Characterization of 1-Uniform dcsl labeling of Product Graphs

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Abstract - A distance compatible set labeling (dcsl) of a connected graph G is an injective set assign- ment $f : V(G) \to 2^X, X$ being a non empty ground set, such that the corresponding induced function $f^{\oplus}: V(G) \times V(G) \to 2^X \backslash \{\varphi\}$ given by $f^{\oplus}(u,v) = f(u) \oplus f(v)$, the symmetric difference of $f(u)$ and $f(v)$, satisfies $|f(u) \oplus f(v)| = k^f_{(u,v)} d_G(u,v)$ for every pair of distinct vertices $u, v \in V(G)$, where $d_G(u,v)$ denotes the path distance between u and v and $k^f_{(u,v)}$ is a constant, not necessarily an integer, depending on the pair of vertices u, v chosen. A dcsl f of G is k-uniform if all the constants of proportionality with respect to f are equal to k, and if G admits such a dcsl then G is called a k-uniform dcsl (k-U dcsl) graph. Let $\mathcal{F}$ be a family of subsets of a set X. A graph G is called a topologically 1-uniform dcsl graph, if $\{f(V(G)$ }, the set of all vertex labeling of G is a topology. In this paper, we discuss the 1-uniform dcsl (1-U dcsl) and characterization of some product graphs.

Index Terms - dcsl graphs, 1-U dcsl graphs, topologically 1-U dcsl graphs.

1 INTRODUCTION

THROUGHOUT this article by a graph means a connected, finite, simple graph. Exclusively for all the terms in Graph theory, the scholar is recommended to [4] and [5]. Prof. Acharya [1] established the concept of vertex set valuation as a set equivalent of number valuation. For a graph $G = (V, E)$ and a non empty set X, Acharya defined a set valuation of G as a one-one set valued function $f : V(G) \to 2^X$, and he defined a set-indexer as a set valuation such that the function $f^{\oplus}: V(G) \times V(G) \to 2^X \backslash \{\varphi\}$ given by $f^{\oplus}(u,v) = f(u) \oplus f(v)$ for every $uv \in E(G)$ is also one-one, where 2^X is the set of all subsets of X and $\oplus$ is the binary operation of taking the symmetric difference of subsets of X.

Definition 1.1. [2] *Let $G = (V, E)$ be any connected graph. A distance compatible set labeling (dcsl) of a graph G is an injective set assignment $f : V(G) \to 2^X, X$ being a nonempty ground set, such that the corresponding induced function $f^{\oplus}: V(G) \times V(G) \to 2^X \backslash \{\varphi$ given by $f^{\oplus}(u,v) = f(u) \oplus f(v)$satisfies $|f^{\oplus}(u,v)| = k^f_{(u,v)} d_G(u,v)$ for every pair of distinct vertices $u, v \in V(G)$, where $d_G(u,v)$denotes the path distance between u and v and $k^f_{(u,v)}$ is a constant, not necessarily an integer, depending on the pair of vertices u, vchosen.*

A dcsl f of G is k-uniform if all the constants of proportionality with respect to f in Definition 1.1 are equal to k, and if G admits such a dcsl then G is called a k-U dcsl graph.

Definition 1.2. [10] *Let (G, f)be a k-U dcsl graph where $f$is the k-U dcsl labeling of G. $G$is called a topologically k-U dcsl graph, if $\{f(V(G)\}$, the collection of vertex labeling of $G$is a topology.*

Definition 1.3. *Given a graph G, the generalized Mycielski graph $\mu(G)$is constructed as follows: Start with G and for every vertex v_i in G add a vertex u_i. Make ui adjacent to all vertices in $N_G(v_i)$. Finally add a vertex w which is adjacent to all u_i.*

Definition 1.4. *The join $G = G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex sets V_1 and V_2 and edge sets X_1 and X_2 is the graph union $G_1 \cup G_2$ to get here with all the edges joining $V_1$and V_2.*

Definition 1.5. *The square G^2 of a graph G has $V(G^2) = V(G)$with u, vadjacent in G^2whenever $d(u,v) \leq 2$ in G.*

Definition 1.6. *The Cartesian product $G_1 \times G_2 = (V, E)$of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph with $V = V_1 \times V_2$, and if $u = (u_1, u_2), v = (v_1, v_2)$ then, $e = uv \in E(G_1 \times G_2)$, whenever $u_1 = v_1$ and $u_2 v_2 \in E(G)$ or $u_2 =$*

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v_2 *and* $u_1v_1 \in E(G_1)$.

Definition 1.7. *Let* $G_1 = (V_1, E_1)$ *and* $G_2 = (V_2, E_2)$ *be two graphs. The tensor product of* G_1 *and* G_2*, denoted by* $G = G_1 \otimes G_2$ *is the graph with vertex set* $V = V_1 \times V_2$ *and two vertices* (u_1, v_1) *and* (u_2, v_2) *in* V *are adjacent in the tensor product* $G_1 \otimes G_2$ *if* $u_1u_2 \in E_1$ *and* $v_1v_2 \in E_2$ *. Equivalently,* $d_{G_1}(u_1, u_2) = 1$ *and* $d_{G_2}(v_1, v_2) = 1$*. The tensor product of two bipartite graphs are not connected.* $G_1 \otimes G_2$ *is a bipartite graph if one of the graphs is bipartite.*

Definition 1.8. *Let* G_1 *and* G_2 *be graphs. The strong product* G_1 *Reject* G_2 *of graphs* G_1 *and* G_2 *is the graph with vertex set* $V(G_1) \times V(G)$ *and* $u = (u_1, v_1)$ *is adjacent with* $v = (u_2, v_2)$ *whenever (* $v_1 = v_2$ *and* u_1 *is adjacent with* u_2*) or (* $u_1 = u_2$ *and* v_1 *is adjacent with* v_2*) or (*u_1 *is adjacent with* u_2 *and* v_1 *is adjacent with* v_2*). The strong product of two non trivial graphs has a triangle.*

Definition 1.9. *The lexicographic product* $G_1 \circ G_2$ *of two graphs* G_1 *and* G_2 *has vertex set* $V(G_1) \times V(G)$ *and two vertices* (u_1, v_1) *and* (u_2, v_2) *are adjacent whenever* $u_1u_2 \in E(G_1)$*, or* $u_1 = u_2$ *and* $v_1v_2 \in E(G_2)$*.* $G_1 \circ G_2$ *of two non-trivial graphs has a triangle.*

Germina and Jinto have established following result [7].

Theorem 1.10. [8] *Paths are topologically 1-U dcsl graphs.*

Theorem 1.11. [3] *Cycle* $C_n, (n > 3)$ *is 1-U dcsl if and only if* n *is even.*

Theorem 1.12. [3] *For* $n > 4$*, no graph* C_n *has a 1-U dcsl* f *such that* $f(u) = \emptyset$ *for some* $u \in C_n$

Theorem 13. [8] C_n *is topologically 1-U dcsl graph if and only if* $n = 4$.

Theorem 1.14. [3] *The Cartesian product* $P_m \times P_n$ *of two paths* P_m *and* P_n *is a 1-U dcsl graph.*

Theorem 1.15. [9] *If* G *be a graph that admits a 1-U dcsl then* G *is triangle free.*

Theorem 1.16. [9] *If* G *be a 1-U dcsl graph, then no two adjacent vertices in* G *receive subsets of the same cardinality.*

In this paper, we discuss the 1-U dcsl labeling of product graphs and characterization of the product graphs. Obviously for a graph to be 1-U dcsl G must be a connected and no two adjacent vertices receive the subsets of the same cardinality. Part of Theorem 1.11 becomes a particular case of the following theorem.

2 MAIN RESULTS

Theorem 2.1. *If* G *is 1-U dcsl graph then* G *is a bipartite graph. The converse need not be true.*

Proof. We prove that if G is not a bipartite then the graph G is not 1-U dcsl graph. Let G be a graph that is not bipartite. Obviously $n \geq 3$ and G has an odd cycle. We can always find an odd cycle C in G. Suppose G is 1-U dcsl graph and f is the corresponding labeling. Let the cycle C be (v_1, v_2, v_k, v_1). With out loss of generality let $|f(v_1)|$ be odd then obviously $|f(v_k)|$ and $|f(v_2)|$ are even. Similarly, we can find two adjacent vertices namely $f(v_{\lfloor \frac{k}{2} \rfloor 1})$ and $f(v_{\lfloor \frac{k}{2} \rfloor 1}) + 1$ have **the** same cardinality. So G cannot be a 1-U dcsl graph. The converse need not be true. $K_{m,n}, m, n > 2$ is not 1-uniform.

Corollary 2.2. *The* Q_n *is 1-U dcsl graph.*

Corollary 2.3. *Cycle* C_n *with* n *odd is not 1-U dcsl graph.*

Corollary 2.4. *Petersen Graph is not 1-U dcsl graph.*

Corollary 2.5. *The Wheel graph* W_n *is not 1-U dcsl graph.*

Theorem 2.6. *Let* G *be a topologically 1-U dcsl graph with dcsl labeling* f *and ground set* X *of cardinality* n*. Then* $diam(G) = n$*. Moreover, ecc* $d(u)$ *=ecc =* $d(v) = n$*, where* $f(u) = \emptyset$ *and* $f(v) = X$.

Proof. It is given that G is a topologically 1-U dcsl graph. Hence, without loss of generality assume $f(u_0) = \emptyset$ and $f(v_0) = X$ for some $u_0, v_0 \in V(G)$. Since G is 1- uniform, for any two vertices u, v.

$|f^{\oplus}(u, v)| = d(u, v)$ for all, $u, v \in V(G)$, which implies, $u, v \in V(G)$Max $d(u, v) =$ Max $|f^{\oplus}(u, v)| =$ Max $|f(u) \oplus f(v)| = n$ and hence, diam G $= n$.

Theorem 2.7. *The Cartesian product* $G_1 \times G_2$ *is* $k-U$ *dcsl graph if and only if* G_1 *and* G_2 *are* $k-U$ *dcsl graphs.*

Proof. Let G_1 and G_2 be two k-U dcsl graphs with the ground sets X and Y respectively. Assume that X and Y are finite sets and $X \cap Y = \emptyset$. Let $V(G_1) = \{u_1, u_2, u_m\}$ and $(G_2) = \{v_1, v_2, v_n\}$. Let $\tau_1 = \{A_1, A_2, A_m\}$ and $\tau_2 = \{B_1, B_2, B_n\}$ be the $k-$U dcsl of $V(G)$ and $V(G)$ respectively. The vertex set of $V(G_1 \times G_2) = \{(u_i, v_j)/i = 1,2, m, j = 1,2, n\}$ Define

$f: V(G_1 \times G_2) \to 2^{X \cup Y}$,

such that

$$f(u_i, v_j) = A_i \cup B_j, i = 1,2, \dots m, j - 1,2, n.$$

f is well defined and $1-1$

$$f(u_i, v_j) = f(u_l, v_k)$$
$$\Leftrightarrow A_i \cup B_j = A_l \cup B_k$$
$$\Leftrightarrow A_i = A_l \text{ and } B_j = B_k$$
$$\Leftrightarrow i = l, j = k$$
$$\Leftrightarrow (u_i, v_j) = (u_l, v_k)$$

Since G_1 and G_2 are connected graph $G_1 \times G_2$ is also a connected graph. Let u_i, v_j and u_l, v_k be two vertices in $G_1 \times G_2$.

Now,

$$d((u_i, v_j), (u_l, v_k)) = d((u_i, v_j), (u_l, v_j)) + d((u_l, v_j), (u_l, v))$$
$$= |A_i \oplus A_l| + |B_j \oplus B_k|$$
$$= |A_i \cup B_j| \oplus |A_l \cup B_k|$$
$$= \frac{1}{k} |f^{\oplus}((u_i, v_j), (u_l, v_k)$$

Hence, for every pair of vertices there exists a shortest path such that

$$|f(u_i, v_j) \oplus f(u_i, v_k)| = |A_i \cup B_j| \oplus |A_l \cup B_k|$$
$$= k \times d((u_i, v_j), (u_l, v))$$

Hence, the Cartesian product $G_1 \times G_2$ of two $k-$ U dcsl graphs is a $k-$ U dcsl graph. Conversely suppose $G_1 \times G_2$ is k-U dcsl graph then G_1 and G_2 being subgraphs of $G_1 \times G_2$ which is k-U dcsl graph. Since whose induced dcsl graph is a dcsl graph.

Corollary 2.8. *The Cartesian product $G_1 \times G_2$ is 1-U dcsl graph if and only if G_1 and G_2 are 1-U dcsl graphs.*

Theorem 2.9. *The Cartesian product $G_1 \times G_2$ is a topologically $k-$U dcsl graphs if and only if G_1 and G_2 are topologically $k-$U dcsl graph.*

Proof. Let $\tau_1 = \{A_1, A_2, A_m\}$ where $A = \emptyset$ and $A_m = X$ and $\tau_2 = \{B_1, B_2, B_n\}$, where $B_1 = \emptyset$ and $B_n = Y$ be the topologically 1-U dcsl of $V(G)$ and $V(G_2)$. τ_1 and τ_2 are topologies defined on X and Y respectively. Invoking Theorem 2.7 $G_1 \times G_2$ is k-U dcsl graph. To prove the set $\tau = \{A_i \cup B_j / i = 1,2, m, j = 1,2, n\}$ is a topology; $A_1 \cup B_1 = \emptyset \in \tau$ and $A_m \cup B_n = X \cup Y \in \tau$. Since X and $Y$are finite set essentially it is enough to prove that for any two sets, τ is closed under union and intersection then τ becomes a topology.

Let $A_i \cup B_j$ and $A_l \cup B_k \in \tau$

$$(A_i \cup B_j) \cup (A_l \cup B_k) = (A_i \cup A_l) \cup (B_j \cup B_k)$$

$\in \tau$, since $A_i \cup A_l \in \tau_1$ and $B_j \cup B_k \in \tau_2$.

Similarly,

$$(A_i \cup B_j) \cap (A_l \cup B_k) = (A_i \cap A_l) \cup (B_j \cap B_k)$$

$\in \tau$, Since $A_i \cap A_l \in \tau_1$ and $B_j \cap B_k \in \tau_2$.

Hence, τ is topology. The converse is an implication of Theorem 2.7.

Corollary 2.10. *The Cartesian product $G_1 \times G_2$ is a topologically* 1-U dcsl *graph if and only if G_1 and G_2 are topologically* 1-U dcsl *graph.*

Corollary 2.11. *The Cartesian product $P_m \times P_n$ of two paths P_m and P_n is a topologically 1-U dcsl graph.*

Proof. By Theorem 2.7 and 2.9 $P_m \times P_n$ is topologically 1-U dcsl graph. Here we determine the exact 1- uniform labeling that gives a topology by the following construction.

Consider two path P_m and P_n. The product graph of P_m and P_n is a grid of m rows and n columns. Let $v_{1j}, v_{2j}, v_{mj}, 1 \leq j \leq n$ be the vertices of n columns of the grid $P_m \times P_n$. Take $X = \{1,2, m + n - 2\}$. Define : $V(G) \to 2^X$ such that

$f(v_{11}) = \emptyset$,

$f(v_{1j}) = f(v_{1j-1}) \cup \{j - 1\}, 2 \leq j \leq n$ and

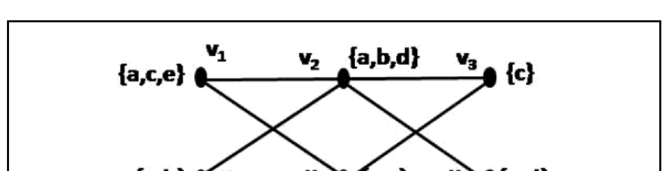
$f(v_{ij}) = f(v_{i-1j}) \cup \{n + i - 2\}, 2 \leq i \leq m, 1 \leq j \leq n$

Claim 1: $P_m \times P_n$ is 1-U dcsl

Obviously $v_{1l_j}, v_{2l_j}, v_{ml_j}, 1 \leq l \leq n$ are the set of vertices in the rows 1, 2, 3, m of $P_m \times P_n$. Consider $v_{ij}, 1 \leq j \leq n$then $|f(v_{1j}) \oplus f(v_{1j+1})| =$for $1 \leq j \leq n$ Hence, $\left|f\left(v_{il_j}\right) \oplus f\left(v_{il_k}\right)\right| = l_k - l_j$, $1 \leq i \leq m$, $1 \leq j < k \leq n$, for all $v_{ij} \in P_m \times P_n$. Hence, the labeling is 1-U dcsl.

Claim 2: $P_m \times P_n$ is topologically 1-U dcsl

Clearly the collection of set labeling contain empty set, and



$f(u) \cup f(v) =$

$$\begin{cases} f(v) or f(u) & \text{if } f(u) \subset f(v) or f(v) \subset f(u). \\ f(u) \cup f(v) & \text{if } f(u) \cap f(v) = \emptyset \end{cases} \quad (1)$$

also,

$f(u) \cap f(v) =$

$$\begin{cases} f(v) or f(u,) & \text{if } f(u) \subset f(v) or f(v) \subset f(u). \\ \emptyset, & otherwise. \end{cases} \quad (2)$$

and which is in $V(p_m \times P_n)$. Then for any two sets, f is closed under union and intersection. Hence, $P_m \times P_n$ is topologically 1-U dcsl.

Corollary 2.12. *The Cartesian product* $C_n \times C_m$ *is topologically 1-U dcsl if and only if* $n = m = 4$.

Proof. $C_4$is a topologically 1-U dcsl graph. By Theorem 2.7 and 2.9 $C_4 \times C_4$ is topologically 1-U dcsl. The converse is true in view of the $Theorem$ 12.

Corollary 2.13. *The Cartesian product* $C_4 \times P_m$ *is a topologically 1-U dcsl graph.*

Theorem 2.14. *If* G *is graph with at* least *three vertices then* G^2 *is not 1-U dcsl graph.*

Proof. If G is connected graph with at least three vertices then there exist a triangle or a path of length 2. So in G^2 there is a triangle. Hence by Theorem 1.15 G^2 is not 1-U dcsl graph.

Lemma 2.15. *Let* $\mathcal{H}$ *be a graph as in fig (l) .Then* $\mathcal{H}$*is not 1-U dcsl graph.*

Proof. Let $\mathcal{H}$ be the graph given as in the fig (l). Let $V(\mathcal{H}) = \{u, u_1, u_2, u_3, v_1, v_2, v_3\}$. Suppose $\mathcal{H}$ is 1-U dcsl graph. Then there exist 1 -uniform labeling f with the ground set X. Without loss of generality let $|f(u)| = k \geq 0$. Obviously $|f(u_i)| = k + 1$ or $k - 1$, for $i = 1,2,3$. Suppose $|f(u_i)| = k + 1$, for $i = 1,2,3$, then $|f(v_i)| = k + 2$ or k for $i = 1,2,3$ implying that f is not 1 -uniform as u and v are adjacent. Similarly we get a contradiction if $|f(u_i)| = k - 1$, for $i = 1,2,3$. So two of the vertices of the set $\{u_1, u_2, u_3\}$ are labeled with cardinality $k + 1$ or $k - 1$. Without loss of generality let $|f(u_1)| = f(u_3) = k + 1$ and $|f(u_2)| = k - 1$. Then $|f(v_1)| = k - 2$ or k and $|f(v_2)| = k$ or $k + 2$. This implies that $||f(v_1)| - |f(v_2)|| = 0$ or ≥ 2, a contradiction. A similar case can be applied for $|f(u_1)| = f(u_2) = k + 1$ and $|f(u_3)| = k - 1$. Thus $\mathcal{H}$ is not a 1-U dcsl graph..

Theorem 2.16. *For any graph* $G, \mu(G)$ *is 1-U dcsl graph if and only if* G *is a trivial graph.*

Proof. Let G is any connected graph with atleast three vertices, then there exist a path P_3 as a subgraph of G. Obviously $\mu(G)$ will contain the graph $\mathcal{H}$ as a subgraph. By the construction (G) , if $u, v \in V()$ then $d_{\mu(G)}(u, v) = d_{\mathcal{H}}(u, v)$. Suppose $\mu(G)$ is 1-U dcsl graph, then $\mathcal{H}$ is 1-U dcsl such that $f^{\oplus}(u, v) = d_{\mu(G)}(u, v) = d_G(u, v)$, for all $u, v \in V(\mathcal{H})$. Which implies $f/\mathcal{H}$ becomes 1-U dcsl for $\mathcal{H}$. This is a contradiction to the fact that $\mathcal{H}$ is not 1-U dcsl graph.

Theorem 2.17. *Tensor product* $G_1 \otimes G_2$ *of two 1-U dcsl graph is not 1-U dcsl graph.*

Proof. By Theorem 2.1 1-U dcsl graph is bipartite. But we have tensor product of two bipartite graphs is disconnected. Hence tensor product of 1-U dcsl graph is not 1-U dcsl graph.

Corollary 2.18. $P_m \otimes P_n, P_m \otimes C_{2n}, C_{2n} \otimes C_{2n}$*is not 1-U dcsl graph.*

Theorem 2.19. *The join of two graphs* $G_1 + G_2$ *is 1-U dcsl graph if and only if* G_1 *and* G_2 *are trivial graphs.*

Corollary 2.20. $P_m + P_n, P_m + C_{2n}, C_{2n} + C_{2n}, K_{1,n} + P_n$ *are not 1-U dcsl graph.*

Theorem 2.21. *The strong product of two graphs* $G_1 \boxtimes G_2$ *is 1-U dcsl graph if and only if either* G_1 *or* G_2 *is a trivial graph.*

Theorem 2.22.*The lexicographic product of two graphs* $G_1 \circ G_2$ *is 1-U dcsl graph if and only if* G_1 *and* G_2 *are 1-U dcsl graphs and either* G_1*or* G_2*is a trivial graph.*

OPEN PROBLEMS:

1. Classify the graphs whose tensor product is 1-U dcsl.
2. Classify the graphs whose tensor product is topologically 1-U dcsl.

3. Find the necessary and sufficient condition for the cartesian product of k-U dcsl graph and m-U dcsl graph to be a U dcsl for some l.
4. Find the necessary and sufficient condition for the cartesian product of topologically k-U dcsl graph and topologically m -U dcsl graph to be a topologically U dcsl for some l.

CONCLUSION

In this paper we characterize the 1-uniform dcsl graph of some product graph such as Cartesian product, tensor product, lexicography product, etc. The main motivation to study distance compatible set-labeling is due to the problem in communication theory. The idea is to let the message be proceeded by some 'address' of B, permitting to decide at each node of the network, in which direction the message should proceed. The message will proceed to the next node if its hamming distance to the destination node B is shorter or, at a constant proportionality distance or, at various fixed constants of proportionality. The most natural way of devising such a scheme is by labeling the nodes by strings of subsets of a set X, which amounts to try to embed the graph in a dcsl-graph.

REFERENCES

[1] Acharya, "Set-valuations of Graphs and Their Applications," MRI Lecture Notesin Applied Mathematics, No.2, Mehta, Research Institute of Mathematics and Mathematical Physics,(1983), Allahabad

[2] B.D. Acharya and Germina, "Distance compatible Set-labeling of graphs," Indian J. Math. And Comp. Sci. 1 (2011) 49-54.

[3] Bindhu K. Thomas and K. A. Germina, "Distance compatible set-labeling index of graphs," Int. J. Contemp. Math. Sciences, Vol. 5, no. 19, 911-919

[4] F. Harary, "*Graph Theory*," Addison Wesley, Reading Massachusetts, (1969)

[5] R. Balakrishnan, K. Ranganathan, "*A Text book of graph theory*," Springer International Edition (2000).

[6] K. A. Germina, Gency Joseph and L. Benedict Michael Raj, "Characterization of 1-U dcsl graphs and learning graphs," International Journal of Pure and Applied Mathematics, Vol. 32, No. 1, 2019, pp 51-58.

[7] K.A. Germina and Jinto James, "Characterization of 1-U dcsl graphs and well-graded family of sets," Advances and Applications in Discrete Mathematics, Vol.15, (2015) 113- 123.

[8] Jinto James, "Study on labeled graphs and their properties," PhD dissertation, Department of Mathematics, Central University of Kerala, India, 2018.

[9] Bindhu K. Thomas and Germina K. A, "Distance Compatible Set-Labeling of Graphs," International Mathematical Forum, Vol. 6, no. 31, (2011), 1513-1520.

[10] Germina, K.A., Jinto James, and Shaini P, "Learning graphs and 1-U dcsl graphs," Discrete Mathematics Algorithm sand Applications, World Scientific, Vol.09, No. 04,(2017) 1750046