Detour Domination In Soft Graph

N. Sarala, K. Manju

Abstract: A set D of vertices of a soft graph (F, A) is said to be a dominating set if every vertex of the subgraph induced by F(x) in V −D is adjacent to a vertex in D. In this paper, the concepts of detour domination in soft graph and some types of detour domination in soft graphs including detour distance, total detour distance, detour dominating set, minimal detour dominating set, upper detour dominating number in soft graphs are introduced and investigating some of their properties and results.

Keywords: detour distance, total detour set, detour dominating set, minimal detour dominating set, upper detour dominating number in soft graphs.

I. INTRODUCTION

Domination is an area in graph theory with an extensive research activity. The concept of domination in graphs is introduced by Haynes et al. (1998). In 2012, Bounds on connected domination in square of a graph is introduced by M.H.Muddabihal and G.Srinivasa. Distance in Graphs was introduced by F. Buckley and F. Harary [2]. The concept of Detour distance in graph was introduced by G.Chartrand, G. L.JohnsandS.Tian [4]. The Upper Detour Domination Number of a Graphwas introduced by J. John and N. Arianayagam[10] . Akram and Nawaz introduced the novel concepts called fuzzy soft graphs and fuzzy vertex induced soft graphs [1] and further more Maji, Biswas and Roy[4] worked on soft set theory. A new notion on soft graph using soft sets was introduced by Rajesh K. Thumbakara and Bobin George[12]. The concept of domination in soft graphs is introduced by Sarala.N, Manju.K [14]. In this paper, we introduce the concept of detour domination in soft graphs and describe some concepts of detour domination in soft graph with related results and investigate some of their properties in soft graph.

2. PRELIMINARIES

Definition: 2.1

Let U be a nonempty finite set of objects called Universe and let E be a nonempty set called parameters. An ordered pair (F, A) is said to be a Soft set over U, where F is a mapping from E into the set of all subsets of the set U. That is F: A → $\rho(U)$. Where $\rho(U)$ denotes the collection of all subsets of U. The set of all Soft sets over U is denoted by S (U).

Definition: 2.2

Let $G = (V, E)$ be a graph and (F, A) be a soft set over V. Then (F, A) is said to be a soft graph of G if the subgraph induced by F(x) in G is a connected subgraph of G for all x ∈ A.

Definition: 2.3

A set D of vertices of a soft graph (F, A) is said to be a dominating set if every vertex of the subgraph induced by F(x) in V −D is adjacent to a vertex in D.

Example: 2.4

Consider a graph G (V, E) shown in the figure 2.1

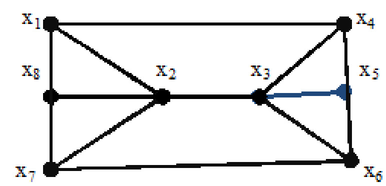

Fig 2.1 *simple graph*

Let A = {v_1, v_2} ⊆ V and (F,A) be a soft set over V with approximate function F : A → P(V) by

F (v_1) = { x_8, x_2, x_3, x_5}, F(v_2) = { x_1, x_4, x_5, x_6, x_7, x_8}

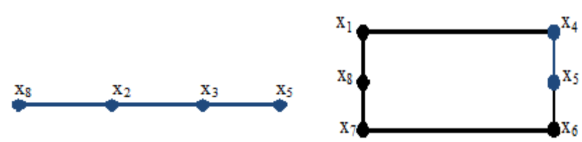

Fig 2.2 *parameterised graph*

F (v_1), Corresponding to parameter v_1 F (v_2), corresponding to parameter v_2

Here {x_5, x_8} is the dominating set of parameterised graph F(v_1) and F(v_2) . Therefore {x_5, x_8} is the dominating \set of soft graph (F, A).

Definition: 2.5

A dominating set D in a soft graph (F, A) is said to be a minimal dominating set if no proper subset of D is a dominating set.

Definition: 2.6

The minimum cardinality of a dominating set of a soft graph (F, A) is called the domination number of (F, A) and is denoted by γ (F, A).

Definition : 2.7

Let $G = (V, E)$ a graph and (F, A) be a soft graph of G. Than (F, A) is said to be a complete graph if the subgraph induced by F(x) is a complete graph for all x ∈ A.

Definition: 2.8

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A Bipartite graph (F, A) is said to be a soft complete Bi-partite graph of G if the subgraph induced by F(x) is complete Bi-partite graph for all x ∈ A.

Definition: 2.9
Let $G = (V, E)$ be a graph with vertex set V is petitioned into two disjoint vertex pair and (F,A) be a soft set over V. Then (F,A) is said to be a soft Bi-partite graph of G if the subgraph induced by F(x) is Bi-partite graph for all x ∈ A.

Definition: 2.10
The distance d(u,v) between two vertices u and v in a connected graph G is the length of the shortest u − v path in G.

Definition: 2.11
The length of the longest u−v path between two vertices u and v in a connected graph G is called detour distance D(u,v).

Definition: 2.12
A vertex v is said to detour dominate a vertex u if u = v or u is detour neighbor of v. A set S of vertices of G is called a detour dominating set if every vertex of G is detour dominated by some vertex in S. A detour dominating set of minimum cardinality is a minimum detour dominating set and its cardinality is the detour domination number γD(G).

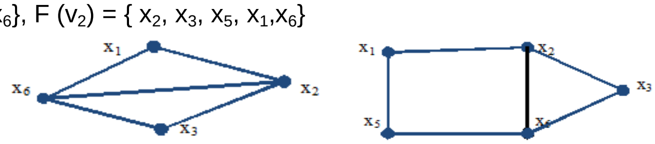
3. DETOUR DOMINATION IN SOFT GRAPH

Definition:3.1
The detour distance D (u,v) of a soft graph (F, A) is the length of the longest u−v path between two vertices u and v in every subgraph induced by F(x) for all x ∈ A.

Definition:3.2
Let (F, A) be a soft graph with at least two vertices. A set D ⊆V is called a total detour set of (F, A) if D is a detour set of (F,A) and the sub graph induced by (F,A) has no isolated vertex. The total detour number tdn(F,A) of (F,A) is the minimum order of ts total detour sets and any total detour set of order tdn(F,A) is called a tdn-set of (F,A).

Definition:3.3
A set D of vertices of a soft graph (F, A) is called a detour dominating set if every vertex of the subgraph induced by F(x) is detour dominated by some vertex of D. A detour dominating set a soft graph (F, A) with minimum cardinality is a minimum detour dominating set and this cardinality is the detour domination number denoted as γ_D (F. A).

Example: 3.4
Consider a soft graph (F,A) be a soft set over V and let A = {v_1, v_2} ⊆ V with approximate function F : A → P(V) F(x) = {y ∈ V : x adjacent to y} for all x ∈ A.That is, F (v_1) = { x_1, x_2, x_3, x_6}, F (v_2) = { x_2, x_3, x_5, x_1,x_6}



F (v_1), Corresponding to parameter v_1, F (v_2), corresponding to parameter v_2
Fig 3.1 *parameterised graph*

Here {x_1, x_3} is the detour dominating set of parameterised graph F (v_1) and F(v_2). Therefore{x_1, x_3} the detour dominating sets of soft graph (F, A).

Definition: 3.5
A detour dominating set D in a soft graph (F, A) is said to be a minimal detour dominating set if no proper subset of D is a detour dominating set and this cardinality is the detour domination number denoted as γ_D(F, A).

Example: 3.6
From example 3.4,{x_1, x_3} is the detour dominating set of parameterised graph F (v_1) and F(v_2).Therefore{x_1, x_3} the detour dominating sets of soft graph (F, A).Therefore γ_D(F, A)= 2

Theorem: 3.7
A detour dominating set D is a minimal detour dominating set if and only if for each vertex u ∈ D either of the following two conditions holds. (i) u is a detour isolate in D. (ii) For all u ∈ D and v ∈ D − {u} ∃ at least a vertex u′ ∈ N_D(u) such that u′ ∉ N_D(u) ∩ N_D(v). Proof:Suppose D is a minimal detour dominating set and neither of the conditions (i) and (ii) hold. That is, if u is not a detour isolate in D, then there exist some vertex v′ ∈ D such that u ∈ N_D(v′). Then S − {u} still remains a detour dominating set. Also for all u ∈ S and v ∈ S − {u} ∃ at least a vertex u′ ∈ N_D(u) such that u′ ∈ N_D(u)∩N_D(v). That is, u′ is detour dominated by some vertex v ∈ D − {u}. Hence, D − {u} is a detour dominating set which is a contradiction. Therefore either of the conditions must hold. Conversely suppose that (i) or (ii) holds. But is not a minimal detour dominating set. Hence, D − {u} is still a detour dominating set. Then ∃ some vertex u such that N_D[u] ⊆ ∪N_D(v), for some v ∈ D − {u}. Consequently ∃ at least a vertex u′ ∈ N_D(u)∩N_D(v). Hence, (ii) does not hold. Also if S − {u} is a detour dominating set, then in that case u is a detour neighbor of at least a vertex of D− {u}. That is, (i) does not hold. Thus neither condition (i) nor (ii) holds which contradicts our assumption. Therefore,D is a minimal detour dominating set.

Theorem: 3.8
If (F, A) is a soft complete bipartite graph and $m = n = 1$ an , $n \geq 2$ then for each induced subgraph of F(x) γ_d(F, A) = 2.
Proof: Let (F, A) be a soft complete bipartite graph. Since each parameterised subgraph of F(x) is complete bipartite graph . Let we take a F(a) be one of the induced subgraph of F(x) with the bipartition V_1 = {a_1,a_2,a_3,…….,a_m} and V_2 ={ b_1,b_2,b_3,…….,b_n} . If m= $n = 1$ $then$ F(a) = K_2. sed Cleary, γ_d (F(a)) = 2.If ,$n \geq 2$. Let D = {$a_{1,1}$} be a minimum detour dominating set $K_{m,n}$. Distance of (a_1,b_1) is longest path of (a_1, b_2, a_2, b_1),(a_1, b_3, a_3, ◎$_1$),(◎$_1$, ◎$_4$, ◎$_4$, ◎$_1$),………… (◎$_1$, ◎◎, ◎◎, ◎$_1$). This is a detour set, and (◎$_1$, ◎$_1$) is dominating set of F(a). This satisfying detour dominating set condition. Hence, ◎ F(a) = 2,which is true for all induced subgraph of F(x). Hence, ◎ (F, A) = 2

Theorem:3.9 For a soft complete Graph (F, A). Then ◎◎ (F, A) = 2 Proof:Let (F, A) be a soft complete graph. Since each parameterised subgraph of F(x) is complete graph. Let we take F(a) be one of the induced subgraph of F(x). F(a) complete graph with n vertices. ◎ = { ◎ ;1 ≤ ◎ ≤ ◎}. Let we take longest path between two vertices (◎$_1$,◎) is ◎$_1$,◎$_2$,◎$_3$,…….,◎◎. And ◎$_1$ is adjacent to ◎ ;2 ≤ ◎ ≤ ◎. Thus detour dominating set condition is satisfying. Hence, ◎◎ F(a) = 2. which is true for all induced subgraph of F(x). Hence, ◎ (F, A) = 2.

Theorem: 3.10
Let (F, A) be a soft graph. Then, every detour dominating set of (F, A) contains all the extreme vertices of (F, A). Proof: Let (F, A) be a soft graph and let D be the set of all extreme vertices of every induced subgraph of F(x). we know that every detour set of (F, A) contains D. As every detour dominating set of (F, A) is a detourset of (F, A), every detour dominating set of (F, A) contains D.

Theorem: 3.11
Let (F, A) be a soft graph. If D, the set of all extreme vertices of (F, A), is a detour dominating set of (F, A), then D is the unique minimum detour dominating set of (F, A). Proof:Suppose S is a detour dominating set of (F, A). Then, ◎◎ (F, A) ≤ D .By Theorem 3.10, ◎ (F, A) ≥ D . Therefore, ◎(F, A) = D and so D is a minimum detour dominating set of (F, A). Again by Theorem 3.10, every minimum detour dominating set of (F, A) contains D and so D is the unique minimum detour dominating set of (F, A).

Theorem: 3.12
Let (F, A) be a soft graph of order ◎ and diameter ◎. Then, ◎◎ (F, A) ≤ ◎ − ◎ + $\left[\frac{d}{3}\right]$.
Proof: Let (F, A) be a soft graph, since each parameterised subgraph of F(x) of order ◎ and diameter ◎. Let we take F(a) be one of the induced subgraph of F(x). Let ◎: (◎ = ◎$_0$, ◎$_1$,……….,◎ = ◎) be a path of length d in F(a). If S = {◎$_{1,2}$,……….,◎$_{◎-1}$} , then ◎ − D is a detour set of F(a). Also, it is a dominating set of < ◎ − (◎ − ◎$_{1,◎-1}$} >. Let ◎′ = ◎$_{2,3}$,……….,◎$_{◎-2}$. ◎ $|P'|$ = ◎ − 3. Let ◎′ be a minimum dominating set of ◎′.We know that $|D'|$ = $\left[\frac{d-3}{3}\right]$. Let ◎ = (◎ − ◎) ∪ ◎′. Clearly, D is a detour dominating set of F(a). Therefore, ◎◎ (F(a)) ≤ |D| = $|(V-S) \cup D'|$ ≤ ◎ − ◎ + 1 + ◎−3 3 = ◎ − ◎ + 1 + $\left[\frac{d-3}{3}\right]$ = ◎ − ◎ +$\left[\frac{d}{3}\right]$, Which is true for all induced subgraph of F(x). Hence, ◎ (F, A) ≤ ◎ − ◎ + $\left[\frac{d}{3}\right]$

Theorem: 3.13
Let (F, A) be a soft graph of order (◎ ≥ 2) and diameter ◎. Suppose minimum degree of soft graph is ◎ ≥ 3. Then, ◎◎ (F, A) ≤ ◎ − ◎ + 1.
Proof: Let (F, A) be a soft graph. Since each parameterised subgraph of F(x) of order ◎≥ 2 and diameter d.Let we take F(a) be one of the induced subgraph of F(x). Let ◎, ∈ ◎.As diam (F(a))=d, There exists a shortest path ◎: (◎ = ◎$_0$, ◎$_1$,……….,◎◎ = ◎) of length d in F(a). If S = {◎$_1$,◎$_2$,……….,◎$_{◎-1}$}, then ◎ − S is a detour set of F(a). since δ ≥ 3, each vertex of S is adjacent to atleast one vertex of $V - S$ is a detour dominating set of F(a) and so γ_d (F(a)) ≤ | V − S| = $p - (d - 1) = p - d + 1$. Which is true for all induced subgraph of F(x). Hence, $_d$ (F, A) ≤ $p - d$ +1

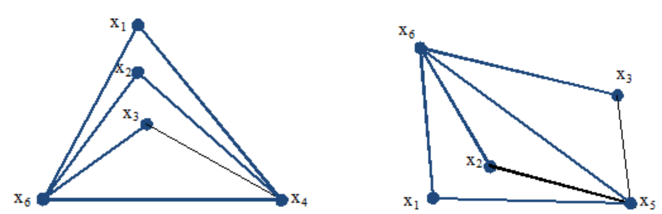
Observation 3.14:
Let (F, A) be a soft graph with p ≥ 2 vertices. Then

1. A minimal detour set of (F, A) which is also a dominating set of is (F, A) a minimal detour dominating set of (F, A).
2. A minimum detour set (or d-set) of G which is also a dominating set of (F, A) is a minimum detour dominating set of (F, A). ($or\ \gamma_d$ − set) of (F, A).
3. Any minimal dominating set of (F, A) which is also a detour set of (F, A) is a minimal detour dominating set of (F, A).
4. Any minimum dominating set (or γ-set) of (F, A) which is also a detour set of (F, A) is a minimum detour dominating set of (F, A). (r γ_d − set) of (F, A).

Theorem: 3.15
For every positive integer k ≥ 2, there exists a soft graph (F, A) with γ_d (F, A) = k.
Proof: Let k ≥ 2. Consider the soft complete bipartite graph, Since each parameterised subgraph of F(x) is complete bipartite graph . Let we take F(a) be one of the induced subgraph of F(x). . F(a) =$K_{1,k}$. Let D = $(K_{1,k}) - \{v\}$ where v is the central vertex. Obviously, D is a detour dominating set of F(a). Therefore, by Theorem: 3.11, D is the unique minimum detour dominating set of F(a) and so γ_d (F(a))= k. Which is true for all induced subgraph of F(x). Hence, $_d$ (F, A) = k.

Definition: 3.16
Let (F, A) be a soft graph. A detour dominating set of (F, A) is called a minimal detour dominating set D of (F, A) if no proper subset of D is a detour dominating set of (F, A).The upper detour dominating number of γ_d^+ (F, A) of (F, A) is the maximum cardinality of a minimal detour dominating set of (F, A).

Example: 3.17
Consider a soft graph (F,A) be a soft set over V and let A = {v_1, v_2} ⊆ V with approximate function F : A → P(V) , Here F(x) = {y ∈ V : x adjacent to y} for all x ∈ A.That is, F (v_1) = { x_1, x_2, x_3, x_4, x_6}, F (v_2) = { x_2, x_3, x_5, x_1,x_6}



F (v_1), Corresponding to parameter v_1,F (v_2), corresponding to parameter v_2

***Fig 3.2** parameterised graph*

Here {x_1, x_2, x_3} is the minimal detour dominating set of parameterised graph F (v_1) and F(v_2). Since no proper subset of is a detour dominating set of {x_1, x_2, x_3}. Therefore {x_1, x_2, x_3} is the upper detour dominating set of soft graph (F, A). The upper detour dominating number of γ_d^+ (F, A) = 3

Theorem: 3.18
For a soft graph, 2 ≤γ_d (F, A) ≤ γ_d^+ (F, A) ≤ p.
Proof:
A detour dominating sets needs at least two vertices so that γ_d (F, A) ≥2.Since every minimal detour dominating set is

also a detour dominating set, γ_d (F, A) ≤ γ_d^+ (F, A). Since the set D is a detour dominating set of (F, A). we have γ_d^+ (F, A) ≤ p .Thus 2≤γ_d (F, A) ≤ γ_d^+ (F, A) ≤ p .

Theorem: 3.19
For a soft complete bipartite graph (F, A) with m = 1 and n = p-1, γ_d^+(F, A) = p -1
Proof:
Let (F, A) be soft complete bipartite graph and each induced subgraph of F(x) are complete bipartite graph with m = 1 and n = p-1. Let D be the set of all end vertices of (F, A).Then by Theorem 3.10, D is a subset of every detour dominating set of (F, A) and so γ_d (F, A) ≥ p -1. It is clear that D is a detour dominating set of (F, A).so that γ_d^+(F, A) = p -1.

Theorem: 3.20
For the soft complete graph (F, A) of order (p ≥ 2), γ_d^+ (F, A) = 2.
Proof:
Let u, v be two vertices. Then D = {u, v}, is a detour dominating set of (F, A) so that γ_d^+ (F, A) ≥2. We have to show that γ_d^+ (F, A) = 2. Suppose that γ_d^+ (F, A) ≥3.Then there exists a minimal detour dominating set D' of (F, A) such that $|D'|$≥3. Since (F, A) is complete, the element of D' are adjacent in (F, A). Then it follows that D' contains a detour dominating set of cardinality two, which is contradiction to D' is a minimal detour dominating set of (F, A). Therefore γ_d^+ (F, A) = 2.

5. CONCLUSION
This paper proposed the concept of detour distance and total detour distance in soft graph and extend our research to introduce detour domination, detour domination number and upper detour domination number in soft graphs and describes certain related properties and results in detour dominating set in soft graph.

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