Negative Binomial - Geeta Distributionand Some Of Its Property

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Abstract: In this paper, we derived a probability mass function of new discrete probability distribution named as negative binomial - Geeta distribution and it is obtained by usingLagrange expansion of second kind. We studied some important characteristics of this distribution such as convolution property, probability generating function, etc. Further, it is shown that the proposed distribution is in the form of modified power series distribution(MPSD). Maximum likelihood method is used to estimate the parameters of the distribution.

Keywords: Negative Binomial Distribution, Geeta Distribution, Second Lagrange Expansion, Recurrence Relations, Convolution Property and Maximum Likelihood Estimation.

1 INTRODUCTION

In statistics, the discrete probability distributions have been used as models throughout the history of statistics. Johnson, Kotz, and Kemp [13]discussed on univariate discrete distributions. Consul and Shenton [2], [3]defined and studied a new class of Lagrange probability distribution with help of Lagrange expansion. There is two kinds of Lagrange expansion namely first kind and second kind with different prospects. In this paper, we have used the second kind of Lagrange expansion. Janardan and Rao [12], Janardan [11] and Consul and Famoye [7]intensely studied about the class of Lagrange probability distributions on the second kind. Every member of the class of the Lagrange probability distributions of the second kind is a member of the class of Lagrange distribution of the first kind. Gupta [10] proposed the modified power series distribution (MPSD). Consul [4],[5] derived Geeta distribution and its properties along with two stochastic models for the Geeta distribution. Consul and Famoye [8] used second kind Lagrange expansion introduced Dev probability distribution and some of its applications in queuing theory. Consul and Famoye [9] developed a new class of discrete probability distribution named Harish probability distribution and also derived some of the properties of the Harish probability distribution with applications in the branching process and queuing theory.Arbous A.G and H.S. Sichel [1] introduced new techniques for the analysis of absenteeism data.Li et al. [17], [18]defined the generalized Lagrangian(GL) distribution class, in which an extra parameter was brought into the probability mass function of GL and some mixture distributions based on Lagrangian probability models. Lindskog et al. [19] derived common Poisson shock models and applications for insurance and credit risk models. In this paper, we derived a new discrete probability distribution by using Lagrange expansion of second kindnamed as Negative binomial and Geeta distribution. Recurrence relations between the moments are provided. Convolution property studies to get probability generating function. The proposed probability model is in the form of Modified power series distribution. Maximum likelihood method is used to estimate the parameters of negative binomial – Geeta distribution.

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2 LAGRANGE EXPANSION OF SECOND KIND

Let $f(z)$ and $g(z)$ be two analytic functions of z, which are successively differentiablefunctions in [0,1] such that $g(0) \neq 0$ and $f(1) = g(1) = 1$. The Lagrange expansion of second kind given by Janardan and Rao [13] is

$$\frac{(1-g'(1))\,f(z)}{1-zg'(z)/g(z)} = \sum_{y=0}^{\infty} \frac{u^y (1-g'(1))}{y!} D^y [(g(z)^y f(z)]\Big|_{z=0}$$

(1)where

$$D = \frac{\partial}{\partial z},\quad f'(z) = \frac{\partial f(z)}{\partial z} \text{ and } g'(1) = \frac{\partial g(z)}{\partial z}\Big|_{z=1}$$

When $u = z = 1$ then the left hand side of equation (1) becomes unity. In equation(1), every term of the series is non-negative hence this series becomes a probability generating function in u and provides the probability mass function of the second Lagrange expansion L_2D .The probability mass function of L_2D is,

$$P(X = x) = \frac{1-g'(1)}{x!} D^x [g^x(z) f(z)]\Big|_{z=0} \quad x = 0,1,\ldots \quad (2)$$

Let $g(z) = q^k (1-pk)^{-k}$ is a pgf of negative binomialdistribution and $f(z) = f_1(z) \times f_2(z)$ where $f_1(z)$ is pgf of negative binomial distribution and $f_2(z)$ is pgf of Geeta distribution then we get

$$f(z) = q^m (1-pz)^{-m} \left(\frac{1-pz}{1-p}\right)^{-k+1}.$$

Under the transformation $u = \frac{z}{g(z)}$ on simplification using fact that $u = z = 1$, we obtain

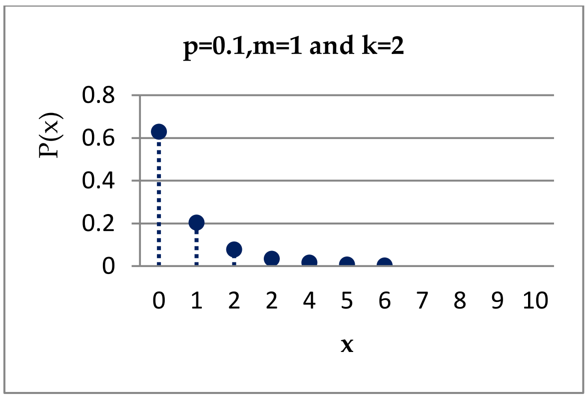
$$1 = \sum_{x=0}^{\infty} \left(1 - kp(1-p)^{-1}\right)(1-p)^{kx+m+k-1} p^x \left\{ \sum_{r=0}^{x} \binom{k+x-r-2}{x-r} \binom{m+kx+r-1}{r} \right\} \quad (3)$$

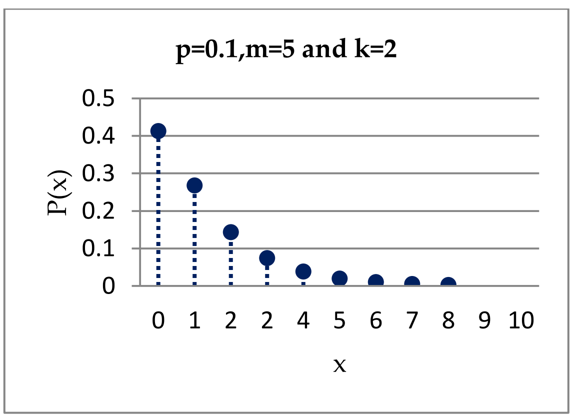
Since every term in the summation on the right hand side is positive and the sum of all the terms is unity, it provides a genuine discrete probability named as Negative Binomial-Geeta (NB-G) distribution and it is given by,

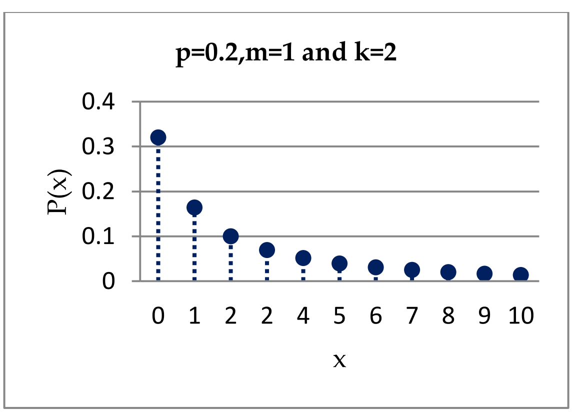
$$P(X=x) = \left(1-kp(1-p)^{-1}\right)(1-p)^{kx+m+k-1} p^{x}\left\{\sum_{r=0}^{x}\binom{k+x-r-2}{x-r}\binom{m+kx+r-1}{r}\right\} \quad (4)$$

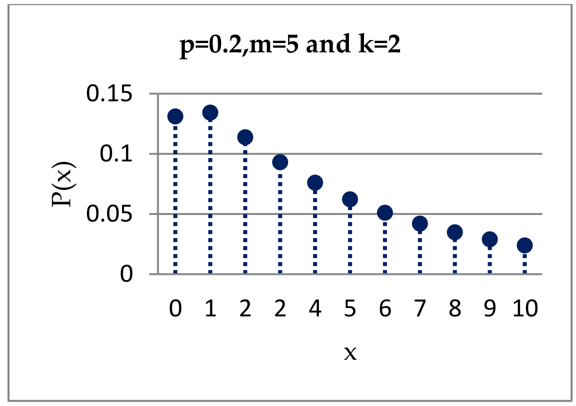
$x = 0,1,2,3,\ldots\ ;\ 0 < p < \frac{1}{3}\ ,\ k = 2\ ,\ m \geq 0$.

2.1 Behaviour of Negative Binomial-Geeta distribution









3NB-G DISTRIBUTION IN THE FORM OF MODIFIED POWER SERIES DISTRIBUTION

Modified power series distribution is given by Gupta [10],

$$P(X=x) = a_x \frac{[\varphi(\theta)]^x}{h(\theta)}\ ,\ x \in T \subset N\ ,$$

Where N is the set of non-negative integers and T is a subset of N . All MPSD are linear exponential and form a sub-class of the Lagrangian distribution.Let take,

$g(p) = (1-p)^{-k}$ and $f(p) = (1-p)^{-m-k+1}$

Under the transformation $p = u\,g(p)$

The second Lagrange expansion is equation (1), we recall the equation,

$$\frac{(1-g'(1))\,f(z)}{1-zg'(z)/g(z)} = \sum_{x=0}^{\infty}\frac{u^x(1-g'(1))}{x!}D^x[(g(z)^x f(z)]\Big|_{z=0}$$

$$u = p(1-p)^k$$

we get

$$\frac{(1-p)^{-k-m+1}}{\left(1-kp(1-p)^{-1}\right)} = \sum_{x=0}^{\infty}\left[p(1-p)^k\right]^x \sum_{r=0}^{x}\binom{k+x-r-2}{x-r}\binom{m+kx+r-1}{r}$$

we get,

$$a_x = \sum_{r=0}^{x}\binom{k+x-r-2}{x-r}\binom{m+kx+r-1}{r}$$

and

$$h(\theta) = h(p) = (1-p)^{-m}(1-p)^{-k+1}\left[1-pk(1-p)^{-1}\right]^{-1}$$

$$h'(\theta) = h(p) = (-m-k+1)(1-p)^{-m-k+1-1} \times (-1)\left[1-pk(1-p)^{-1}\right]^{-1} + (1-p)^{-m-k+1}(-1)\left[1-pk(1-p)^{-1}\right]^{-1-1} \times \left[-k(1-p)^{-1}-kp(1-p)^{-2}\right]$$

$$\phi(\theta) = \phi(p) = p(1-p)^k$$

$$\phi'(\theta) = \phi'(p) = (1-p)^k + p(1-p)^{k-1}(-1)$$

Mean

$$E(X) = \frac{\phi(p)}{\phi'(p)}\frac{h'(p)}{h(p)}$$

$$E(X) = \frac{p(1-p)^k}{(1-p)^k - p(1-p)^{k-1}}\left[\frac{m+k-1}{1-p} + \frac{-k(1-p)^{-1}-kp(1-p)^{-2}}{1-pk(1-p)^{-1}}\right] \quad (5)$$

Variance

$$\sigma^2 = \mu_2 = \frac{\phi(p)}{\phi'(p)}\frac{d\mu}{d(p)}$$

$$\mu_{r+1} = \frac{\phi(p)}{\phi'(p)}\frac{d\mu_r}{d(p)} + r\mu_2\mu_{r-1}, r = 2,3,...$$

Where $\mu_0 = 1,\ \mu_1 = 0$ and $\phi'(p), h'(p)$ denote the derivatives of $\phi(p)$ and $h(p)$ respectively with respect to p.

Here we using Mathematica software then we get,

$$\sigma^2 = \mu_2 = \frac{(1-p)^k p[(m-1)(p-1)+k(-2+(m-1)p)]}{(kp+p-1)^3\left((1-p)^k - k(1-p)^{-1+k}p\right)} \quad (6)$$

Since $0 < p < \frac{1}{3}$ and $\sigma^2 - \mu > 0$ the variance σ^2 is larger than the mean μ for all values of the parameters m, k and p.

4 RECURRENCE RELATIONOF CENTRAL MOMENTS

Let the j-th moment about the origin be denoted by μ'_j so that $\mu'_0 = 1$

$$\mu'_j = \sum_{x=0}^{\infty} x^j P(X = x)$$

$$= \sum_{x=0}^{\infty} x^j \left(1-kp(1-p)^{-1}\right)(1-p)^{m+k-1}\left(p(1-p)^k\right)^x \times \sum_{r=0}^{x}\binom{k+x-r-2}{x-r}\binom{m+kx+r-1}{r}$$

On differentiation both side with respect to pand simplifying the resulting expressions, we get the recurrence relation between the moments about the origin as

$$\mu'_{j+1} = \frac{p(1-p)^k}{(1-p)^k - pk(1-p)^{k-1}}\frac{d\mu'_j}{dp} + \frac{p(1-p)^k}{(1-p)^k - pk(1-p)^{k-1}}\left[\frac{m+k-1}{1-p} + \frac{k(1-p)^{-1}+kp(1-p)^{-2}}{1-kp(1-p)^{-1}}\right]\mu'_j$$

$$\mu'_{j+1} = \frac{\phi(p)}{\phi'(p)}\frac{d\mu'_j}{dp} + \mu'_1\mu'_j$$

5 CONVOLUTION PROPERTY

Theorem: Let $X_i,\ i = 1,2,3,...,n$ be n independent and identically distributed random variables having the Negative binomial-Geeta distribution (NB-G) in (3) with parameter (m,k,p). The distribution of $Y = \sum_{i=1}^{n} X_i$ is negative binomial-Geeta distribution with parameters (nm, nk, p).

Proof: since the pgf of the NB-G is given by

$$E\left[u^X\right] = H(u) = q^m(1-pz)^{-m}\left(\frac{1-pz}{1-p}\right)^{-k+1} \quad (7)$$

Where $z = q^k(1-pk)^{-k}$

Thepgf of the random variable Y becomes

$$E\left[u^Y\right] = E\left[u^{X_1+X_2+X_3+...+X_n}\right] = \prod_{i=1}^{n}\left[u^{X_i}\right]$$

$$= \prod_{i=1}^{n} H(u) = [H(u)]^n$$

$$E\left[u^Y\right] = q^{nm}(1-pz)^{-nm}\left(\frac{1-pz}{1-p}\right)^{-n(k-1)}$$

Where $z = q^k(1-pk)^{-k}$

Since the above pgf of the same form asgiven in (7), with a difference that m and $k-1$ are replaced by nm and $n(k-1)$ respectively, the random variable Y represents a NB-G distribution given by

$$P(Y=y) = \left(1-nkp(1-p)^{-1}\right)(1-p)^{nm+nk-1}\left(p(1-p)^{nk}\right)^y \times \left\{\sum_{r=0}^{y}\binom{nk+y-r-2}{y-r}\binom{nm+nky+r-1}{r}\right\}$$

$y = 0,1,2,3,...$; $0 < p < \frac{1}{3}$, $nk = 2$, $nm \geq 0$.

6 PARAMETER ESTIMATION

Maximum Likelihood Estimation

The log-likelihood function for the negative binomial-Geeta probability model can be written in the form

$$\ln L = n\ln\left(1 - kp(1-p)^{-1}\right) + n(m+k-1)\ln(1-p) + k\sum_{i=1}^{n} x_i \ln(1-p) + \sum_{i=1}^{n} x_i \ln p + \sum_{i=1}^{n}\ln\left[\sum_{r=0}^{x_i}\binom{k+x_i-r-2}{x_i-r}\binom{m+kx_i+r-1}{r}\right] \quad (8)$$

On differentiating the above with respect to p and m . Here we know parameter $k = 2$, we have the maximum likelihood (ML) equation as,

$$\frac{\partial \log L}{\partial p} = 0, \quad \frac{\partial \log L}{\partial m} = 0$$

$$\frac{\partial \log L}{\partial p} = \frac{n\left(-k(1-p)^{-1} - (kp)(1-p)^{-2}\right)}{\left(1 - kp(1-p)^{-1}\right)} - \frac{n(m+k-1-k\bar{x})}{1-p} + \frac{n\bar{x}}{p}$$

$$\frac{\partial \log L}{\partial m} = n\log(1-p) + \frac{\partial}{\partial m}\left\{\sum_{i=1}^{n}\ln\left[\sum_{r=0}^{x_i}\binom{k+x_i-r-2}{x_i-r}\binom{m+kx_i+r-1}{r}\right]\right\}$$
$$- n\log(1-p) + \sum_{i=1}^{n}\sum_{rr=}^{x_i}\frac{1}{\binom{k+x_i-r-2}{x_i-r}\binom{m+kx_i+r-1}{r}}\binom{k+x_i-r-2}{x_i-r}$$
$$\times \frac{1}{\Gamma(r+1)}\left[\frac{\Gamma(m+kx_i+r)'\,\Gamma(m+kx_i) - \Gamma(m+kx_i)'\,\Gamma(m+kx_i+r)}{(\Gamma(m+kx_i))^2}\right]$$

There is no closed form solution form ML estimates of p and m .So that, the solution for parameters may be solved numerically by using Newton Raphson method.

7 CONCLUSION

In this paper, we introduced a new discrete probability distribution namely Negative binomial and Geeta distribution by using Lagrange expansion of second kind. Recurrence relations between the moments are provided. Further convolution propertyis worked out to get probability generating function. Modified power series distribution is shown in this model. Maximum Likelihood Method is used to estimate the parameters of the distribution

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