Evaluation On Suitability Of Available Reaeration Equations For River Tungabhadra, Karnataka, India And Derivation Of Refined Equation For The Same.

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Abstract: Re-aeration of a stream is very paramount in improving the ability of the stream to self-purify. It's compulsory to assess the coefficient of re-aeration in order to assess the oxygen dissolved in the stream or river. The coefficient of re-aeration is usually given as a result of many stream quantities like shear stress velocity, flow depth, mean stream velocity, Froude number, flow depth, and bed slope. Previous years have seen the development of several experimental equations. About 13 of the most widely known empirical equations used to predict re-aeration have been checked to assess how applicable they can be in the river system of the Tungabhadra, Karnataka, India at different places. Between March 2017 and February 2018, a wide array of data has been taken from the field from 6 separate locations in the river to assess the coefficient of re-aeration by applying the mass balance method. Different error assessment measures have been used to know how the re-aeration equations would perform. The error assessments are as follows; mean multiplicative error (MME), correlation statistics, normalized mean error (NME), and standard error (SE). It can be deduced from the results that the equation derived for prediction by Jha et al. (Refinement of predictive re-aeration equations for a typical Indian river. Hydrological Process. 2001;15(6):1047-1060) for an average river in India produced well-matching values of correlation coefficient, *r*, NME, SE, and MME. In addition, an equation for predictive purposes has been refined for the Tungabhadra river with the help of an algorithm that reduces the error assessments; the least-squares algorithm.

Index Terms: Dissolved oxygen, DOBT, Reaeration Coefficient, Tungabhadra River.

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1 Introduction Palatino

With respect to internal disturbance and mixing, oxygen transfer has undergone a lot of research and study in recent times. The effect of low levels of dissolved oxygen (DO) or conditions void of aeration is evident in an ecosystem lacking balance. The process of moving oxygen physically from the atmosphere to the body of water in contact with it when its DO levels decreases below normal or saturation level at a particular temperature is called Re-aeration. This is the major method through which DO is recovered by a body of water. From the sensitivity analysis of the coefficient of re-aeration done by Jain and Jha [1], the approximate value of DO content is altered greatly by a small difference in the *ka* values. Hence, it's extremely necessary that the assessment of the coefficient of re-aeration is accurate.

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Transfer of gases in natural waters are described popularly using two theories. While the two theories are used in estuaries, streams, and lakes, the one used more in still water bodies like lakes is the stagnant film theory while for moving water bodies like streams, the one more used is the surface renewal model. The two-film theory also called two-resistance theory [2-5] is of the idea that there is maximum resistance when a substance moves from phase to phase in two layers bound by a laminar where molecular diffusion is the means for transfer of mass. The accumulative resistance to the transfer of gases is a product of the separate resistances in the boundary layers of both liquid and gas. The liquid film resistance often controls the substance transfer rate, especially for those gases with high Henry's constant like nitrogen, oxygen, and carbon dioxide.

The surface renewal model is described as having for its constituents, water parcels that are carried to the surface for a while. Exchange occurs at the surface after which the parcels are carried away from the surface, and heavy liquid is added. Before the condition, the two-film theory foresaw, the gas dissolved must pass through the film. This earned it the name Penetration theory. The method was modified by Danckwerts [6] by supposing that the components of the fluid randomly get to and exit the interface, and a statistical distribution describes their contact. This is what is called the surface renewal theory. The equation which controls oxygen transfer generally can be expressed thus;

$$V\,(dc/dt) = K_1 A_s (C_s - C) \quad (1)$$

Where A_s is the surface area of water body (m^2), V is the volume of water body (m^3), K_1 is the mass transfer velocity in liquid laminar layer ($m\ d^{-1}$), C is the oxygen concentration in water ($mg\ L^{-1}$) and C_s is the saturation concentration of oxygen ($mg\ L^{-1}$).

In situations where there's no narrow air-water boundary, the volume is $V=A_sH$, where H is the mean depth (m). Hence, equation (1) is written as

$$(dc/dt) = K_a (C_s - C) \quad (2)$$

where K_a=the re-aeration rate coefficient (d^{-1}), which is equivalent to $K_a = K_1/H$.

The equations expressed above will give an idea of the dynamics of oxygen re-aeration. The difference between the real value and saturation value of the water's DO concentration is what determines the magnitude and direction of the transfer of mass.

$$K_{a\,(T)} = K_{a\,(20)}\theta^{\,T-20} \quad (3)$$

Where θ = 1.024 for pure water. In the rest of this paper, K_a (20^0) is used as K_a.

Many researchers have derived formulas for the prediction of re-aeration in rivers and streams. O'Connor and Dobbins [7] came up with a formula based on the surface renewal theory. They proposed that the rate of surface renewal could be estimated using the ratio of average stream velocity to depth. For trial-based assessment of *Ka*, three simple methods have been employed, and they are the distributed equilibrium method [11, 12], the tracer method [13] and the DO balance technique (DOBT) [8-10]. There have been many experimental equations derived from creating a relationship between Ka and variables like shear stress velocity (U^*), mean velocity (U), channel energy slope (S), mean flow depth (H) and Froude number (Fr). The equations that are mostly used for prediction, which are also used to compare and assess the current work's performance are expressed in Table 1.

Table 1. some frequently of used predictive re-aeration equation:

Investigator	Empirical formula
Connor and William Dobbins [14]	$Ka=3.901\ U^{0.5}\ H^{-1.5}$
Churchill.et.al. [9]	$Ka=5.010\ U^{0.969}\ H^{-1.673}$
Orlob & Krenkel.et.al [15]	$Ka=173\ (SU)^{0.404}\ H^{-0.66}$
Owens.et.al [16]	$Ka=5.35\ U^{0.67}\ H^{-1.85}$
Langbein and Durum [17]	$Ka=5.14\ U\ H^{-1.33}$
Cadwallader and McDonell [18]	$Ka=186\ (SU)^{0.5}\ H^{-1.0}$
Edward L. Thackston[19]	$Ka=24.9\ (1 + Fr^{0.5})\ UH^{-1.0}$
J.D.Parkhurst & R.D.Pomeroy.et.al[20]	$Ka=3\ (1 + 0.17\ Fr^2)(SU)^{0.375}\ H^{-1.0}$
Tassioglou & Wallace.et.al [13]	Ka=31,200 SU for Q <0.28 m3/s
	Ka=15200 SU for Q >.28 m3/s
Smoot [21]	$Ka=543\ S^{0.6236}\ U^{0.5325}\ H^{-0.7258}$
Moog [22]	$Ka=1740\ U^{0.46}\ S^{0.79}\ H^{0.74}$ for $S < 0.00$
Jha et.al [23]	$Ka=5.791\ U^{0.50}\ H^{-0.25}$
Jha et.al [24]	$Ka=0.603286\ U^{0.4}\ S^{-1}\ H^{0.154}$ for $Fr < 1$

Analysis of these equations shows that as written above, there has always been a relationship between *Ka* and various stream variables. A lot of effort has been put into deriving the experimental equations, yet, general formula for the coefficient of re-aeration that is accurate does not exist. With this continuing improbability, the need arises to check the authenticity of the commonest equations for prediction achieving the in-stream estimations to analyze K_a. This study has the objective of coming up with an easy equation for prediction for K_a employing various stream variables. The Tungabhadra river, India was the location for field measurements to get values for K_a using the DOBT. [8].

In addition, the prediction of K_a for the Tungabhadra river was made using 13 of the most widely known equations. Different error measurements have been employed in assessing the performance of all the equations for prediction. The different error measurements are a mean multiplicative error (MME), standard error (SE), correlation statistics, and normalized mean error (NME). Also, a new modified experimental equation that reduces the error and gives a better coefficient of correlation is being derived using the database.

2 Study area

The Tungabhadra river, which passes through Harihara taluk, Davangere district, Karnataka, India has been chosen for the purpose of this study. The stream in this area is thick with industrial pollution, and the downstream side is heavy with domestic waste. Tungabhadra is a confluence formed by the two tributaries; Tunga and Bhadra. The confluence is formed at Koodil at an altitude of 610mt. It is above Tungabhadra and MSL moving along Andhra Pradesh and Karnataka before joining the Krishna river. The Harihara rea has semi-arid conditions which are characterized by moderate to high summer with heavy rainfall and normal winter with heavy rainfall.

Three villages situated on the downstream side of the Harihara were chosen for this research. They are as follows; Nalawagal, Airani, and Nadiharalahalli. These villages have the same socio-economic and industrial settings. The communal waste from these three villages all pour directly into the stream and Harihara Poly fiber which manufactures rayon grade pulp discharge of about 30000litres per day and the Rayon industry discharges 10000litres per day [27]. The left bank of the Tungabhadra river, near Kumarapattanam, is where both industries are situated. Sample stations were chosen based on the total mixture of discharge along with the depth and width of the stream used. Figure 1 shows the segments of sampling stations.

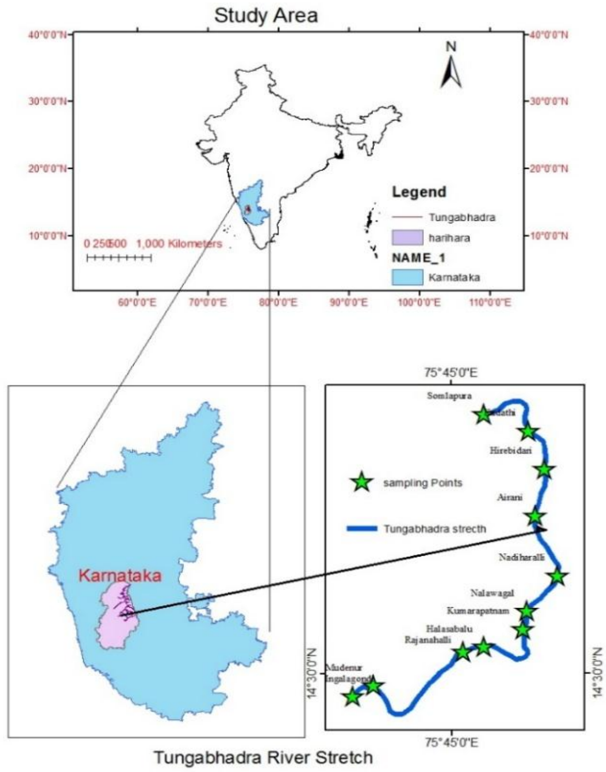


Figure 1. location of river Tungabhadra selected for the study.

3 Employing the DOBT technique to calculate reaeration constant

Seeing as communal waste is cleared into the upstream part of the chosen river without treatment, there will be a reduction in the BOD by decaying of the organic matter dissolved through microbial activity via using the DO in the water and particles from the waste would settle. Initially, after the waste is discharged, there is a higher rate of BOD removal induced by settling of organic matter that degrades easily and subsequent decay. At the farther part of the river going downstream, the rate of removal is lower due to the slower rate of degradation of stronger organic matter. The BOD decay model is quite simple and is gotten by

$$L=L_o\, e^{-kr\,(x/U)} \qquad (4)$$

Where L is the BOD concentration (mg L -1), L0 is the initial BOD concentration (mg L -1), and kr is the total BOD rate of removal induced by decaying and settling (d). a simplified model by Streeter and Phelps developed for a single source of BOD is derived by

$$D=D_0\, e^{-Ka(x/u)} + [k_d L_0/(K_a-K_r)][e^{-kr(x/u)} - e^{-(x/u)}] \qquad (5)$$

where *D* is the DO deficit (mg L^{-1}), *D*0 is the initial DO deficit (mg L^{-1}), *k*d is the de-oxygenation rate constant (d^{-1}), *k*a is the re-aeration rate constant (d^{-1}), *x* is the distance travelled (km) and *U* is the velocity (km/d).

To calculate the constant for kinematics like the rate of BOD degrading, rate of deoxygenation and rate of reaeration, a stretch of 19 km to 33km has been chosen, and this is downstream of the Harihara to the Airani village. This area gets industrial waste from Harihara Poly fiber and Grasim Rayon industry with domestic waste released into Nalawagal, Nadiharalahali and Airani stream. The stream water has no other source of release or blockage. Six stations have been chosen for sampling based on total mixing of waste along depth and width of stream water taken. By utilizing a basic BOD decay model, (Equation 4), plotting L against (x/U) will result in a straight line, and Kr[25] will be the slope. In the ongoing work, the total rate of BOD loss was the mean higher values of k in the initial stretch. The deoxygenation constant *Kd* was derived from *Kr* and mean lower values of *k*.

The DOBT is employed in measuring the reaeration constant, ka. for this purpose, the DO mass balance equation (Equation (5)) is used. All the sinks and sources of DO apart from the reaeration are assessed in this method. The ka is then derived based on the difference in the reaeration required to produce the concentration of the DO seen at the end of the reach, i.e., downstream. This method can be effectively employed in comparison to the tracer method and distributed equilibrium method. The “light and dark” bottle technique was employed to get the rates of respiration and photosynthesis. But, it was discovered that there was no difference caused by the algae in river water and that they were not much. This was caused by the processes of respiration and photosynthesis. Each set of data had their Ka values measured by the estimation of every variable apart from Ka and using the Newton-Raphson algorithm.

Table 2 shows the values of Ka whose measurements were taken during different months before and after the monsoon seasons for different flow conditions of the body of water. Each of the values of Ka is a representation of the mean value of six sets of data each month.

Figure 2. segmentation scheme to calculate K_a river.

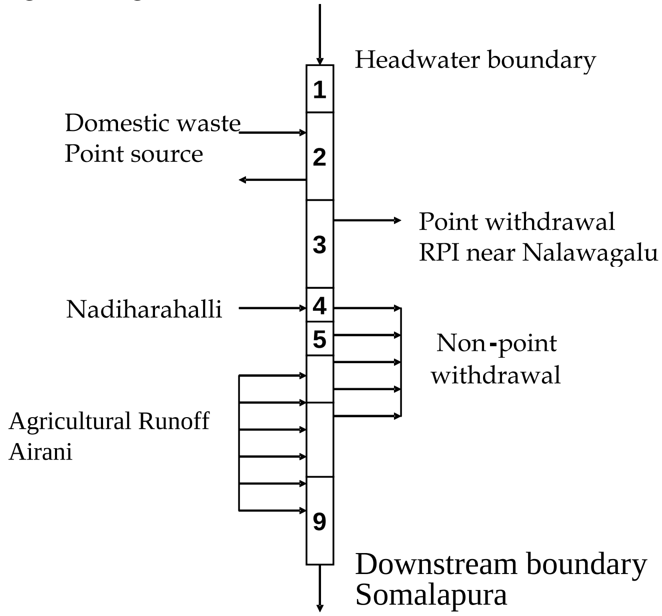


Table 2. K_a values measured by DOBT for different flow condition of river Tungabhadra.

sl.no	velocity	depth	Bed slope	flow	Ka	
	m/s	in m		m3/s	observed	predicted
January	0.51	0.48	0.00126	1.71	6.35	6.23
February	0.59	0.61	0.00126	1.64	6.1	5.81
March	0.62	0.58	0.00126	2.56	6.577	6.23
April	0.71	0.6	0.00126	3.548	6.6	6.66
may	0.73	0.73	0.00126	5.49	5.9	5.92
June	0.55	0.55	0.00126	3.497	6.57	5.96
July	0.56	0.58	0.00126	4.52	6.0794	5.81
October	0.57	0.55	0.00126	5.292	6.167	6.10
November	0.59	0.95	0.00126	5.46	4.1	4.26
December	0.6	0.58	0.00126	5.52	5.9	6.09

4 Errors of measurement/assessment in predicted equations for reaeration

All the predictive equations in Table 1 have had their performance assessed using data based on differential errors as follows; correlation coefficient (*r*), NME, MME, and SE. Related background work can be found in Jha et al. [23]. The following equations can be used to determine the NMEs and standard;

SE=$SQRT\ \{\sum_{i=1}^{N}((Kp-Km)^2$)/N} (6)

NME=(100%)/100 $\sum_{i=1}^{N}((Kp-Km)^2$)/Km (7)

Where

N= number of reaerations, K_p= predicted value, K_m=measured value.

The MME is thought to give better outcomes for measuring the impact of mistakes and is given the definition

MME={i=1N(In(Kp-Km)2)/N} (8)

The strength and data significance of the relationship between two or more random water quality variables can be estimated using the correlation analysis. In this ongoing research, values for the coefficient, r, have been derived using Pearson's equation of product-moment correlation.

R= $(1-S_{ee}/S_{yy})^{0.5}$ (9)

Where See= sum of the square difference between the derived values and the observed values. Syy= sum of squares of departures of values of K that were observed and from the average observed values.

5 Result Assessment

Values for the coefficient of the re-aeration rate (Ka) were derived for every equation of re-aeration prediction given in table 1 using the stream velocity, sets of slope data and flow depth from the Tungabhadra river. Figure 3 shows a plot of the mean multiplicative and standard errors for different predictive equations which were garnered from Equations (6) and (8). Figure 4 shows the plot of NME derived from equation (7). The correlation coefficient, r, plot for every analytical equation is seen in figure 5.

Figure 3. standard error and mean multiplicative error for predictive equation.

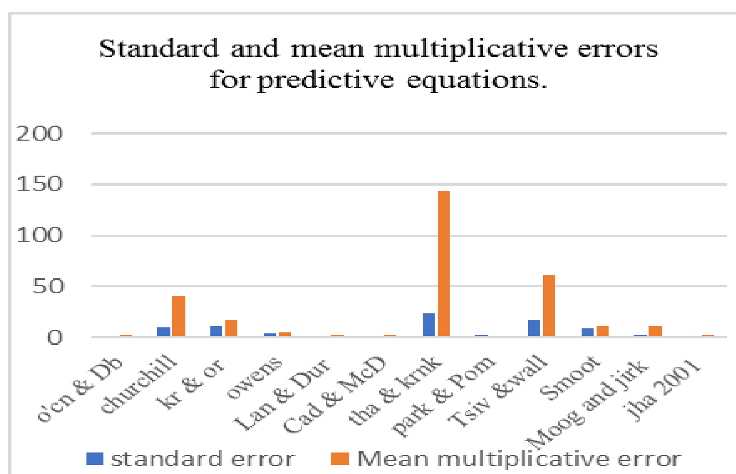


Figure 4. Normal mean error for predictive equation

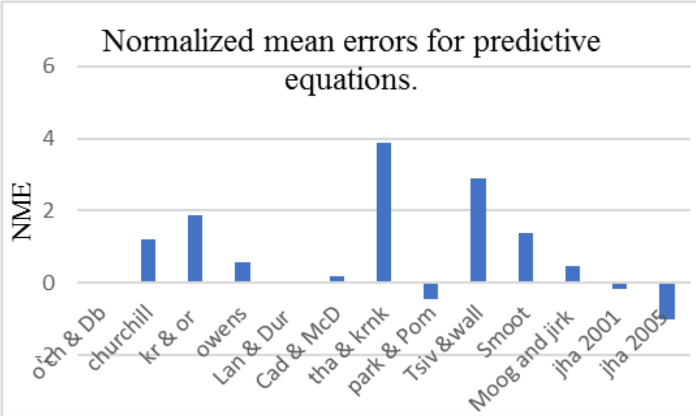


Figure 5. Coefficient of correlation r value for predictive equation

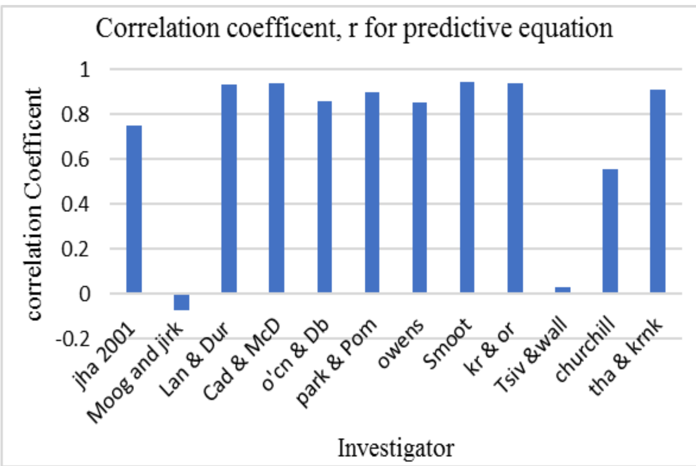
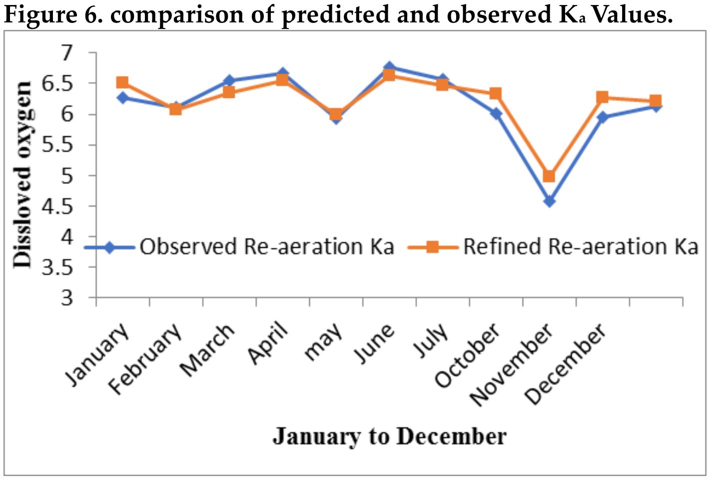


Figure 6. comparison of predicted and observed K_a Values.



It has been seen that Jha et al. 's re-aeration equation revealed the best relationship with values that were measured with respect to NME, SE, and MME (NME= -0.1664, SE= 1.0589, MME= 2.3). Parkhurst and Pomeroy's equation follows [20] (SE= 2.7, MME= 1.732, NME= -0.45). But as regards correlation coefficient, Parkhurst and Pomeroy's equation agreed better (r= 0.9) than that of Jha et al. [23] (r= 0.7). Values for SE, MME, and NME got from equations postulated by Moog, and Jirka [22] (SE= 2.5 NME= 0.46 and MME= 10.7) are closer to those gotten by Jha et al. [23] and Parkhurst and Pomeroy [20] but has a correlation coefficient that is quite low. (r= 0.7). The NME values are very low for the equations derived by O'Connor and Dobbins [7] (SE= 0.74, NME= 0.05 and MME= 2.9) and Langbein and Durum [17] (SE= 0.5, NME= -0.0019 and MME= 2.9) though MME and SE values are a little close to Jha et al. 's equation. Equations proposed by other researchers have error values that partly agree with the values observed. The researchers are as follows; Churchill et al.,[9] Krenkel and Orlob,[15], Tsivoglou and Wallace,[13], Owens et al.,[16] Cadwallader and McDonnell,[18] Smoot and Thackston and Krenkel [19].

Results garnered are assumed to be inclined to bigger values and errors that are squared just as differential errors are given by the SE and NME. The MME value employs the ratio of measured and predicted values and is thought to possess the greatest accuracy as a parameter for error assessment. [22]. The model being used is seen as giving good results if the value of MME is close to unity. In the ongoing work, 2.3, which is the MME value for Jha et al. 's equation [23] is the closest to unity. The values for equations proposed by Langbein and Durum, [17], Owens[16] and Cadwallader and McDonell [18] which are 2.68, 5, and 3.24 respectively follow. The MME values are 2.96 for the equations O'Connor and Dobbins proposed. [14]. Every other equation remaining have MME values of 2.3. It may come to notice that apart from Jha et al. 's equation,[23], every other predictive equation was derived for non-Indian rivers. Seeing as this ongoing study is being carried out on a river in India, it makes sense that the values of the study are in line with Jha et al. 's predictive equation. [23] whose re-aeration dynamics matches that of the characteristic geographical and climatic conditions in India. Using the broad field survey done in this ongoing study as a basis, a better predictive equation has been derived for re-aeration of the Tungabhadra river using the only velocity and flow depth variables. There is an interrelationship between velocity and flow depth and other stream variables like Froude number, slope and friction velocity. Hence, these other stream variables have been removed from the current model being developed. The Newton-Raphson and the least-squares method have been used to derive the equation that follows.

$$Ka=5.710234\, U^{0.361443}\, H^{-0.510369} \tag{9}$$

The error values derived from equation (9) with the values measured in the ongoing work appear to be refined in relation to the equations in table 1. These are the values: SE= 0.34; NME= -0.064; MME= 2.5 and the coefficient of correlation r= 0.95. Figure 6 shows a comparison between observed values of ka from 10 sets of data representing field data and values of ka predicted using the improved predictive equation (9). Encouraging outcomes are derived and bring to fore, an improved equation for prediction with better performance.

6 Conclusion

Using the most widely-known re-aeration constant predictive equations in records have produced error assessment results that show that Jha et al. ’s equation [23] agrees best with ka values been that have been observed. But, Jha et al. ’s equations [24] which were derived using slope as its basis, were discovered to produce values that are extremely high. Values derived from Parkhurst and Pomeroy’s equations [20], Moog and Jirka’s [2] and Langbein and Durum’s equations [17] were also closely in line with values measured.

The predictive re-aeration equation derived for the Tungabhadra river, which is improved shows better outcomes and may be used to replace broader field research and may also be suitable for other rivers with climatic, geographical and hydraulic conditions similar to the Tungabhadra river. Correlation statistics and MME give a better standard of measurement for equations for re-aeration.

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