M/M/1 Queue With Working Vacation, Server Failure And Customer's Impatience

V.N.Rama Devi, A.Ankamma Rao, K.Chandan

Abstract: This paper analyses an M/M/1 queueing model with working vacation, server failure and customer's impatience. Arrival times are considered as Poisson and the service times of both normal and working vacation modes are assumed as exponentially distributed. The customer may balk upon his arrival in two cases of working vacation as well as normal condition and the balking times are assumed to follow exponential distribution. Instant repair facility is assumed for system breakdowns. R-K method is applied using Mat lab software to calculate system performance measures and also tested various parameters influence on these constants through numerical illustrations.

Index terms: Balking, Customer's impatience, Poisson Process , RK method , Server failure, Transient analysis and Working vacation.

1 INTRODUCTION

Queueing models with working vacations have several applications in many fields such as in Educational Institutions, Industrial sectors, agricaltural fields and Transportations etc. Queueing systems with server vacations by using pgf technique was first introduced by Levy and Yechiali. Servi and Finn introduced a class of semi-vacation policies, one which is working vacation (WV), they studied an M/M/1 queue with working vacations. Working vacation queues typically refer to queues where the server utilizes its time on vacation for other purposes. Xu, X and Tian, N (2009) work on Performance analysis of an M/M/1 working vacation queue with setup times. G.Kannadasan and N. Sathiyamoorth anlyze the M/M/1 queue with working vacation in fuzzy environment. Joshuo Patterson and Andrzej korzeniowski works on M/M/1 model with unreliable service and a working vacation. Yue et al (2011) discussed a two-phase M/M/1 queueing system with impatient customers, multiple vacations, customer balking and reneging. Avi-Itzhak, B. and Naor, p. (1963) discussed some queueing problems with the service station subject to breakdowns in 1963. V.N.Rama Devi and Dr.K.Chandan studied the optimal of a two-phase M/M/1 queueing system with server startup, N-Policy, unreliable server and Balking. Queueing systems with impatient customers have been studied by Ancker C.J., Gafarian A., "Some queueing problems with balking and reneging: I" and Rakesh Kumar, Sumeet Kumar Sharma (2012), "An M/M/1/N Queueing Model with Retention of Reneged Customers and Balking". The main objective of this paper is to analyze an M/M/1 queueing model with working vacation, server failure and customer's impatience. Also carried out the numerical results for various parameter effects on the system.

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2 THE SYSTEM AND ASSUMPTIONS

We consider the M/M/1 queueing model with unreliable server, customer's impatience and working vacations in Transient state with the following assumptions:

1. Customers are assumed to arrive according to a Poisson process with mean arrival rate λ where service is on FCFS basis.
2. Customers are given service such that service times are assumed to be exponentially distributed with means $1/\mu$.
3. At the end of a service, if there is no customer in the system, the server starts WV and the vacation times are assumed to be exponentially distributed with parameter φ. During a WV, service is provided according to a Poisson distribution with parameter η. If the server finds a customer at a WV completion instant, it returns to a regular busy period. The server provides first essential service to all existing customers in an individual manner.
4. The server may get breakdowns and assumed to follow a Poisson process, with rates α_1 in the WV service mode and γ_1 in the normal service mode. When the server fails, it is immediately repaired at repair rates α_2 and γ_2 in WV and normal service modes respectively, and these repair times are assumed to follow an exponential distribution. After repair the server immediately resumes the concerned service.
5. Customers are assumed to be annoyed and balk from the system. The probabilities of balking in WV and normal modes are taken as b_0 and b_1.

Notations

We use the following notations to represent transient probabilities for the system to be in various modes

$$p_i^{(1)}(t) = p\begin{pmatrix}\text{i customers in the system when server} \\ \text{is in Working Vacation}\end{pmatrix}; \quad i = 1,2, \dots S$$

$$p_i^{(2)}(t) = p\begin{pmatrix}\text{i customers in the system when server is} \\ \text{in Working Vacation and brokendown}\end{pmatrix}; \quad i = 1,2, \dots, S$$

$$p_i^{(3)}(t) = p\begin{pmatrix}\text{i customers in the system when srver is} \\ \text{doing Normal service}\end{pmatrix}; \quad i = 0,1, \dots . S$$

$$p_i^{(4)}(t) = p\begin{pmatrix}\text{i customers in the system when srver is} \\ \text{doing Normal service and brokendown}\end{pmatrix}; \quad i = 1,2, \dots . S$$

The Transient state equations governing the system size probabilities at any arbitrary point of time are given by the following Differential Equations:

$$\frac{dp_0^{(1)}(t)}{dt} = -(\lambda b_0 + \theta)p_0^{(1)}(t) + \eta p_1^{(1)}(t) + \mu p_2^{(1)}(t); \qquad (1)$$

$$\frac{dp_{0,j}^{(1)}(t)}{dt} = -(\lambda b_0 + \theta + \eta + \alpha_1)p_{0,j}^{(1)}(t) + \lambda b_0 p_{0,j-1}^{(1)}(t) + \eta p_{0,j+1}^{(1)}(t) + \alpha_2\, p_{1,j}^{(1)}(t);\ \ 1 \le j \le S \qquad (2)$$

$$\frac{dp_s^{(1)}(t)}{dt} = -(\theta + \eta + \alpha_1)p_s^{(1)}(t) + \lambda b_0 p_{s-1}^{(1)}(t) + \alpha_2\, p_s^{(1)}(t); \qquad (3)$$

$$\frac{dp_j^{2}(t)}{dt} = -(\lambda b_0 + \alpha_2)p_j^{(2)}(t) + \lambda b_0 p_{j-1}^{(2)}(t) + \alpha_1\, p_j^{(2)}(t)\,; \qquad 1 \le j \le S \qquad (4)$$

$$\frac{dp_s^{(2)}(t)}{dt} = -(\alpha_2)p_s^{(2)}(t) + \lambda b_0 p_s^{(2)}(t) + \alpha_1\, p_s^{(2)}(t)\,; \qquad (5)$$

$$\frac{dp_0^{(3)}(t)}{dt} = -(\lambda b_1)p_0^{(3)}(t) + \theta p_0^{(1)}(t)\,; \qquad (6)$$

$$\frac{dp_j^{3}(t)}{dt} = -(\lambda b_1 + \mu + \gamma_1)p_j^{(3)}(t) + \lambda b_1 p_{j-1}^{(3)}(t) + \theta p_j^{(1)}(t) + \mu p_{j+1}^{(3)}(t) + \gamma_2\, p_j^{(4)}(t);\ 1 \le j \le s-1 \qquad (7)$$

$$\frac{dp_s^{(3)}(t)}{dt} = -(\mu + \gamma_1)p_s^{(3)}(t) + \lambda b_1 p_{s-1}^{(3)}(t) + \theta p_s^{(1)}(t) + \gamma_2\, p_s^{(4)}(t); \qquad (8)$$

$$\frac{dp_j^{(4)}(t)}{dt} = -(\lambda b_1 + \gamma_2)p_j^{(4)}(t) + \lambda b_1 p_{j-1}^{(4)}(t) + \gamma_1\, p_j^{(3)}(t);\ 1 \le j \le S-1 \qquad (9)$$

$$\frac{dp_s^{(4)}(t)}{dt} = -(\lambda b_1 + \gamma_2)p_s^{(4)}(t) + \lambda b_1 p_{s-1}^{(4)}(t) + \gamma_1\, p_s^{(3)}(t) \qquad (10)$$

3 PERFORMANCE MEASURES

Some performance measures are calculated to predict the system behaviour using the probabilities obtained through Runge-Kutta method:

1. P(Server being doing service in WV mode) =WVS(t) = $\sum p_j^{(1)}(t)$
2. P(Server being doing service in Normal mode) =WVS(t) = $\sum p_j^{(3)}(t)$
3. P(Server being broken down at time t)=B(t) =$\sum p_j^{(2)}(t) + \sum p_j^{(4)}(t)$
4. L(t)=Expected length of customers in the system at time (t)= $\sum\sum_{n=0}^{S} n * p_n^{i}(t)$
5. **5.** Waiting time in the system at time t =W(t)= $L(t)\Big/(\lambda * (1 - p_{\max.customers}(t))$

4 NUMERICAL RESULTS

The obtained results are numerically evaluated based on specific parameters. The effect of various parameters on the system performance measures such as the expected number of customers in the system and mean waiting time in the system are studied. MATLAB software is used to develop the computational program. The effect of different parameters in the system of performance measures is summarized in Tables 1-7 in Appendix. In all numerical computations, the model parameters are taken as

N = 4, S=6, **λ** =0.4, $\mu = 0.9$, $\theta = 0.5$, $\eta = 0.6, \alpha_1 = 0.001, \alpha_2 = 0.002, \gamma_1 = 0.002, \gamma_2 = 0.003, b_0 = 0.001, b_1 = 0.002$, T=2 and h=0.5.

5 CONCLUSION

In this model, we have derived performance measures of a Two-phase M/M/1 queue with working vacation, server failure and customer's impatience behavior.

- System length and waiting time are increasing wrt increase in parameters λ, θ, b_0 , b_1.
- System length and waiting time are decreasing wrt increase in parameters μ, η.
- The other parameters $\alpha_2, \gamma_2, \alpha_1, \gamma_1$ have minor effect on System length and waiting time.

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Appendix

Table1: *Effect of λ on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$\lambda = 0.4$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\lambda = 0.41$	I(t)	1	0.778815	0.606586	0.472504	0.368142	0.28693
	W(t)	0	0.221169	0.393379	0.527469	0.631879	0.713185
	B(t)	0	5.49E-08	2.33E-07	5.51E-07	1.02E-06	1.64E-06
	L(t)	0	0.000182	0.00034	0.00049	0.000637	0.000781
	W(t)	0	0.000444	0.000829	0.001195	0.001554	0.001906
$\lambda = 0.42$	I(t)	1	0.778815	0.606587	0.472507	0.368148	0.28694
	W(t)	0	0.221168	0.393377	0.527465	0.631873	0.713178
	B(t)	0	5.62E-08	2.38E-07	5.64E-07	1.05E-06	1.68E-06
	L(t)	0	0.000186	0.000348	0.000502	0.000652	0.0008
	W(t)	0	0.000446	0.000829	0.001196	0.001555	0.001906
$\lambda = 0.43$	I(t)	1	0.778815	0.606588	0.47251	0.368154	0.28695
	W(t)	0	0.221168	0.393376	0.527461	0.631868	0.713171
	B(t)	0	5.75E-08	2.44E-07	5.78E-07	1.07E-06	1.72E-06
	L(t)	0	0.000191	0.000356	0.000513	0.000668	0.000819
	W(t)	0	0.000447	0.000829	0.001197	0.001556	0.001908

Table2: *Effect of μ on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$\mu = 0.9$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\mu = 0.91$	I(t)	1	0.778815	0.606585	0.472502	0.368138	0.286922
	W(t)	0	0.221169	0.393381	0.527471	0.631882	0.713189
	B(t)	0	5.35E-08	2.27E-07	5.36E-07	9.92E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000476	0.000619	0.000757
	W(t)	0	0.000443	0.000827	0.001191	0.001546	0.001894
$\mu = 0.92$	I(t)	1	0.778815	0.606586	0.472503	0.368139	0.286924
	W(t)	0	0.221169	0.39338	0.52747	0.63188	0.713185
	B(t)	0	5.35E-08	2.27E-07	5.35E-07	9.89E-07	1.59E-06
	L(t)	0	0.000177	0.00033	0.000475	0.000616	0.000753
	W(t)	0	0.000443	0.000826	0.001187	0.001539	0.001882
$\mu = 0.93$	I(t)	1	0.778815	0.606586	0.472504	0.368141	0.286926
	W(t)	0	0.221169	0.39338	0.527469	0.631877	0.713181
	B(t)	0	5.35E-08	2.26E-07	5.34E-07	9.86E-07	1.58E-06
	L(t)	0	0.000177	0.00033	0.000474	0.000613	0.000748
	W(t)	0	0.000443	0.000825	0.001184	0.001532	0.001871

Table3: *Effect of θ on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$\theta = 0.5$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192

	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\theta = 0.51$	I(t)	1	0.774931	0.600552	0.465472	0.360857	0.279853
	W(t)	0	0.225053	0.399415	0.534504	0.63917	0.720271
	B(t)	0	5.37E-08	2.29E-07	5.42E-07	1.01E-06	1.62E-06
	L(t)	0	0.000178	0.000332	0.000481	0.000627	0.000771
	W(t)	0	0.000444	0.000831	0.001202	0.001568	0.001928
$\theta = 0.52$	I(t)	1	0.771067	0.594579	0.458547	0.353721	0.272961
	W(t)	0	0.228916	0.405388	0.54143	0.646311	0.727174
	B(t)	0	5.39E-08	2.30E-07	5.47E-07	1.02E-06	1.64E-06
	L(t)	0	0.000178	0.000334	0.000484	0.000633	0.00078
	W(t)	0	0.000445	0.000834	0.00121	0.001582	0.00195
$\theta = 0.53$	I(t)	1	0.767223	0.588665	0.451726	0.346728	0.266239
	W(t)	0	0.232761	0.411302	0.548254	0.653311	0.733908
	B(t)	0	5.41E-08	2.31E-07	5.52E-07	1.03E-06	1.66E-06
	L(t)	0	0.000178	0.000335	0.000487	0.000639	0.000789
	W(t)	0	0.000445	0.000837	0.001218	0.001597	0.001973

Table4: *Effect of η on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$\eta = 0.6$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\eta = 0.61$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527473	0.631885	0.713194
	B(t)	0	5.35E-08	2.27E-07	5.37E-07	9.94E-07	1.60E-06
	L(t)	0	0.000177	0.00033	0.000476	0.00062	0.000761
	W(t)	0	0.000443	0.000826	0.001191	0.00155	0.001902
$\eta = 0.62$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393382	0.527473	0.631886	0.713195
	B(t)	0	5.34E-08	2.26E-07	5.36E-07	9.92E-07	1.60E-06
	L(t)	0	0.000177	0.00033	0.000475	0.000619	0.000759
	W(t)	0	0.000442	0.000824	0.001188	0.001547	0.001899
$\eta = 0.63$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393382	0.527474	0.631887	0.713196
	B(t)	0	5.34E-08	2.26E-07	5.35E-07	9.91E-07	1.60E-06
	L(t)	0	0.000176	0.000329	0.000474	0.000617	0.000758
	W(t)	0	0.000441	0.000822	0.001185	0.001543	0.001895

Table5: *Effect of α_1 on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$\alpha_1 = 0.001$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\alpha_1 = 0.0011$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.74E-08	2.39E-07	5.59E-07	1.03E-06	1.64E-06

	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001906
$\alpha_1 = 0.0012$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	6.13E-08	2.51E-07	5.80E-07	1.06E-06	1.68E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001906
$\alpha_1 = 0.0013$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	6.51E-08	2.63E-07	6.02E-07	1.09E-06	1.72E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001906

Table6: *Effect of α_2 on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$\alpha_2 = 0.002$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\alpha_2 = 0.0021$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\alpha_2 = 0.0022$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\alpha_2 = 0.0023$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905

Table7: *Effect of γ_1 on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$\gamma_1 = 0.002$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\gamma_1 = 0.0021$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.43E-08	2.32E-07	5.54E-07	1.03E-06	1.67E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001906
$\gamma_1 = 0.0022$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.50E-08	2.38E-07	5.70E-07	1.06E-06	1.73E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001906
$\gamma_1 = 0.0023$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.57E-08	2.43E-07	5.86E-07	1.10E-06	1.79E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762

	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001906

Table8: *Effect of γ_2 on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$\gamma_2 = 0.003$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\gamma_2 = 0.0031$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\gamma_2 = 0.0032$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$\gamma_2 = 0.0033$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905

Table9: *Effect of b_0 on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$b_0 = 0.001$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192
	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$b_0 = 0.0011$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.221168	0.393375	0.527462	0.631869	0.713173
	B(t)	0	5.74E-08	2.39E-07	5.59E-07	1.03E-06	1.64E-06
	L(t)	0	0.000191	0.00035	0.000496	0.000638	0.000777
	W(t)	0	0.000477	0.000874	0.001241	0.001596	0.001943
$b_0 = 0.0012$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.221166	0.393369	0.527451	0.631854	0.713154
	B(t)	0	6.13E-08	2.51E-07	5.80E-07	1.06E-06	1.68E-06
	L(t)	0	0.000204	0.000368	0.000515	0.000656	0.000792
	W(t)	0	0.000511	0.000919	0.001288	0.001639	0.00198
$b_0 = 0.0013$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.221164	0.393363	0.52744	0.631839	0.713135
	B(t)	0	6.51E-08	2.63E-07	6.02E-07	1.09E-06	1.72E-06
	L(t)	0	0.000218	0.000386	0.000534	0.000673	0.000807
	W(t)	0	0.000544	0.000965	0.001334	0.001682	0.002017

Table10: *Effect of b_1 on system performance measures under values of 't'*

	Parameters/Time	0	1	2	3	4	5
$b_1 = 0.002$	I(t)	1	0.778815	0.606585	0.472501	0.368136	0.28692
	W(t)	0	0.22117	0.393381	0.527472	0.631884	0.713192

	B(t)	0	5.35E-08	2.27E-07	5.38E-07	9.95E-07	1.60E-06
	L(t)	0	0.000177	0.000331	0.000478	0.000621	0.000762
	W(t)	0	0.000444	0.000828	0.001194	0.001553	0.001905
$b_1 = 0.0021$	I(t)	1	0.778815	0.606587	0.472507	0.368148	0.28694
	W(t)	0	0.221169	0.39338	0.52747	0.631881	0.713188
	B(t)	0	5.43E-08	2.32E-07	5.54E-07	1.03E-06	1.67E-06
	L(t)	0	0.00018	0.000339	0.000492	0.000644	0.000793
	W(t)	0	0.000449	0.000847	0.001231	0.00161	0.001982
$b_1 = 0.0022$	I(t)	1	0.778815	0.606589	0.472513	0.36816	0.28696
	W(t)	0	0.221169	0.39338	0.527468	0.631877	0.713183
	B(t)	0	5.50E-08	2.38E-07	5.70E-07	1.06E-06	1.73E-06
	L(t)	0	0.000182	0.000346	0.000507	0.000666	0.000824
	W(t)	0	0.000454	0.000866	0.001267	0.001666	0.002059
$b_1 = 0.0023$	I(t)	1	0.778816	0.606591	0.472519	0.368172	0.28698
	W(t)	0	0.221169	0.393379	0.527466	0.631874	0.713178
	B(t)	0	5.57E-08	2.43E-07	5.86E-07	1.10E-06	1.79E-06
	L(t)	0	0.000184	0.000354	0.000521	0.000689	0.000854
	W(t)	0	0.00046	0.000884	0.001303	0.001722	0.002136