psychology, 35(4), 244–259. https://doi.org/10.1037/teo0000025

[33] Stylianou, D. A. (2013). An Examination of Connections in Mathematical Processes in Students' Problem Solving: Connections between Representing and Justifying. Journal of Education and Learning, 2(2), 23–35. https://doi.org/10.5539/jel.v2n2p23

[34] Susanti, E. (2015). Proses Berpikir Siswa Dalam Membangun Koneksi Ide-ide Matematis Pada Pemecahan Masalah Matematika, (1), 2–3.

[35] Susanti, E. (2018). Cognitif Prosess Students of Mathematical Problem Solving in Productive Connectivity Thinking. In INCOMED 2017 (Ed.) (p. 5). Atlantis Press.

[36] Tasni, Nurfaidah, et all. (2017). Obstacles to students' productive connective thinking in solving mathematical problems. JIP MIPA UPI, 22(2). https://doi.org/http://dx.doi.org/10.1826